Problem Set - Maximum likelihood

1. Let $X_1, X_2, \ldots, X_N$ be independent random variables, all with a binomial distribution.

   (a) Find the maximum likelihood estimator for p.

   (b) Is the maximum likelihood estimator the minimum variance unbiased estimator?

   (c) Let $N = 100$, $\sum_{i=1}^{N} x_i = 40$, and $\sum_{i=1}^{N} x_i^2 = 48$. Calculate the Maximum likelihood estimator (MLE).

2. Let $X_1, X_2, \ldots, X_N$ represent a random sample from a Poisson distribution with parameter λ.

   (a) Find the maximum likelihood estimator for λ and its asymptotic distribution.

   (b) Suppose we are interested in the probability of a count of zero, $\theta = \Pr[k = 0] = \exp(-\lambda)$. Find the maximum likelihood estimator for θ and its asymptotic distribution.

3. Wooldridge exercise 15.3.

4. Suppose that $X_1, X_2, \ldots, X_N$ are IID draws from a normal distribution with mean μ and variance σ^2 .

   (a) What is the MLE of μ? Is it unbiased? Is it consistent?

   (b) What is the MLE of σ^2? Is it unbiased? Is it consistent?

   (c) Let $\hat{\sigma}^2$ be the maximum likelihood estimator of σ^2. Also, consider the alternative estimator $\bar{\sigma}^2 = \frac{1}{N-1} \sum_{i=1}^{N} (x_i - \bar{x})^2$, where $\bar{x}$ is the sample average of the x_i. Suppose that the true mean and variance are $\mu = 1$ and $\sigma^2 = 2$. Using Stata, randomly draw 5 values from $N(1, 2)$, and calculate $\hat{\sigma}^2$ and $\bar{\sigma}^2$ in this sample. Repeat 1000 times, and save the estimates in two Stata variables. Calculate the mean and variance of the estimates, and the approximate mean squared error (i.e. the mean squared difference between the estimate and the true value). Which estimator appears to be "better"?