$\boxed{9\text{-}01}$

$$\max_{u\in\mathbb{R}} \int_0^2 (2x - 3u - \alpha u^2)\,dt \qquad \dot{x} = x + u \qquad x(0) = 5 \qquad x(2) \text{ free}$$

$\alpha > 0$

$$H(t,x,u,p) = 2x - 3u - \alpha u^2 + p(x+u)$$

$$H'_x = 2 + p \Rightarrow \underline{\dot{p} = -2 - p}$$

Thus $\dot{p} + p = -2 \qquad \frac{d}{dp}(pe^t) = -2e^t \qquad pe^t = -2e^t + C \qquad \underline{p = Ce^{-t} - 2}$

$$p(2) = Ce^{-2} - 2 = 0 \qquad \underline{C = 2e^2} \Rightarrow \boxed{p(t) = 2(e^{2-t} - 1)} \qquad \underline{p \geq 0 \ \forall t}$$

$$H'_u = -3 - 2\alpha u + p = 0 \Rightarrow u = \frac{p-3}{2\alpha} \qquad u(t) = \frac{e^{2-t} - 1 - \frac{3}{2}}{\alpha} \qquad \boxed{u(t) = \frac{e^{2-t} - \frac{5}{2}}{\alpha}}$$

$$\dot{x} = x + \frac{e^{2-t} - \frac{5}{2}}{\alpha} \qquad \dot{x} - x = \frac{e^{2-t} - \frac{5}{2}}{\alpha}$$

$$\frac{d}{dt}(xe^{-t}) = \frac{e^{2-2t} - \frac{5}{2}e^{-t}}{\alpha} \qquad xe^{-t} = \frac{\frac{e^{2-2t}}{-2} + \frac{5}{2}e^{-t}}{\alpha} + C \qquad \underline{x = \frac{5}{2\alpha} + Ce^{t} - \frac{1}{2\alpha}e^{2-t}}$$

$$x(0) = \frac{5}{2\alpha} + C - \frac{1}{2\alpha}e^2 = 5 \qquad \underline{C = 5 + \frac{e^2}{2\alpha} - \frac{5}{2\alpha} = \frac{e^2 + 10\alpha - 5}{2\alpha}}$$

$$\boxed{x(t) = \frac{5}{2\alpha} + \frac{e^2 + 10\alpha - 5}{2\alpha}e^t - \frac{1}{2\alpha}e^{2-t}}$$

Thus, $\underline{(x(t), u(t)) = \left(\frac{5}{2\alpha} + \frac{e^2 + 10\alpha - 5}{2\alpha}e^t - \frac{1}{2\alpha}e^{2-t},\ \frac{e^{2-t} - \frac{5}{2}}{\alpha}\right) \text{ and } p(t) = 2(e^{2-t} - 1)}$