SEMINAR 6 - ECON 4140

Problem 4-05

$$\int_0^1\left(\int_0^1 x e^y \, dy\right)dx = \int_0^1\left[\Big|_0^1 x e^y\right]dx = \int_0^1 x(e-1)\,dx = \Big|_0^1 \tfrac{1}{2}x^2(e-1) = \underline{\underline{\tfrac{1}{2}(e-1)}}$$

Problem 4-06

$$\int_0^1\int_0^4 (\sqrt{xy} + 2x + y)\,dy\,dx = \int_0^1\left[\Big|_0^4 \sqrt{x}\,\tfrac{2}{3}y^{\frac{3}{2}} + 2xy + \tfrac{1}{2}y^2\right]dx = \int_0^1 \tfrac{16}{3}\sqrt{x} + 8x + 8\,dx = \Big|_0^1 \tfrac{32}{9}x^{\frac{3}{2}} + 4x^2 + 8x = \underline{\underline{12 + \tfrac{32}{9}}}$$

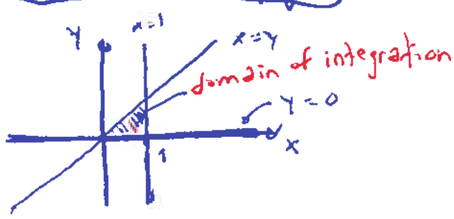
Problem 4-07

$$\int_0^1\int_0^1 x\sqrt{x^2+y}\,dx\,dy \qquad A = \int_0^1 x\sqrt{x^2+y}\,dx \qquad u = x^2+y \quad \frac{du}{dx} = 2x \quad dx = \frac{du}{2x}$$

$$\Rightarrow A = \int_y^{1+y} \tfrac{1}{2}\sqrt{u}\,du = \Big|_y^{1+y} \tfrac{1}{2}u^{\frac{3}{2}}\,\tfrac{2}{3} = \tfrac{1}{3}\left[(1+y)^{\frac{3}{2}} - y^{\frac{3}{2}}\right]$$

$$\int_0^1 A\,dy = \int_0^1 \tfrac{1}{3}\left[(1+y)^{\frac{3}{2}} - y^{\frac{3}{2}}\right]dy = \Big|_0^1 \tfrac{2}{15}(1+y)^{\frac{5}{2}} - \tfrac{2}{15}y^{\frac{5}{2}} = \tfrac{2}{15}2^{\frac{5}{2}} - \underbrace{\tfrac{2}{15}}_{-\frac{2}{15}} - \tfrac{2}{15} = \underline{\underline{\tfrac{2}{15}(\sqrt{32}-2)}}$$

Problem on webpage



$$\Rightarrow \int_0^1\left(\int_0^x x e^y\,dy\right)dx = \int_0^1\left[\Big|_0^x x e^y\right]dx = \int_0^1 xe^x - x\,dx$$

$$= \Big|_0^1 xe^x - \int_0^1 e^x dx \underbrace{-\int_0^1 \tfrac{1}{2}x^2}_{} = e - \Big|_0^1 e^x \underbrace{}_{-\frac{1}{2}} = e - e + 1 \underbrace{}_{-\frac{1}{2}} = \underline{\underline{\tfrac{1}{2}}}$$

~~Opposite~~ $\cancel{\int_0^1\left(\int_0^1 x e^y\,dx\right)dy = \int_0^1\left[\Big|_0^1 \tfrac{1}{2}x^2 e^y\right]dy = \int_0^1 \tfrac{1}{2}y^2 e^y\,dy = \Big|_0^1 \tfrac{1}{2}y^2 e^y - \int_0^1 y e^y\,dy}$

$\cancel{= \tfrac{1}{2}e - \left[\Big|_0^1 y e^y - \int_0^1 e^y\,dy\right] = \tfrac{1}{2}e - e + \Big|_0^1 e^y = -\tfrac{1}{2}e + e - 1 = \tfrac{1}{2}e - 1}$

~~Order of integration relevant!~~

Opposite: We then get $\int_0^1\left[\int_0^1 xe^y\,dx - \int_0^y xe^y\,dx\right]dy = \int_0^1\left(\int_y^1 xe^y\,dx\right)dy$ Order of integration not relevant!

$$= \int_0^1\left[\Big|_y^1 \tfrac{1}{2}x^2 e^y\right]dy = \int_0^1 \tfrac{1}{2}e^y - \tfrac{1}{2}y^2 e^y\,dy = \tfrac{1}{2}(e-1) - \tfrac{1}{2}\left[\Big|_0^1 y^2 e^y - \int_0^1 2ye^y\,dy\right]$$

$$= \tfrac{1}{2}(e-1) - \tfrac{1}{2}\left[e - 2\left(\Big|_0^1 ye^y - \int_0^1 e^y\,dy\right)\right] = -\tfrac{1}{2} + e - \Big|_0^1 e^y = \underline{\underline{\tfrac{1}{2}}}$$

Same number from both orders of integration

$\boxed{4-03}$

$$g(x) = \int_{\pi}^{2\pi} \frac{\sin(xt)}{t}\, dt$$

$$g'(x) = \int_{\pi}^{2\pi} \frac{\cos(xt)\cdot t}{t}\, dt = \int_{\pi}^{2\pi} \cos(xt)\, dt$$

$$g'(1) = \int_{\pi}^{2\pi} \cos(t)\, dt = \Big|_{\pi}^{2\pi} \sin t = \sin 2\pi - \sin \pi = 0 - 0 = \underline{\underline{0}}$$