K.S, November 06

Exam, ECON4140/ECON4145 Mathematics 3, 7/12-04

Problem 1

See model answers to Problem 5, Seminar 5.

Problem 2

(a) $g(x, y)$ is concave for $p \in [-1, 0]$, $q \in [-1, 0]$. (Use the Hessian. It is useful to consider some special values of p and q to confirm the result. For example $p = q = -1$ and $p = q = -2$.)

(b) $f(x, y)$ is concave iff $x \geq 1/4$.

(c) It follows from (b) that the Lagrangian is concave in (x, y) for $x \geq 1/4$.

(d) $(x, y) = (3/2, -1/4)$ with $\lambda_1 = \lambda_2 = 0$ solves the problem.

Problem 3

(a) $u^*(t) = \dfrac{5(4e^t + e^{-3t/2})}{4e^T - e^{-3T/2}}$, $x^*(t) = \dfrac{10(e^t - e^{-3t/2})}{e^T - e^{-3T/2}}$. (Hint: You derive $\dot{p} = -p$, so $p = Ae^{-t}$, and $\dot{x}^* - x^* = 4Ae^{-3t/2}$.)

(b) $u^*(t) = \frac{1}{2}x_0 e^t$, $x^*(t) = x_0 e^t$. (Here $p(t) = 0$ for all t.)

Problem 4

Rather hard. (Hint: Prove that $\mathbf{Ax} = \mathbf{Bx}$ for all 2×2-vectors $\mathbf{x}$.)

mat3x04f 24.11.2006 644