Double integrals; brief preview

Fact: For all "sufficiently nice" (i.e., Math 3-relevant) functions $g(x,y)$, the expressions

$$\int_a^b \underbrace{\left[\int_c^d g(x,y)\,dy\right]}_{\text{a function } u(x)} dx \qquad \text{and}$$

$$\int_c^d \underbrace{\left[\int_a^b g(x,y)\,dx\right]}_{\text{a function } v(y)} dy \qquad \text{are } \underline{\text{equal}},$$

But sometimes, one is easier to compute than the other.

Ex: $$\int_{-1}^{1} \underbrace{\int_3^4 y e^y \sin(xy)\,dy}_{\text{hard to compute}}\,dx$$

$$= \int_3^4 \underbrace{\int_{-1}^{1} y e^y \sin(xy)\,dx}_{=0}\,dy$$

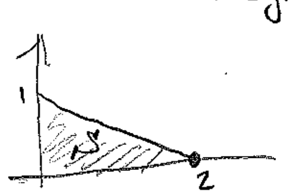
$$\int_{-1}^{1} y e^y \sin(xy)\,dx = -e^y\left[\cos(xy)\right]_{x=-1}^{x=1} = 0.$$

What more ?

→ The (double) integral as limits of sums:
the Riemann integral

↳ $\int_{-\infty}^{\infty} e^{-\frac{1}{2}x^2}\,dx = ?$ as application

→ Double integrals over other sets. E.g



The double integral over S:
x runs from 0 to 2. For each x,
y runs from 0 to $1-\frac{1}{2}x$

$$\int_0^2 \int_0^{1-x/2} \ldots\ldots \, dy \, dx$$

How to change order here?

→ The triple integral