Answers to DIY questions, Lecture 4

Ragnar Nymoen

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DYI Exercise 1

In this case e_i^* is

$$e_i^* = e_i - \bar{e} = e_i - \frac{1}{n}\sum_{i=1}^{n} e_i$$

Set first $n = 3$, for simplicity, and consider e_1^*

$$e_1^* = e_1 - \frac{1}{3}e_1 - \frac{1}{3}e_2 - \frac{1}{3}e_3 = (1 - \frac{1}{3})e_1 - \frac{1}{3}e_2 - \frac{1}{3}e_3$$

$$\begin{aligned} var(e_1^*) &= var((1 - \frac{1}{3})e_1 - \frac{1}{3}e_2 - \frac{1}{3}e_3) = \\ &= \left(1 - \frac{1}{3}\right)^2 \sigma^2 + \left(\frac{1}{3}\right)^2 \sigma^2 + \left(\frac{1}{3}\right)^2 \sigma^2 \\ &= \sigma^2 \left[1 - \frac{2}{3} + \left(\frac{1}{3}\right)^2 + \left(\frac{1}{3}\right)^2 + \left(\frac{1}{3}\right)^2\right] \\ &= \sigma^2 \left[1 - \frac{2}{3} + \frac{1}{3}\right] = \sigma^2(1 - \frac{1}{3}) \end{aligned}$$

By induction we get for e_i^*

$$\begin{aligned} var(e_i^*) &= \sigma^2(1 - 2\frac{1}{n} + \frac{1}{n^2}) + \sigma^2\frac{n-1}{n^2} = \\ &= \sigma^2\left[1 - \frac{2}{n} + \frac{1}{n}\right] = \sigma^2\left[1 - \frac{1}{n}\right] \end{aligned}$$

DYI Exercise 2

R^2 from the original and regression and the regression with scaled varaibles will be the same. This is because

$$\hat{e}_i^* = \omega_y \hat{e}_i$$

and

$$y_i^* = \omega_y y_i$$

$$R^{*2} = 1 - \frac{\sum_{i=1}^{n} \hat{e}_i^{*2}}{\sum_{i=1}^{n}(y_i^* - \overline{y^*})^2} = 1 - \frac{\omega_y^2 \sum_{i=1}^{n} \hat{e}_i^2}{\omega_y^2 \sum_{i=1}^{n}(y_i - \overline{y})^2} = R^2$$

DYI Exercise 3

$$E(\hat{\sigma}^2) = \frac{1}{n-2}\sigma^2 E\left(\frac{\sum_{i=1}^{n} \hat{e}_i^2}{\sigma^2}\right) = \frac{1}{n-2}\sigma^2(n-2) = \sigma^2$$