Answers to DIY questions, Lecture 4

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DYI Exercise 1

1.

$$
\begin{aligned}
Cov\,(\varepsilon_i, \varepsilon_j) &= E\left[(\varepsilon_i - E(\varepsilon_i)(\varepsilon_j - E(\varepsilon_j)\right] = \\
&= E\left[(\varepsilon_i - E(\varepsilon_i))\varepsilon_j - (\varepsilon_i - E(\varepsilon_i))E(\varepsilon_j)\right] \\
&= E\left[(\varepsilon_i\varepsilon_j - E(\varepsilon_i)\varepsilon_j\right] = E(\varepsilon_i\varepsilon_j) - E(\varepsilon_i)E(\varepsilon_j) \\
&= E(\varepsilon_i\varepsilon_j)
\end{aligned}
$$

by the use of the rules for expectation and assumption b: $E(\varepsilon_i) = 0$. So assumption d. can be written as $E(\varepsilon_i\varepsilon_j) = 0 \ \forall \ i \neq j$.

2.

$$
\begin{aligned}
E(Y_i) &= E\left[\alpha + \beta_1(X_i - \bar{X}) + \varepsilon_i\right] = \alpha + \beta_1(X_i - \bar{X}) + E(\varepsilon_i) \\
&= \alpha + \beta_1(X_i - \bar{X}) \quad \text{, using b: } E(\varepsilon_i) = 0
\end{aligned}
$$

Of course, by the same reasoning:

$$
E(Y_i) = \beta_0 + \beta_1 X_i
$$

if we use the original form of the equation: $Y_i = \beta_0 + \beta_1 X_i + \varepsilon_i$.

$$
\begin{aligned}
Var(Y_i) &= Var\left[\alpha + \beta_1(X_i - \bar{X}) + \varepsilon_i\right] \\
&= Var(\varepsilon_i) = \sigma^2 \ \forall i, \text{ using c.}
\end{aligned}
$$

$$
\begin{aligned}
Cov(Y_i, Y_j) &= E\left[(Y_i - E(Y_i))(Y_j - E(Y_j))\right] \\
&= E\left[(\varepsilon_i\varepsilon_j)\right] \\
&= 0 \ \forall i \neq j \text{ using d.}
\end{aligned}
$$