Lecture 14: Reliability of inference (2 of 2 lectures)

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5 March 2013

This lecture (#14):

- General mis-specification test (RESET)
- Autocorrelation
- Parameter stability and structural break.
- HGL Ch 6.3.5, and 9.3-9.4. and parts of 9.5.1. (BN: kap 8)

RESET I

- As we have seen: heteroskedasticity can be the result of "wrong functional form" for the conditional expectation.
- The regression specification error test (RESET) was historically proposed as test of functional form
- In modern interpretations (e.g., HGL Ch 6.3.5) RESET is presented as a general test of mis-specification.
- RESET is calculated from the auxiliary regression

$$Y_i = a_0 + a_1 X_i + a_2 \hat{Y}_i^2 + a_3 \hat{Y}_i^3 + v_i, \quad i = 1, 2, ..., n, \tag{1}$$

  where the fitted values $\hat{Y}_i$ are from the regression model which is under test.
- It is an F test of the joint null: $H_0: \; a_1 = 0$ and $a_2 = 0$.

Autocorrelation I

- Autocorrelation is specific to econometric models of time series data, where we have a unique ordering of the observations
- Under the classical assumptions of the regression model, there is no autocorrelation, which we write as:

$$E(\varepsilon_t \varepsilon_{t-j} \mid X_t) = 0 \quad j = 1, 2, \ldots$$

The case of residual autocorrelation is

$$E(\varepsilon_t \varepsilon_{t-j} \mid X_t) \neq 0 \quad \text{for one or more } j$$

Consequences of autocorrelation I

- The consequences of autocorrelation depends of the status of the regressor in

$$Y_t = \beta_0 + \beta_1 X_t + \varepsilon_t \tag{2}$$

- If we can maintain the IID assumption for $\{Y_t, X_t\}$, $t = 1, 2, \ldots,$ X_t is uncorrelated with ε_t and **all** past and future disturbances $\varepsilon_{t\pm j}$. In this case X_t is *strictly exogenous*.
- For time series data, the assumption of strict exogeneity is often unrealistic: It is more relevant to assume that X_t is independent of current and future disturbances ε_{t+j} $(j = 0, 1, 2, \ldots)$ but that it depends on past disturbances ε_{t-j} $(j = 1, 2, \ldots)$.
- In this case X_t is a *predetermined variable*.

Consequences of autocorrelation II

- When X_t is exogenous, the consequences of autocorrelation is the same as of heteroskedasticity.
- When X_t is predetermined, the consequences are more damaging: We lose unbiasedness and consistency. **In class**, and more in the lecture about dynamic regression.

$$X_t = \beta_0 + \beta_1 X_t + \varepsilon_t$$

	Disturbances ε_t are:			
X_t	heteroskedastic		autocorrelated	
	OLS		OLS	
X_t	$\hat{\beta}_1$	$\widehat{Var}(\hat{\beta}_1)$	$\hat{\beta}_1$	$\widehat{Var}(\hat{\beta}_1)$
exogenous	unbiased consistent	wrong	unbiased consistent	wrong[1]
predetermined	biased consistent	wrong	biased inconsistent	wrong[1]

1. Underestimates if positive autocorrelation Overestimates with negative

Testing the hypothesis of no autocorrelation I

- Informal test: The empirical correlations between $\hat{\varepsilon}_t$ and $\hat{\varepsilon}_{t-j}$ are almost always informative: It is called the sample (residual) autocorrelation function (ACF) in most software and books.
- The most important formal test is based on the auxiliary regression

$$\hat{\varepsilon}_t = a_0 + a_1\hat{\varepsilon}_{t-1} + a_2\hat{\varepsilon}_{t-2} + a_3X_t + v_t \tag{3}$$

In this example the null hypothesis is

$$H_0 : a_1 = a_2 = 0\,.$$

and the alternative is that $\hat{\varepsilon}_t$ is correlated with $\hat{\varepsilon}_{t-1}$ or $\hat{\varepsilon}_{t-2}$.

Testing the hypothesis of no autocorrelation II

- H_0 is tested with the use of a $F(2, T-4)$ test, since there are 4 parameters in the auxiliary regression in the unrestricted case.
- (The X_t is included for formal statistical reasons (similarity of tests))
- This Lagrange-multiplier test of autocorrelation generalizes to lower/higher order autocorrelation. And to multiple regression models where one of the regressors are Y_{t-1}.
- Historically, an important test of autocorrelation has been the Durbin-Watson statistic (D-W). Less used now.
- See HGL Ch 9.3 and 9.4; BN 8.3.2 about these tests

Example: Norwegian PCM

$$\pi_i = \underset{(1.453)}{10.5} - \underset{(0.423)}{1.83}\, U_t$$

$$197\mathbf{9} - 2005 \ (T = 27),\ R^2 = 0.44826$$

$$\chi^2_{normality}(2) = 1.0925[0.5791] \text{ (J-B test)}$$
$$F_{het}(2,24) = 2.6057[0.0946] \ \ (X^2 \text{ version})$$
$$F_{arch}(1,25) = 7.5486[0.0110]*$$
$$F_{ar\ 1-2}(2,23) = 13.800[0.0001] **$$
$$F_{RESET}(2,23) = 2.4797[0.1059]$$

Sample Autocorrelation function from lag 1 to 4:
0.71767 0.46817 0.32912 0.16728 —-> Positive autocorrelation

Inference and estimation under autocorrelation I

- In the same way as for HET, there at robust, autocorrelation consistent estimators of $Var(\hat{\beta}_1)$, often dubbed *HAC* estimators of variance.
- If it is relevant to regard the autocorrelation as a part of the model: Specify a model of the autocorrelation in ε_t and re-do the estimation of β_0 and β_1 under that assumption.
- Example: Let the model be specified as

$$Y_t = \beta_0 + \beta_1 X_t + \varepsilon_t \tag{4}$$

$$\varepsilon_t = \rho \varepsilon_{t-1} + \varepsilon^{'}_t, \; -1 < \rho < 1 \tag{5}$$

where v_t is a disturbance with classical properties.

- (5) is called a first order *autoregressive process*, $AR(1)$.

Inference and estimation under autocorrelation II

- It is useful to apply the Koyck-transformation: Write (4) for period $t-1$ and multiply by ρ:

$$\rho Y_{t-1} = \rho\beta_0 + \rho\beta_1 X_{\mathbf{t-1}} + \rho\varepsilon_{\mathbf{t-1}}$$

and subtract this from (4):

$$Y_t - \rho Y_{t-1} = \beta_0(1-\rho) + \beta_1(X_t - \rho X_{t-1}) + \varepsilon_t^{'} \quad (6)$$

- If ρ is a known parameter, OLS estimation of (6) gives the BLUE estimator of β_1. This is the WLS (or GLS) estimator for the case of known ρ.

Inference and estimation under autocorrelation III

- If ρ is unknown. We can re-arrange (6)

$$Y_t = \beta_0(1-\rho) + \rho Y_{t-1} + \beta_1 X_t + \beta_1 \rho X_{t-1} + \varepsilon_t^{'} \quad (7)$$

  and estimate β_1 consistently by using OLS on (7).
- In the dynamic model (7) we estimate 4 parameter, while (4) and (5) only contained 3 parameters.
- (4) and (5) can be estimated more efficiently by an iterative procedure:
  1. Estimate (4) as if there is no autocorrelation.
  2. Use the OLS residuals $\hat{\varepsilon}_t$ and $\hat{\varepsilon}_{t-1}$ to estimate (5) by OLS and obtain a first estimate, $\hat{\rho}^{(1)}$ of ρ
  3. Obtain a first WLS estimate $\hat{\beta}_1^{(1)}$ by regressing $\left(Y_t - \hat{\rho}^{(1)} Y_{t-1}\right)$ on $\left(X_t - \hat{\rho}^{(1)} X_{t-1}\right)$

Inference and estimation under autocorrelation IV

4. Use the residuals from 3. to obtain a new estimate $\hat{\rho}^{(2)}$ of ρ, and $\hat{\beta}_1^{(2)}$ of β_1 and continue until convergence

- More generally, if as in our NPC example, a regression model suffers from autocorrelation, it is not advisable to re-formulate the regression model as if the alternative of the mis-specification test is true.
- There are many possible explanations of the significant F_{ar}, and is advisable to re-specify the model to account for the systematic variation in Y_t rather than to simply add an AR-process to the static regression equation.

Parameter constancy I

- Parameter constancy and invariance are important properties of regression models:
  - Without parameter constancy: forecasting future values of Y_t with the use of a regression model leads to systematic errors in forecasts
  - Without invariance: Predicting the effects of a change in an explanatory variable on Y becomes unreliable (the Lucas critique is a special case as we shall see)
- Like other model properties constancy/and invariance are relative concepts. In practice: A higher degree of constancy of parameters are better than little stability.
- A detected significant instability is called a *structural break*.

Parameter constancy II

- Testing for structural breaks in the regression model over a sample of time series data can be done by F-distributed *Chow-tests* and recursive plots of estimated parameters

Chow tests of structural breaks I

Two-sample Chow test

Recall from Lecture 13, found that

$$F_{Chow2} = \frac{RSS_R - RSS_U}{RSS_U}\frac{T-2(k+1)}{(k+1)} \sim F(k+1, T-2(k+1)).$$

can be used to test that the same conditional expectation model (regression) holds for two sub-samples $(t = 1, 2, \dots, T_1)$ and $(t = T_1 + 1, T_1 + 2, \dots, T)$.

Forecast Chow test

If $T - T_1 < k + 1$, not enough observations in the second sample we use another approach.

RSS_R is still based on the full sample, but RSS_U is now only based on the first T_1 observations.

Chow tests of structural breaks II

Therefore it is called the **Forecast Chow-test**:

$$F_{ChowF} = \frac{RSS_R - RSS_U}{RSS_U}\frac{T_1 - (k+1)}{T - T_1} \sim F(T - T_1, T_1 - (k+1)).$$

for the null hypothesis that the regression equation is the same in the first sample and in the full sample.

Both tests assume that σ^2 is a constant parameter (homoskedasticity)

Therefore it is good practice to get an impression about the stability of σ^2, either by graphs and/or formal tests. **Examples in class**

$$\pi_i = \underset{(1.453)}{10.5} - \underset{(0.423)}{1.83}\, U_t$$

$$1979 - 2005\ (T = 27),\ R^2 = 0.44826$$

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$$F_{RESET}(2,23) = 2.4797[0.1059]$$
$$F_{Chow-F}(6,25) = 1.7974[0.1405]$$