Effects of autocorrelation (extra to "Reliability of inference" part)

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$$Y_t = \beta_0 + \beta_1 X_t + \varepsilon_t$$

	Disturbances ε_t are:			
X_t	heteroskedastic		autocorrelated	
	OLS		OLS	
X_t	$\hat{\beta}_1$	$\widehat{Var}(\hat{\beta}_1)$	$\hat{\beta}_1$	$\widehat{Var}(\hat{\beta}_1)$
exogenous	unbiased consistent	wrong	unbiased consistent	wrong[1]
predetermined	biased consistent	wrong	biased inconsistent	wrong[1]

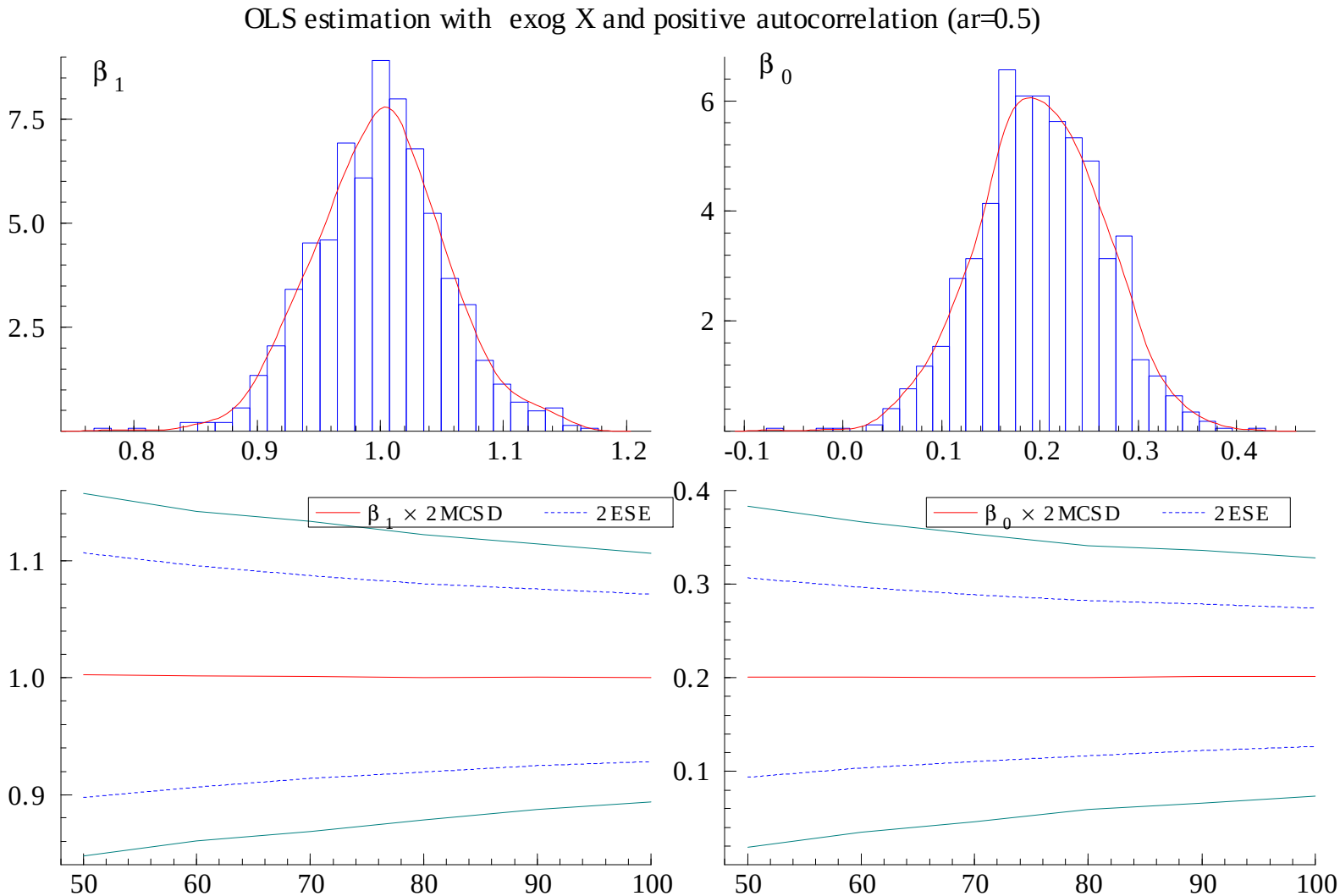
Ilustrate effects of autocorrelation with strictly exogeous and pre-determined X_t

X_t strictly exogeous

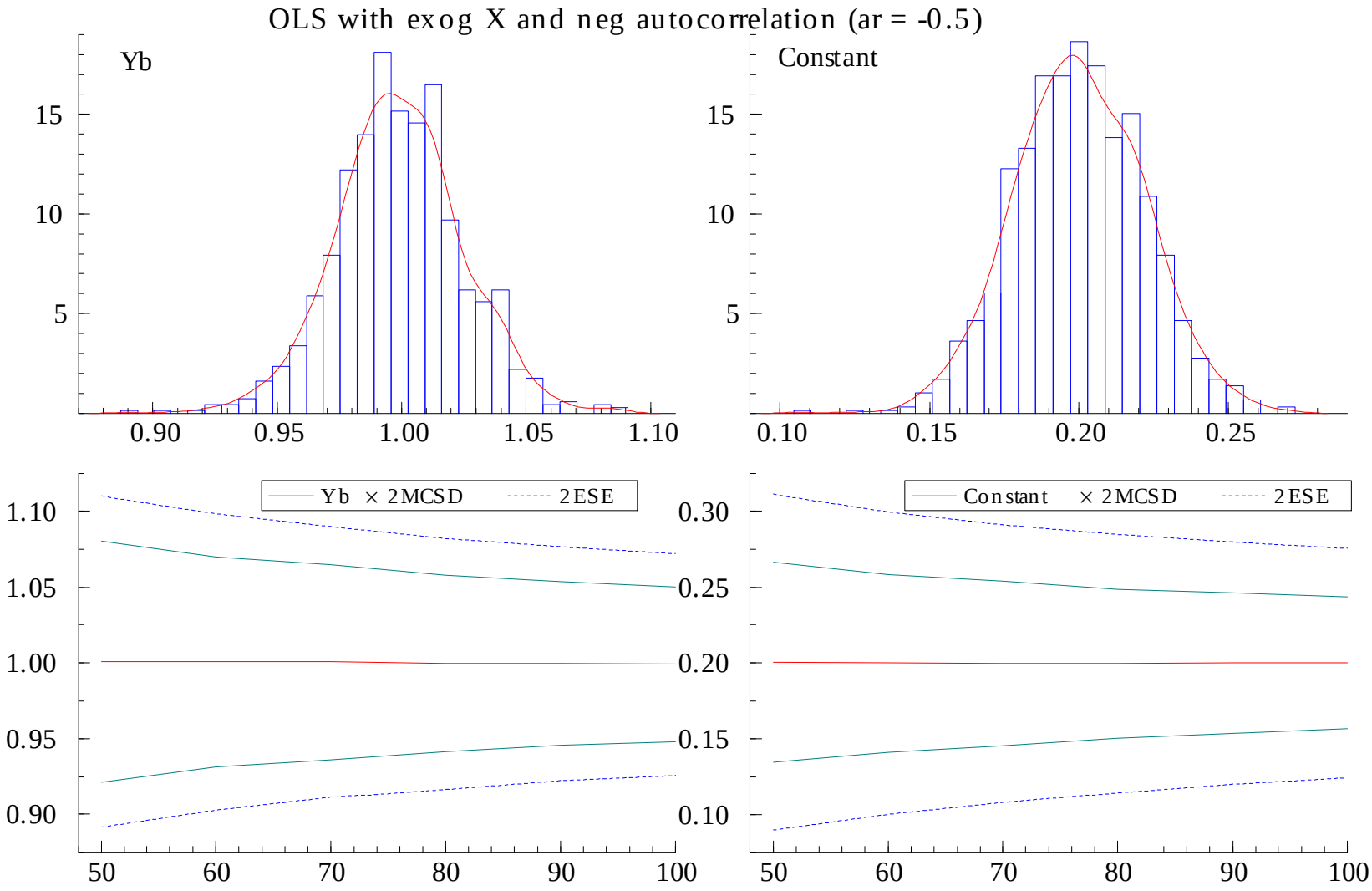
$$
\begin{aligned}
Y_t &= 0.2 + 1X_t + \varepsilon_t \\
a)\ \varepsilon_t &= 0.5\varepsilon_{t-1} + \varepsilon_t^{'} \quad \varepsilon_t^{'} \sim NIID(0, 0.1) \\
b)\ \varepsilon_t &= -0.5\varepsilon_{t-1} + \varepsilon_t^{'} \\
X_t &= 0.75X_{t-1} + v_t \quad v_t \sim NIID(0, 0.5)
\end{aligned}
$$

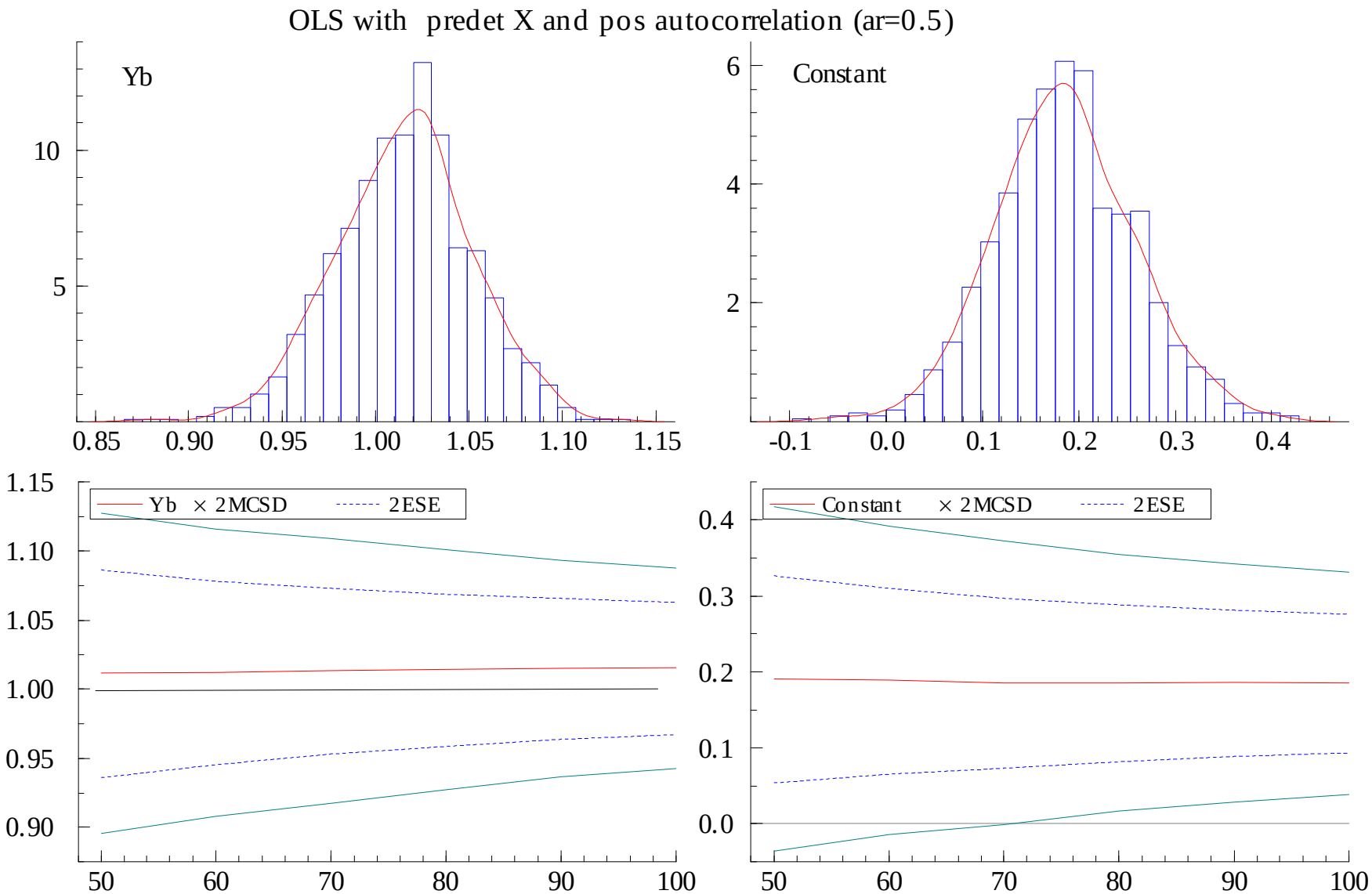
X_t predetermined exogeous:

$$
\begin{aligned}
Y_t &= 0.2 + 1X_t + \varepsilon_t \\
a)\ \varepsilon_t &= 0.5\varepsilon_{t-1} + \varepsilon_t^{'} \quad \varepsilon_t^{'} \sim NIID(0, 0.1) \\
b)\ \varepsilon_t &= -0.5\varepsilon_{t-1} + \varepsilon_t^{'} \\
X_t &= 0.4Y_{t-1} + +0.35X_{t-1} + v_t
\end{aligned}
$$


OLS estimation with exog X and positive autocorrelation (ar=0.5)
β 1
β 0
β 1 × 2MCSD
2ESE
β 0 × 2MCSD
2ESE

- $\pm 2MCSD$ is true estimate of $\sqrt{Var(\hat{\beta}_1)}$under autocorrelation
- $\pm 2ESE$ is $\sqrt{Var(\hat{\beta}_1)}$ from Stata when assuming no autocorrelation


OLS with exog X and neg autocorrelation (ar = -0.5)
Yb
Constant
15
10
5
0.90
0.95
1.00
1.05
1.10
0.10
0.15
0.20
0.25
Yb × 2MCSD
2ESE
Constant × 2MCSD
2ESE
1.10
1.05
1.00
0.95
0.90
0.30
0.25
0.20
0.15
0.10
50
60
70
80
90
100


OLS with predet X and pos autocorrelation (ar=0.5)
Yb
Constant
Yb × 2MCSD
2ESE
Constant × 2MCSD
2ESE