CHAPTER 9

Exercise Solutions

EXERCISE 9.1

(a) If $FFRATE_t = 2$ for $t = 1, 2, 3, 4$, then

$$\begin{aligned} INVGWTH_4 &= 4 - 0.4FFRATE_4 - 0.8FFRATE_3 - 0.6FFRATE_2 - 0.2FFRATE_1 \\ &= 4 - 0.4 \times 2 - 0.8 \times 2 - 0.6 \times 2 - 0.2 \times 2 \\ &= 0 \end{aligned}$$

(b) If $FFRATE_t = 2.5$ for $t = 5$ and $FFRATE_t = 2$ for $t = 6, 7, 8, 9$, then:

For $t = 5$,

$$\begin{aligned} INVGWTH_5 &= 4 - 0.4FFRATE_5 - 0.8FFRATE_4 - 0.6FFRATE_3 - 0.2FFRATE_2 \\ &= 4 - 0.4 \times 2.5 - 0.8 \times 2 - 0.6 \times 2 - 0.2 \times 2 \\ &= -.2 \end{aligned}$$

For $t = 6$,

$$\begin{aligned} INVGWTH_6 &= 4 - 0.4FFRATE_6 - 0.8FFRATE_5 - 0.6FFRATE_4 - 0.2FFRATE_3 \\ &= 4 - 0.4 \times 2 - 0.8 \times 2.5 - 0.6 \times 2 - 0.2 \times 2 \\ &= -.4 \end{aligned}$$

For $t = 7$,

$$\begin{aligned} INVGWTH_7 &= 4 - 0.4FFRATE_7 - 0.8FFRATE_6 - 0.6FFRATE_5 - 0.2FFRATE_4 \\ &= 4 - 0.4 \times 2 - 0.8 \times 2 - 0.6 \times 2.5 - 0.2 \times 2 \\ &= -.3 \end{aligned}$$

For $t = 8$,

$$\begin{aligned} INVGWTH_8 &= 4 - 0.4FFRATE_8 - 0.8FFRATE_7 - 0.6FFRATE_6 - 0.2FFRATE_5 \\ &= 4 - 0.4 \times 2 - 0.8 \times 2 - 0.6 \times 2 - 0.2 \times 2.5 \\ &= -.1 \end{aligned}$$

For $t = 9$,

$$\begin{aligned} INVGWTH_9 &= 4 - 0.4FFRATE_9 - 0.8FFRATE_8 - 0.6FFRATE_7 - 0.2FFRATE_6 \\ &= 4 - 0.4 \times 2 - 0.8 \times 2 - 0.6 \times 2 - 0.2 \times 2 \\ &= 0 \end{aligned}$$

Since *FFRATE* was increased from 2% to 2.5% in period 5 and then returned to its original level, we use the impact and delay multipliers to examine the effect of the increase. Using the notation β_0, β_1, β_2 and β_3 for the impact and delay multipliers, and noting that the increase was 0.5, the effect of the increase in periods 5, 6, 7 and 8 is given by $0.5\beta_0$, $0.5\beta_1$, $0.5\beta_2$ and $0.5\beta_3$, respectively. The estimates of these values are -0.2, -0.4, -0.3 and -0.1. Examining the forecasts given above, we find that, relative to the initial value of *INVGWTH* of 0% (when $t = 4$), *INVGWTH* has declined by 0.2, 0.4, 0.3, and 0.1, in periods 5, 6, 7 and 8, respectively. Thus, our forecasts agree with the estimates we get from using the impact and delay multipliers. Since the delay multiplier for period 4 is zero $(\beta_4 = 0)$, *INVGWTH* returns to its original level of 0% in period 9.

Exercise 9.1 (continued)

(c) If $FFRATE_t = 2.5$ for $t = 5, 6, 7, 8, 9$, then:

For $t = 5$,

$$\begin{aligned} INVGWTH_5 &= 4 - 0.4FFRATE_5 - 0.8FFRATE_4 - 0.6FFRATE_3 - 0.2FFRATE_2 \\ &= 4 - 0.4 \times 2.5 - 0.8 \times 2 - 0.6 \times 2 - 0.2 \times 2 \\ &= -.2 \end{aligned}$$

For $t = 6$,

$$\begin{aligned} INVGWTH_6 &= 4 - 0.4FFRATE_6 - 0.8FFRATE_5 - 0.6FFRATE_4 - 0.2FFRATE_3 \\ &= 4 - 0.4 \times 2.5 - 0.8 \times 2.5 - 0.6 \times 2 - 0.2 \times 2 \\ &= -.6 \end{aligned}$$

For $t = 7$,

$$\begin{aligned} INVGWTH_7 &= 4 - 0.4FFRATE_7 - 0.8FFRATE_6 - 0.6FFRATE_5 - 0.2FFRATE_4 \\ &= 4 - 0.4 \times 2.5 - 0.8 \times 2.5 - 0.6 \times 2.5 - 0.2 \times 2 \\ &= -.9 \end{aligned}$$

For $t = 8$,

$$\begin{aligned} INVGWTH_8 &= 4 - 0.4FFRATE_8 - 0.8FFRATE_7 - 0.6FFRATE_6 - 0.2FFRATE_5 \\ &= 4 - 0.4 \times 2.5 - 0.8 \times 2.5 - 0.6 \times 2.5 - 0.2 \times 2.5 \\ &= -1 \end{aligned}$$

For $t = 9$,

$$\begin{aligned} INVGWTH_9 &= 4 - 0.4FFRATE_9 - 0.8FFRATE_8 - 0.6FFRATE_7 - 0.2FFRATE_6 \\ &= 4 - 0.4 \times 2.5 - 0.8 \times 2.5 - 0.6 \times 2.5 - 0.2 \times 2.5 \\ &= -1 \end{aligned}$$

Since *FFRATE* increased from 2% to 2.5% in period 5, and was then kept at its new level, we use the impact and interim multipliers to examine the effect of the increase. The impact and interim multipliers are β_0, $(\beta_0 + \beta_1)$, $(\beta_0 + \beta_1 + \beta_2)$, and $(\beta_0 + \beta_1 + \beta_2 + \beta_3)$ for periods 5, 6, 7 and 8, respectively. With an increase of 0.5, the estimated effects in periods 5, 6, 7 and 8 are given by $0.5b_0 = -0.2$, $0.5(b_0 + b_1) = -0.6$, $0.5(b_0 + b_1 + b_2) = -0.9$ and $0.5(b_0 + b_1 + b_2 + b_3) = -1$. Examining the forecasts given above, we find that, relative to the initial value of *INVGWTH* of 2% (when $t = 4$), *INVGWTH* has declined by 0.2, 0.6, 0.9, and 1 in periods 5, 6, 7 and 8, respectively. Thus, our forecasts agree with the estimates we get from using the impact and interim multipliers. The interim multipliers for $t = 8$ and $t = 9$ are the same as the total multiplier, namely, -1, and a value of $INVGWTH = -1$ becomes the new equilibrium value.

EXERCISE 9.2

(a) Overall, advertising has a positive impact on sales revenue. There is a positive effect in the current week and in the following two weeks, but no effect after 3 weeks. The greatest impact is generated after one week. The total effect of a sustained $1 million increase in advertising expenditure is given by

$$\text{total multiplier} = b_0 + b_1 + b_2 = 1.842 + 3.802 + 2.265 = 7.909$$

(b) The null and alternative hypotheses are $H_0 : \beta_i = 0$ against $H_1 : \beta_i \neq 0$, and the *t*-value is calculated from $t = b_i / \text{se}(b_i)$ for $i = 0,1,2$. Relevant information for the significance tests is given in the following table. The 1% and 5% critical values for a two-tail test are $t_{(0.995,99)} = 2.626$ and $t_{(0.975,99)} = 1.984$, respectively. The 1% and 5% critical values for a one-tail test are $t_{(0.99,99)} = 2.365$ and $t_{(0.95,99)} = 1.660$, respectively. We use ** to denote significance at a 5% level and *** to denote significance at the 1% level. No * implies a lack of significance at these levels. We find that b_0 is insignificant for both types of test and for both significance levels; b_1 is significant at the 5% level for a two-tail test, and significant at the 1% level using a one-tail test; b_2 is insignificant at the 5% level for a two-tail test, and significant at the 5% level using a one-tail test.

Coefficient	Standard Error	*t*-Value	Two-tail *p*-value	One-tail *p*-value
b_0	1.1809	1.560	0.122	0.061
b_1	1.4699	2.587	0.011**	0.006***
b_2	1.1922	1.900	0.060	0.030**

(c) Using $t_c = t_{(0.975,99)} = 1.984$, the 95% confidence interval for the impact multiplier is given by

$$b_0 \pm t_c \times \text{se}(b_0) = 1.842 \pm 1.984 \times 1.181 = (-0.501, 4.185)$$

The one-period interim multiplier is $b_0 + b_1 = 1.842 + 3.802 = 5.644$, with standard error given by

$$\text{se}(b_0 + b_1) = \sqrt{\widehat{\text{var}}(b_0) + \widehat{\text{var}}(b_1) + 2\widehat{\text{cov}}(b_0, b_1)}$$

$$= \sqrt{1.3946 + 2.1606 + 2 \times (-1.0406)}$$

$$= \sqrt{1.474} = 1.2141$$

The 95% confidence interval for the one-period interim multiplier is

$$\left(b_0 + b_1\right) \pm t_c \times \text{se}(b_0 + b_1) = 5.644 \pm 1.984 \times 1.214 = (3.235, 8.053)$$

Exercise 9.2(c) (continued)

The total multiplier is $b_0 + b_1 + b_2 = 1.842 + 3.802 + 2.265 = 7.909$, with standard error given by

$$
\begin{aligned}
\text{se}(b_0 + b_1) &= \sqrt{\widehat{\text{var}}(b_0) + \widehat{\text{var}}(b_1) + \widehat{\text{var}}(b_2) + 2\widehat{\text{cov}}(b_0, b_1) + 2\widehat{\text{cov}}(b_0, b_2) + 2\widehat{\text{cov}}(b_1, b_2)} \\
&= \sqrt{1.3946 + 2.1606 + 1.4214 + 2 \times (-1.0406) + 2 \times 0.0984 + 2 \times (-1.0367)} \\
&= \sqrt{1.0188} = 1.009
\end{aligned}
$$

The 95% confidence interval for the total multiplier is given by

$$\left(b_0 + b_1 + b_2\right) \pm t_c \times \text{se}(b_0 + b_1 + b_2) = 7.909 \pm 1.984 \times 1.009 = (5.907, 9.911)$$

EXERCISE 9.3

(a) For the first allocation,

$$\begin{aligned}\widehat{SALES}_{106} &= \hat{\alpha} + b_0 ADV_{106} + b_1 ADV_{105} + b_2 ADV_{104} \\ &= 25.34 + 1.842\times 6 + 3.802\times 1.358 + 2.265\times 1.313 \\ &= 44.53\end{aligned}$$

$$\begin{aligned}\widehat{SALES}_{107} &= \hat{\alpha} + b_0 ADV_{107} + b_1 ADV_{106} + b_2 ADV_{105} \\ &= 25.34 + 3.802\times 6 + 2.265\times 1.358 \\ &= 51.23\end{aligned}$$

$$\begin{aligned}\widehat{SALES}_{108} &= \hat{\alpha} + b_0 ADV_{108} + b_1 ADV_{107} + b_2 ADV_{106} \\ &= 25.34 + 2.265\times 6 \\ &= 38.93\end{aligned}$$

For the second allocation,

$$\begin{aligned}\widehat{SALES}_{106} &= \hat{\alpha} + b_0 ADV_{106} + b_1 ADV_{105} + b_2 ADV_{104} \\ &= 25.34 + 3.802\times 1.358 + 2.265\times 1.313 \\ &= 33.48\end{aligned}$$

$$\begin{aligned}\widehat{SALES}_{107} &= \hat{\alpha} + b_0 ADV_{107} + b_1 ADV_{106} + b_2 ADV_{105} \\ &= 25.34 + 1.842\times 6 + 2.265\times 1.358 \\ &= 39.47\end{aligned}$$

$$\begin{aligned}\widehat{SALES}_{108} &= \hat{\alpha} + b_0 ADV_{108} + b_1 ADV_{107} + b_2 ADV_{106} \\ &= 25.34 + 3.802\times 6 \\ &= 48.15\end{aligned}$$

For the third allocation,

$$\begin{aligned}\widehat{SALES}_{106} &= \hat{\alpha} + b_0 ADV_{106} + b_1 ADV_{105} + b_2 ADV_{104} \\ &= 25.34 + 1.842\times 2 + 3.802\times 1.358 + 2.265\times 1.313 \\ &= 37.16\end{aligned}$$

$$\begin{aligned}\widehat{SALES}_{107} &= \hat{\alpha} + b_0 ADV_{107} + b_1 ADV_{106} + b_2 ADV_{105} \\ &= 25.34 + 1.842\times 4 + 3.802\times 2 + 2.265\times 1.358 \\ &= 43.39\end{aligned}$$

$$\begin{aligned}\widehat{SALES}_{108} &= \hat{\alpha} + b_0 ADV_{108} + b_1 ADV_{107} + b_2 ADV_{106} \\ &= 25.34 + 3.802\times 4 + 2.265\times 2 \\ &= 45.08\end{aligned}$$

Exercise 9.3(a) (continued)

The total sales from each of the 3 allocations are 134.69, 121.10 and 125.63, respectively. Thus, the first allocation leads to the largest sales forecast over the 3 weeks. This outcome occurs because the first allocation allows time for the full effect of the \$6 million expenditure to be realized.

The second allocation, in which the marketing executive spends all \$6 million in $t = 107$, provides the highest sales revenue in $t = 108$. The coefficient for the first lag is higher than the coefficients of the other lags, suggesting that the effect of advertising on sales revenue is greatest one week after the advertising expenditure is made.

(b) The estimated variance of the forecast error $f = SALES_{108} - \widehat{SALES}_{108}$ for the first allocation is

$$\begin{aligned}\widehat{\text{var}}(f) &= \hat{\sigma}^2 + \widehat{\text{var}}(\hat{\alpha}) + 6^2\widehat{\text{var}}(b_2) + 2\times 6\times \widehat{\text{cov}}(\hat{\alpha}, b_2)\\ &= 2.3891 + 2.5598 + 36\times 1.4214 + 12\times(-0.7661)\\ &= 42.9261\end{aligned}$$

$$\text{se}(f) = \sqrt{42.9261} = 6.850$$

The 95% confidence interval for the first allocation is

$$\widehat{SALES}_{108} \pm t_c \times \text{se}(f) = 38.93 \pm 1.984\times 6.850 = (25.34, 52.52)$$

The estimated variance of the forecast error for the second allocation is

$$\begin{aligned}\widehat{\text{var}}(f) &= \hat{\sigma}^2 + \widehat{\text{var}}(\hat{\alpha}) + 6^2\widehat{\text{var}}(b_1) + 2\times 6\times \widehat{\text{cov}}(\hat{\alpha}, b_1)\\ &= 2.3891 + 2.5598 + 36\times 2.1606 + 12\times(-0.1317)\\ &= 81.1501\end{aligned}$$

$$\text{se}(f) = \sqrt{81.1501} = 9.008$$

The 95% confidence interval for the second allocation is

$$\widehat{SALES}_{108} \pm t_c \times \text{se}(f) = 48.15 \pm 1.984\times 9.008 = (30.28, 66.02)$$

The estimated variance of the forecast error for the third allocation is

$$\begin{aligned}\widehat{\text{var}}(f) &= \hat{\sigma}^2 + \widehat{\text{var}}(\hat{\alpha}) + 4^2\widehat{\text{var}}(b_1) + 2^2\widehat{\text{var}}(b_2) + 2\times 4\times \widehat{\text{cov}}(\hat{\alpha}, b_1)\\ &\qquad + 2\times 2\times \widehat{\text{cov}}(\hat{\alpha}, b_2) + 2\times 2\times 4\times \widehat{\text{cov}}(b_2, b_1)\\ &= 2.3891 + 2.5598 + 16\times 2.1606 + 4\times 1.4214 + 8\times(-0.1317)\\ &\qquad + 4\times(-0.7661) + 16\times(-1.0367)\\ &= 24.4989\end{aligned}$$

$$\text{se}(f) = \sqrt{24.4989} = 4.950$$

Exercise 9.3(b) (continued)

The 95% confidence interval for the third allocation is

$$\widehat{SALES}_{108} \pm t_c \times \text{se}(f) = 45.08 \pm 1.984 \times 4.950 = (35.26, 54.90)$$

The most favorable allocation is the second or the third. If maximizing expected profits at $t = 108$ is the objective, then the second allocation is best. However, a risk averse marketing executive may prefer the third allocation because its expected profit is only slightly less than that for the second allocation, and it has a much lower standard error of forecast error. This is reflected in the forecast intervals, where sales for the second allocation could be as low as 30.28, whereas for the third allocation the lower limit of the forecast interval is 35.26.

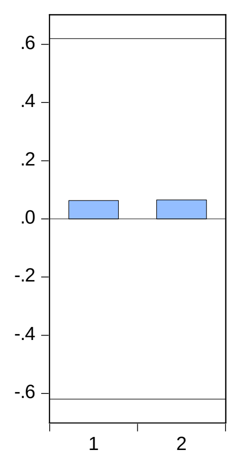
EXERCISE 9.4

(a) Using hand calculations

$$r_1 = \frac{\sum_{t=2}^{T} \hat{e}_t \hat{e}_{t-1}}{\sum_{t=1}^{T} \hat{e}_t^2} = \frac{0.0979}{1.5436} = 0.0634, \qquad r_2 = \frac{\sum_{t=3}^{T} \hat{e}_t \hat{e}_{t-2}}{\sum_{t=1}^{T} \hat{e}_t^2} = \frac{0.1008}{1.5436} = 0.0653$$

(b) (i) The test statistic for testing $H_0: \rho_1 = 0$ against the alternative $H_1: \rho_1 \neq 0$ is $Z = \sqrt{T} r_1 = \sqrt{10} \times 0.0634 = 0.201$. Comparing this value to the critical Z values for a two tail test with a 5% level of significance, $Z_{(0.025)} = -1.96$ and $Z_{(0.975)} = 1.96$, we do not reject the null hypothesis and conclude that r_1 is not significantly different from zero.

(ii) The test statistic for testing $H_0: \rho_2 = 0$ against the alternative $H_1: \rho_2 \neq 0$ is $Z = \sqrt{T} r_2 = \sqrt{10} \times 0.0653 = 0.207$. Comparing this value to the critical Z values for a two tail test with a 5% level of significance, $Z_{(0.025)} = -1.96$ and $Z_{(0.975)} = 1.96$, we do not reject the null hypothesis and conclude that r_2 is not significantly different from zero.



The significance bounds are drawn at $\pm 1.96/\sqrt{10} = \pm 0.62$. With this small sample, the autocorrelations are a long way from being significantly different from zero.

EXERCISE 9.5

(a) The first three autocorrelations are

$$r_1 = \frac{\sum_{t=2}^{250}\left(G_t - \overline{G}\right)\left(G_{t-1} - \overline{G}\right)}{\sum_{t=1}^{250}\left(G_t - \overline{G}\right)^2} = \frac{162.9753}{333.8558} = 0.4882$$

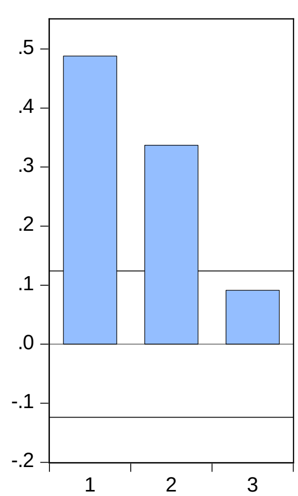
$$r_2 = \frac{\sum_{t=3}^{250}\left(G_t - \overline{G}\right)\left(G_{t-2} - \overline{G}\right)}{\sum_{t=1}^{250}\left(G_t - \overline{G}\right)^2} = \frac{112.4882}{333.8558} = 0.3369$$

$$r_3 = \frac{\sum_{t=4}^{250}\left(G_t - \overline{G}\right)\left(G_{t-3} - \overline{G}\right)}{\sum_{t=1}^{250}\left(G_t - \overline{G}\right)^2} = \frac{30.5802}{333.8558} = 0.0916$$

To test whether the autocorrelations are significantly different from zero, the null and alternative hypotheses are $H_0: \rho_k = 0$ and $H_1: \rho_k \neq 0$, and the test statistic is given by $z_k = \sqrt{T} r_k = 15.8114 r_k$. At a 5% level of significance, the critical values are ± 1.96; thus, we reject the null hypothesis if $|z_k| > 1.96$. The test results are provided in the table below.

Autocorrelations	*z*-statistic	Critical value	Decision
$r_1 = 0.4882$	7.719	± 1.96	Reject H_0
$r_2 = 0.3369$	5.327	± 1.96	Reject H_0
$r_3 = 0.0916$	1.448	± 1.96	Do not reject H_0

The significance bounds for the correlogram are $\pm 1.96/\sqrt{250} = \pm 0.124$. It leads us to the same conclusion as the hypothesis tests.



Exercise 9.5 (continued)

(b) The least squares estimates for θ_1 and δ are

$$\hat{\theta}_1 = \frac{\sum_{t=2}^{250}\left(G_t - \bar{G}_1\right)\left(G_{t-1} - \bar{G}_{-1}\right)}{\sum_{t=2}^{250}\left(G_{t-1} - \bar{G}_{-1}\right)^2} = \frac{162.974}{333.1119} = 0.4892$$

$$\begin{aligned}\hat{\delta} &= \bar{G}_1 - \hat{\theta}\bar{G}_{-1} \\ &= 1.662249 - 0.48925 \times 1.664257 \\ &= 0.8480\end{aligned}$$

The estimated value $\hat{\theta}$ is slightly larger than r_1 because the summation in the denominator for r_1 has one more squared term than the summation in the denominator for $\hat{\theta}$. The means are also slightly different.

EXERCISE 9.6

(a) A one percentage point increase in the mortgage rate in period t relative to what it was in period $t-1$ decreases the number of new houses sold between periods t and $t-1$ by 53,510 units.

A 99% confidence interval for the coefficient of $DIRATE_{t-1}$ is

$$b_2 \pm t_c \text{se}(b_2) = -53.51 \pm 2.599 \times 16.98 = (-97.64, -9.38)$$

With 99% confidence, we estimate that a one percentage point increase in the mortgage rate in period t relative to what it was in period $t-1$ decreases the number of new houses sold by a number between 9,380 and 97,640.

(b) The two tests that can be used are a *t*-test on the significance of the coefficient of $\hat{e}_{t-1}$ and the Lagrange multiplier test given by $T \times R^2$. The null and alternative hypotheses are $H_0: \rho = 0$ and $H_1: \rho \neq 0$. The *LM* test value is given by

$$LM = T \times R^2 = 218 \times 0.1077 = 23.48$$

The 1% critical value from a $\chi^2_{(1)}$- distribution is 6.635. Since the test statistic is greater than the critical value, we reject the null hypothesis and conclude that there is evidence of autocorrelation.

Testing the significance of the coefficient of $\hat{e}_{t-1}$, we find

$$t = \frac{-0.3306 - 0}{0.0649} = -5.09$$

The 1% critical values are $t_{(0.995,215)} = \pm 2.60$; since the *t*-statistic is less than -2.60, we reject the null hypothesis and conclude that there is evidence of autocorrelation.

(c) The 99% confidence interval for the coefficient of $DIRATE_{t-1}$ is given as:

$$\hat{\beta}_2 \pm t_c \text{se}(\hat{\beta}_2) = -58.61 \pm 2.599 \times 14.10 = (-95.25, -21.97)$$

Ignoring autocorrelation gave a lower value for the coefficient of interest and a slightly larger standard error, resulting in a confidence interval with a similar lower bound but a larger upper bound. When autocorrelation is ignored, our inferences about the coefficient could be misleading because the wrong standard error is used.

EXERCISE 9.7

(a) Under the assumptions of the AR(1) model, $\text{corr}(e_t, e_{t-k}) = \rho^k$. Thus,

(i) $\text{corr}(e_t, e_{t-1}) = \rho = 0.9$

(ii) $\text{corr}(e_t, e_{t-4}) = \rho^4 = 0.9^4 = 0.6561$

(iii) $\sigma_e^2 = \dfrac{\sigma_v^2}{1-\rho^2} = \dfrac{1}{1-0.9^2} = 5.263$

(b) (i) $\text{corr}(e_t, e_{t-1}) = \rho = 0.4$

(ii) $\text{corr}(e_t, e_{t-4}) = \rho^4 = 0.4^4 = 0.0256$

(iii) $\sigma_e^2 = \dfrac{\sigma_v^2}{1-\rho^2} = \dfrac{1}{1-0.4^2} = 1.190$

When the correlation between the current and previous period error is weaker, the correlations between the current error and the errors at more distant lags die out relatively quickly, as is illustrated by a comparison of $\rho_4 = 0.6561$ in part (a)(ii) with $\rho_4 = 0.0256$ in part (b)(ii). Also, the larger the correlation ρ, the greater the variance σ_e^2, as is illustrated by a comparison of $\sigma_e^2 = 5.263$ in part (a)(iii) with $\sigma_e^2 = 1.190$ in part (b)(iii).

EXERCISE 9.8

(a) The forecasts for inflation are

$$\begin{aligned}\widehat{INF}_{2009Q4} &= 0.1001 + 0.2354\times 1.0 + 0.1213\times 0.5 + 0.1677\times 0.1 \\ &\quad + 0.2819\times(-0.3) - 0.7902\times(-0.2) \\ &= 0.4864\end{aligned}$$

$$\begin{aligned}\widehat{INF}_{2010Q1} &= 0.1001 + 0.2354\times 0.4864 + 0.1213\times 1.0 + 0.1677\times 0.5 \\ &\quad + 0.2819\times 0.1 - 0.7902\times(-0.2) \\ &= 0.6060\end{aligned}$$

$$\begin{aligned}\widehat{INF}_{2010Q2} &= 0.1001 + 0.2354\times 0.6060 + 0.1213\times 0.4864 + 0.1677\times 1.0 \\ &\quad + 0.2819\times 0.5 - 0.7902\times(-0.4) \\ &= 0.9265\end{aligned}$$

(b) The standard errors of the forecast errors are

For 2009Q4

$$\hat{\sigma}_1^2 = \hat{\sigma}_v^2 = 0.225103$$

$$\hat{\sigma}_1 = 0.47445$$

For 2010Q1

$$\hat{\sigma}_2^2 = \hat{\sigma}_v^2\left(1+\hat{\theta}_1^2\right) = 0.225103\times(1+0.2354^2) = 0.237577$$

$$\hat{\sigma}_2 = 0.4874$$

For 2010Q2

$$\hat{\sigma}_3^2 = \hat{\sigma}_v^2\left(\left(\hat{\theta}_1^2+\hat{\theta}_2\right)^2 + \hat{\theta}_1^2 + 1\right) = 0.225103\times\left((0.2354^2 + 0.1213)^2 + 0.2354^2 + 1\right)$$

$$= 0.244606$$

$$\hat{\sigma}_3 = 0.4946$$

(c) The forecast intervals are

$$\widehat{INF}_{2009Q4} \pm t_{(0.975,84)}\times\hat{\sigma}_1 = 0.4864 \pm 1.9897\times 0.4745 = (-0.4577, 1.4305)$$

$$\widehat{INF}_{2010Q1} \pm t_{(0.975,81)}\times\hat{\sigma}_2 = 0.6060 \pm 1.9897\times 0.4874 = (-0.3638, 1.5758)$$

$$\widehat{INF}_{2010Q2} \pm t_{(0.975,81)}\times\hat{\sigma}_3 = 0.9265 \pm 1.9897\times 0.4946 = (-0.0576, 1.9106)$$

These forecast intervals are relatively wide, containing both negative and positive values. Thus, the forecasts we calculated in part (a) do not provide a reliable guide to what inflation will be in those quarters.

EXERCISE 9.9

(a) The ARDL model can be written as

$$\left(1-\theta_1 L-\theta_2 L^2-\theta_3 L^3-\theta_4 L^4\right) y_t=\delta+\delta_0 x_t$$

$$y_t=\left(1-\theta_1 L-\theta_2 L^2-\theta_3 L^3-\theta_4 L^4\right)^{-1} \delta+\left(1-\theta_1 L-\theta_2 L^2-\theta_3 L^3-\theta_4 L^4\right)^{-1} \delta_0 x_t$$

$$y_t=\alpha+\left(\beta_0+\beta_1 L+\beta_2 L^2+\beta_3 L^3+\beta_4 L^4\right) x_t$$

from which we obtain

$$\alpha=\left(1-\theta_1 L-\theta_2 L^2-\theta_3 L^3-\theta_4 L^4\right)^{-1} \delta$$

$$\left(1-\theta_1 L-\theta_2 L^2-\theta_3 L^3-\theta_4 L^4\right)^{-1} \delta_0=\left(\beta_0+\beta_1 L+\beta_2 L^2+\beta_3 L^3+\beta_4 L^4\right)$$

Thus,

$$\alpha=\frac{\delta}{1-\theta_1-\theta_2-\theta_3-\theta_4}$$

and

$$\delta_0=\left(1-\theta_1 L-\theta_2 L^2-\theta_3 L^3-\theta_4 L^4\right)\left(\beta_0+\beta_1 L+\beta_2 L^2+\beta_3 L^3+\beta_4 L^4\right)$$

$$\begin{aligned}
\delta_0 L^0+0L+0L^2+0L^3+0L^4 = {} & \beta_0+\beta_1 L+\beta_2 L^2+\beta_3 L^3+\beta_4 L^4 \\
& -\theta_1\beta_0 L-\theta_1\beta_1 L^2-\theta_1\beta_2 L^3-\theta_1\beta_3 L^4 \\
& -\theta_2\beta_0 L^2-\theta_2\beta_1 L^3-\theta_2\beta_2 L^4 \\
& -\theta_3\beta_0 L^3-\theta_3\beta_1 L^4 \\
& -\theta_4\beta_0 L^4
\end{aligned}$$

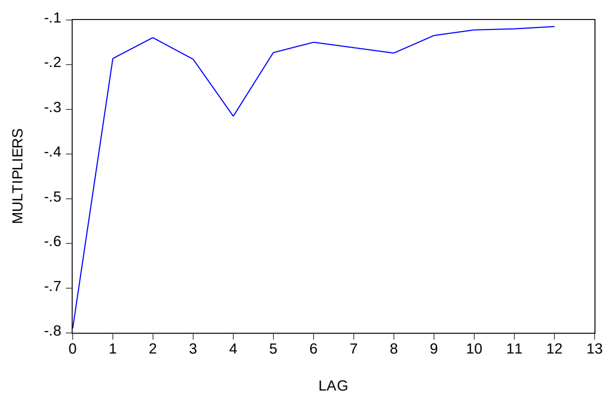
Equating coefficients of like powers in the lag operator yields

$$\begin{array}{lcl}
\beta_0-\delta_0=0 & & \beta_0=\delta_0 \\
\beta_1-\theta_1\beta_0=0 & & \beta_1=\theta_1\beta_0 \\
\beta_2-\theta_1\beta_1-\theta_2\beta_0=0 & & \beta_2=\theta_1\beta_1+\theta_2\beta_0 \\
\beta_3-\theta_1\beta_2-\theta_2\beta_1-\theta_3\beta_0=0 & \Rightarrow & \beta_3=\theta_1\beta_2+\theta_2\beta_1+\theta_3\beta_0 \\
\beta_4-\theta_1\beta_3-\theta_2\beta_2-\theta_3\beta_1-\theta_4\beta_0=0 & & \beta_4=\theta_1\beta_3+\theta_2\beta_2+\theta_3\beta_1+\theta_4\beta_0 \\
\beta_5-\theta_1\beta_4-\theta_2\beta_3-\theta_3\beta_2-\theta_4\beta_1=0 & & \beta_5=\theta_1\beta_4+\theta_2\beta_3+\theta_3\beta_2+\theta_4\beta_1 \\
\vdots & & \vdots \\
\beta_s-\theta_1\beta_{s-1}-\theta_2\beta_{s-2}-\theta_3\beta_{s-3}-\theta_4\beta_{s-4}=0 & & \beta_s=\theta_1\beta_{s-1}+\theta_2\beta_{s-2}+\theta_3\beta_{s-3}+\theta_4\beta_{s-4}
\end{array}$$

Exercise 9.9 (continued)

(b) The estimated weights up to 12 lags and their graph are given below.

Weight	Estimate
0	–0.790
1	–0.186
2	–0.140
3	–0.188
4	–0.315
5	–0.173
6	–0.150
7	–0.162
8	–0.174
9	–0.135
10	–0.122
11	–0.120
12	–0.115



The multipliers are negative at all lags. In absolute value terms, an unemployment change has its greatest effect immediately, and then drops away quickly at lag 1. It increases again at lags 3 and 4, and then drops away again. After that the effect is small, although there is a slight increase at lag 8. The increases at lags 4 and 8 suggest a quarterly effect.

(c) If the unemployment rate is constant in all periods, then $DU = 0$ in all periods and the estimated inflation rate is

$$\hat{\alpha} = \frac{\hat{\delta}}{1-\hat{\theta}_1-\hat{\theta}_2-\hat{\theta}_3-\hat{\theta}_4}$$

$$= \frac{0.1001}{1-0.2354-0.1213-0.1677-0.2819}$$

$$= 0.517$$

EXERCISE 9.10

(a) The forecasts for *DURGWTH* are

$$\widehat{DURGWTH}_{2010Q1} = 0.0103 - 0.1631\times(0.1) + 0.7422\times(0.6) + 0.3479\times(0.9)$$
$$= 0.7524$$

$$\widehat{DURGWTH}_{2010Q2} = 0.0103 - 0.1631\times(0.7524) + 0.7422\times(0.8) + 0.3479\times(0.6)$$
$$= 0.6901$$

(b) Since this model has the same lags as the example in Section 9.8 of POE4, the formulas given in that section for the lag weights are relevant. They are

$$\beta_0 = \delta_0 \qquad \beta_1 = \delta_1 + \theta_1\beta_0 \qquad \beta_s = \theta_1\beta_{s-1} \quad s \geq 2$$

The lag weights for up to 12 quarters are as follows.

Lag	Estimate
0	0.7422
1	0.2268
2	− 0.0370
3	0.0060
4	-9.8×10^{-4}
5	1.6×10^{-4}
6	-2.6×10^{-5}
7	4.3×10^{-6}
8	-6.9×10^{-7}
9	1.1×10^{-7}
10	-1.9×10^{-8}
11	3.0×10^{-9}
12	-4.9×10^{-10}

(c) The one and two-quarter delay multipliers are

$$\hat{\beta}_1 = \frac{\partial DURGWTH_t}{\partial INGRWTH_{t-1}} = 0.2268$$

$$\hat{\beta}_2 = \frac{\partial DURGWTH_t}{\partial INGRWTH_{t-2}} = -0.0370$$

These values suggest that if income growth increases by 1% and then returns to its original level in the next quarter, then growth in the consumption of durables will increase by 0.227% in the next quarter and decrease by 0.037% two quarters later.

Exercise 9.10(c) (continued)

The one and two-quarter interim multipliers are

$$\hat{\beta}_0 + \hat{\beta}_1 = 0.7422 + 0.2268 = 0.969$$

$$\hat{\beta}_0 + \hat{\beta}_1 + \hat{\beta}_2 = 0.969 - 0.0370 = 0.932$$

These values suggest that if income growth increases by 1% and is maintained at its new level, then growth in the consumption of durables will increase by 0.969% in the next quarter and increase by 0.932% two quarters later.

Since the coefficients in the table in part (b) become negligible by the time lag 12 is reached, the total multiplier can be obtained by summing all the coefficients in that table. Doing so yields

$$\sum_{j=0}^{\infty} \hat{\beta}_j = 0.9373$$

This value suggests that if income growth increases by 1% and is maintained at its new level, then, at the new equilibrium, growth in the consumption of durables will increase by 0.937%.

EXERCISE 9.11

(a) To write the AR(1) in lag operator notation, we have

$$e_t = \rho e_{t-1} + v_t$$
$$e_t - \rho e_{t-1} = v_t$$
$$(1-\rho L)e_t = v_t$$

(c) Since

$$(1-\rho L)(1-\rho L)^{-1} = 1$$

we can show that $(1-\rho L)^{-1} = 1+\rho L+\rho^2 L^2+\rho^3 L^3+\cdots$ by showing

$$(1-\rho L)(1+\rho L+\rho^2 L^2+\rho^3 L^3+\cdots) = (1+\rho L+\rho^2 L^2+\rho^3 L^3+\cdots) - (\rho L+\rho^2 L^2+\rho^3 L^3+\cdots)$$
$$=1$$

Thus, we have

$$(1-\rho L)e_t = v_t$$
$$e_t = (1-\rho L)^{-1} v_t$$
$$= (1+\rho L+\rho^2 L^2+\rho^3 L^3+\cdots)v_t$$
$$= v_t + \rho v_{t-1} + \rho^2 v_{t-2} + \rho^3 v_{t-3} + \cdots$$

EXERCISE 9.12

(a)

Coefficient Estimates and AIC and SC Values for Finite Distributed Lag Model

	$q=0$	$q=1$	$q=2$	$q=3$	$q=4$	$q=5$	$q=6$
$\hat{\alpha}$	0.4229	0.5472	0.5843	0.5828	0.6002	0.5990	0.5239
$\hat{\beta}_0$	–0.3119	–0.2135	–0.1974	–0.1972	–0.1940	–0.1940	–0.1830
$\hat{\beta}_1$		–0.1954	–0.1693	–0.1699	–0.1726	–0.1728	–0.1768
$\hat{\beta}_2$			–0.0707	–0.0713	–0.0664	–0.0662	–0.0828
$\hat{\beta}_3$				0.0021	0.0065	0.0062	0.0192
$\hat{\beta}_4$					–0.0222	–0.0225	–0.0475
$\hat{\beta}_5$						0.0015	–0.0169
$\hat{\beta}_6$							0.0944
AIC	–3.1132	–3.4314	–3.4587	–3.4370	–3.4188	–3.3971	–3.4416
AIC*	–0.2753	–0.5935	–0.6208	–0.5991	–0.5809	–0.5592	–0.6037
SC	–3.0584	–3.3492	–3.3490	–3.2999	–3.2543	–3.2052	–3.2223
SC*	–0.2205	–0.5113	–0.5111	–0.4620	–0.4165	–0.3673	–0.3844

Note: AIC* = $\text{AIC}-1-\ln(2\pi)$ and SC* = $\text{SC}-1-\ln(2\pi)$

The AIC is minimized at $q=2$ while the SC is minimized at $q=1$.

(b) (i) A 95% confidence interval for β_0 is given by

$$\hat{\beta}_0 \pm t_{(0.975,88)}\text{se}\left(\hat{\beta}_0\right) = -0.1974 \pm 1.987\times 0.0328 = (-0.2626,-0.1322)$$

(ii) The null and alternative hypotheses are

$$H_0:\beta_0+\beta_1+\beta_2=-0.5 \qquad H_1:\beta_0+\beta_1+\beta_2>-0.5$$

The test statistic is

$$t=\frac{b_0+b_1+b_2-(-0.5)}{\text{se}\left(b_0+b_1+b_2\right)}=\frac{0.062656}{0.034526}=1.815$$

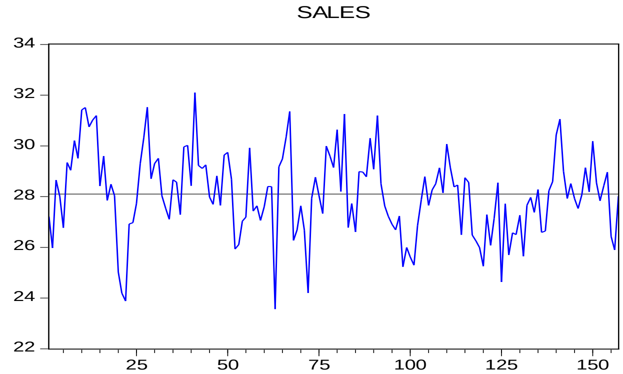
The critical value is $t_{(0.95,88)}=1.662$. Since $t=1.815>1.662$, we reject the null hypothesis and conclude that the total multiplier is greater than –0.5. The *p*-value is 0.0365.

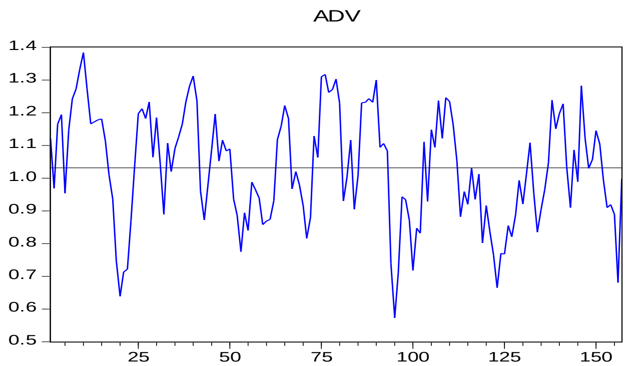
(iii) The estimated normal growth rate is $\hat{G}_N = 0.58427/0.437344 = 1.336$. The 95% confidence interval for the normal growth rate is

$$\hat{G}_N \pm t_{(0.975,88)}\text{se}\left(\hat{G}_N\right) = 1.336 \pm 1.987\times 0.0417 = (1.253,1.419)$$

EXERCISE 9.13

(a) The graphs for *SALES* and *ADV* follow. Both appear not to be trending and both fluctuate around a constant mean.





(b)

Lag	SC	$SC + (1 + \ln(2\pi))$	Total Multiplier
0	0.5949	3.433	6.020
1	0.4269	3.265	7.275
2	0.3756	3.214	8.067
3	0.3736	3.211	8.634
4	0.4015	3.239	8.863
5	0.4288	3.267	8.595

The total multiplier is sensitive to lag length up to lag 3; for lag 3 and longer lags there is little variation.

Exercise 9.13 (continued)

(c) Of the six possible lag lengths, the SC reaches a minimum when the lag length equals three. The estimates for this lag length appear below.

The lag structure is such that the greatest impact from advertising on sales is felt immediately and the lag weights decline thereafter, with the exception of the weight at lag 3 which is greater than that at lag 2. The declining lag weights are sensible. We expect the effect of advertising to diminish over time; however, the increase at lag 3 is not expected.

All the lag weights are not significantly different from zero at a 1% level; at the 5% level, only the lag weight at lag 2 is not significantly different from zero.

Dependent Variable: SALES
Method: Least Squares
Date: 05/27/11 Time: 08:16
Sample: 6 157
Included observations: 152

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	19.21618	0.688875	27.89502	0.0000
ADV	2.756383	0.805025	3.423970	0.0008
ADV(-1)	2.473420	0.997601	2.479368	0.0143
ADV(-2)	1.526656	1.019415	1.497581	0.1364
ADV(-3)	1.877676	0.819567	2.291060	0.0234

(d) (i) The one-week delay multiplier is:

$$b_1 = \frac{\partial \widehat{SALES}_t}{\partial ADV_{t-1}} = 2.4734$$

The 99% confidence interval for the one-week delay multiplier is

$$b_1 \pm t_{(0.995,147)}\text{se}(b_1) = 2.4734 \pm 2.610 \times 0.8050 = (0.373, 4.574)$$

(ii) One-week interim multiplier:

$$b_1 + b_2 = 2.7564 + 2.4734 = 5.2298$$

The 99% confidence interval for one-week delay multiplier is

$$(b_1 + b_2) \pm t_{(0.995,147)}\text{se}(b_1 + b_2) = 5.2298 \pm 2.610 \times 0.8249 = (3.077, 7.383)$$

(iii) Two-week delay multiplier:

$$b_2 = \frac{\partial \widehat{SALES}_t}{\partial ADV_{t-2}} = 1.5267$$

The 99% confidence interval for the two-week delay multiplier is

$$b_2 \pm t_{(0.995,147)}\text{se}(b_1) = 1.5267 \pm 2.610 \times 1.0194 = (-0.488, 3.541)$$

Exercise 9.13(d) (continued)

(iv) Two-week interim multiplier:

$$b_0 + b_1 + b_2 = 2.7564 + 2.4734 + 1.5267 = 6.7565$$

The 95% confidence interval for the two-week interim multiplier is

$$\left(b_0 + b_1 + b_2\right) \pm t_{(0.975,147)}\text{se}(b_0 + b_1 + b_2) = 6.7565 \pm 1.976 \times 0.8387 = (5.099, 8.414)$$

(e) A \$1 million increase in advertising expenditure in each week will increase sales by β_0 in the first week, by $\beta_0 + \beta_1$ in the second week, and by $\beta_0 + \beta_1 + \beta_2$ in the third week. Thus, the total increase over 3 weeks is $3\beta_0 + 2\beta_1 + \beta_2$. Its estimate is

$$b_0 + (b_0 + b_1) + (b_0 + b_1 + b_2) = 2.7564 + 5.2298 + 6.7565 = 14.743$$

with $\text{se}(3b_0 + 2b_1 + b_2) = 1.7035$. We wish to test

$$H_0: 3\beta_0 + 2\beta_1 + \beta_2 \le 6 \quad \text{versus} \quad H_1: 3\beta_0 + 2\beta_1 + \beta_2 > 6$$

The value of the *t*-statistic is

$$t = \frac{14.7426 - 6}{1.7035} = 5.13$$

Since $5.13 > t_{(0.95,147)} = 1.655$, we reject H_0 and conclude that the CEO's strategy will increase sales by more than \$6 million over the 3 weeks.

(f) The estimated equation is

$$\widehat{SALES}_t = 19.2162 + 2.7564 ADV_t + 2.4734 ADV_{t-1} + 1.5267 ADV_{t-2} + 1.8777 ADV_{t-3}$$

For forecasting 1, 2, 3 and 4 weeks into the future we set $t = 158, 159, 160$ and then 161. The required sample values of *ADV* are $ADV_{155} = 0.889,\ ADV_{156} = 0.681,\ ADV_{157} = 0.998$.

The forecast values for each part are presented in the table below:

	Forecast Values (\$millions)			
	$t = 158$	$t = 159$	$t = 160$	$t = 161$
(i)	24.394	22.018	21.090	19.216
(ii)	35.419	31.912	27.197	26.727
(iii)	27.150	27.248	27.847	27.850

In the first set of forecasts, *SALES* gradually declines as the effect of the advertising expenditure during the sample period wears off, with the forecast in the last period equal to the intercept. In the second set of forecasts, the large initial expenditure on advertising leads to a large initial increase in *SALES* which then declines over the forecast horizon. Having a uniform expenditure of \$1 million in each year leads to *SALES* that are more uniform and which achieve a value equal to the intercept plus the total multiplier in the final period (27.850 = 19.216 + 8.634).

EXERCISE 9.14

(a) The estimated model is

$$\ln(AREA_t) = 3.8241 + 0.7746\ln(PRICE_t) - 0.2175\ln(PRICE_{t-1}) - 0.0026\ln(PRICE_{t-2})$$
$$\text{(se)} \quad (0.1006) \; (0.3129) \qquad (0.3185) \qquad (0.3221)$$

$$+ 0.5868\ln(PRICE_{t-3}) - 0.0143\ln(PRICE_{t-4})$$
$$(0.3153) \qquad (0.2985)$$

The interim and delay elasticities are reported in the table below.

Lag	Delay Elasticities	Interim Elasticities
0	0.7746	0.7746
1	–0.2175	0.5572
2	–0.0026	0.5546
3	0.5868	1.1414
4	–0.0143	1.1271

Only b_0, the coefficient of $\ln(P_t)$, is significantly different from zero at a 5% level of significance. All coefficients for lagged values of $\ln(P_t)$, namely, b_1, b_2, b_3, b_4, are not significant at a 5% level. This result is symptomatic of collinearity in the data. When collinearity exists, least squares cannot distinguish between the individual effects of each independent variable, resulting in large standard errors and coefficients which are not significantly different from zero.

Interpreting the delay multipliers, if the price is increased and then decreased by 1% in period *t*, there is an immediate increase of 0.77% in area planted. In period $t+1$, that is one period after the price shock, there is a decrease in area planted of 0.22%. In period $t+2$ there is practically no change in the area planted. In period $t+3$ there is an increase in area planted by 0.59% and in period $t+4$ there is a decrease of 0.01%.

The interim multipliers represent the full effect in period $t+s$ of a sustained 1% increase in price in period *t*. Thus, if the price increases by 1% in period *t*, there is an immediate increase in the area planted of 0.77%. The total increase when period $t+1$ is reached is 0.56%, at period $t+2$ it is 0.55%, at period $t+3$ it is 1.14% and, after $t+4$ periods there is a 1.13% increase.

The different signs attached to the delay multipliers, the relatively large weight at $t-3$, and the interim multipliers that decrease and then increase are not realistic for this example. The pattern is likely attributable to imprecise estimation.

Exercise 9.14 (continued)

(b) Using the straight line formula the lag weights are

$$\begin{array}{ll} \beta_0 = \alpha_0 & i = 0 \\ \beta_1 = \alpha_0 + \alpha_1 & i = 1 \\ \beta_2 = \alpha_0 + 2\alpha_1 & i = 2 \\ \beta_3 = \alpha_0 + 3\alpha_1 & i = 3 \\ \beta_4 = \alpha_0 + 4\alpha_1 & i = 4 \end{array}$$

Substituting these weights into the original model gives

$$\begin{aligned} \ln(AREA_t) &= \alpha + \alpha_0 \ln(PRICE_t) + (\alpha_0 + \alpha_1)\ln(PRICE_{t-1}) + (\alpha_0 + 2\alpha_1)\ln(PRICE_{t-2}) \\ &\quad + (\alpha_0 + 3\alpha_1)\ln(PRICE_{t-3}) + (\alpha_0 + 4\alpha_1)\ln(PRICE_{t-4}) + e_t \\ &= \alpha + \alpha_0\left(\ln(PRICE_t) + \ln(PRICE_{t-1}) + \ln(PRICE_{t-2}) + \ln(PPRICE_{t-3}) + \ln(PRICE_{t-4})\right) \\ &\quad + \alpha_1\left(\ln(PRICE_{t-1}) + 2\ln(PRICE_{t-2}) + 3\ln(PRICE_{t-3}) + 4\ln(PRICE_{t-4})\right) + e_t \\ &= \alpha + \alpha_0 z_{t0} + \alpha_1 z_{t1} + e_t \end{aligned}$$

where

$$\begin{aligned} z_{t0} &= \ln(PRICE_t) + \ln(PRICE_{t-1}) + \ln(PRICE_{t-2}) + \ln(PPRICE_{t-3}) + \ln(PRICE_{t-4}) \\ z_{t1} &= \ln(PRICE_{t-1}) + 2\ln(PRICE_{t-2}) + 3\ln(PRICE_{t-3}) + 4\ln(PRICE_{t-4}) \end{aligned}$$

(c) The least square estimates for α_0 and α_1 are $a_0 = 0.4247$ and $a_1 = -0.0996$.

(d) The estimated lag weights are

$$\begin{aligned} \hat\beta_0 &= a_0 = 0.42467 \\ \hat\beta_1 &= a_0 + a_1 = 0.42467 - 0.09963 = 0.3250 \\ \hat\beta_2 &= a_0 + 2a_1 = 0.42467 - 2\times 0.09963 = 0.2254 \\ \hat\beta_3 &= a_0 + 3a_1 = 0.42467 - 3\times 0.09963 = 0.1258 \\ \hat\beta_4 &= a_0 + 4a_1 = 0.42467 - 4\times 0.09963 = 0.0261 \end{aligned}$$

These lag weights satisfy expectations as they are positive and diminish in magnitude as the lag length increases. They imply that the adjustment to a sustained price change takes place gradually, with the biggest impact being felt immediately and with a declining impact being felt in subsequent periods. The linear constraint has fixed the original problem where the signs and magnitudes of the lag weights varied unexpectedly.

Exercise 9.14 (continued)

(e) The table below reports the delay and interim elasticities under the new equation.

Lag	Delay Elasticities	Interim Elasticities
0	0.4247	0.4247
1	0.3250	0.7497
2	0.2254	0.9751
3	0.1258	1.1009
4	0.0261	1.1270

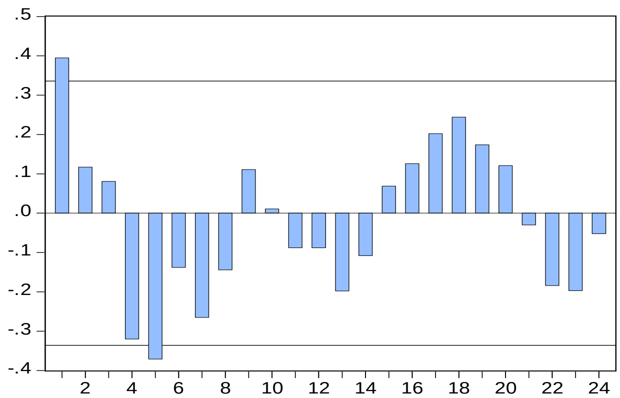
These delay multipliers are all positive and steadily decrease as the lag becomes more distant. This result, compared to the positive and negative multipliers obtained earlier, is a more reasonable one. It is interesting that the total effect, given by the 4-year interim multiplier, is almost identical in both cases, and the 3-year interim multipliers are very similar. The earlier interim multipliers are quite different however, with the restricted weights leading to a smaller initial impact.

EXERCISE 9.15

The least-squares estimated equation is

$$\widehat{\ln(AREA)} = 3.8933 + 0.7761\ln(PRICE)$$
$$(0.0613)\ (0.2771) \qquad \text{least squares se's}$$
$$(0.0624)\ (0.3782) \qquad \text{HAC se's}$$

(a) The correlogram for the residuals is



The significant bounds used are $\pm 1.96/\sqrt{34} = \pm 0.336$. Autocorrelations 1 and 5 are significantly different from zero.

(b) The null and alternative hypotheses are $H_0: \rho = 0$ and $H_0: \rho \neq 0$, and the test statistic is $LM = 5.4743$, yielding a *p*-value of 0.0193. Since the *p*-value is less than 0.05, we reject the null hypothesis and conclude that there is evidence of autocorrelation at the 5 percent significance level.

(c) The 95% confidence intervals are:

(i) Using least square standard errors

$$b_2 \pm t_{(0.975,32)} \times \text{se}(b_2) = 0.7761 \pm 2.0369 \times 0.2775 = (0.2109, 1.3413)$$

(ii) Using HAC standard errors

$$b_2 \pm t_{(0.975,32)} \times \text{se}(b_2) = 0.7761 \pm 2.0369 \times 0.3782 = (0.0057, 1.5465)$$

The wider interval under HAC standard errors shows that ignoring serially correlated errors gives an exaggerated impression about the precision of the least-squares estimated elasticity of supply.

(d) The estimated equation under the assumption of AR(1) errors is

$$\widehat{\ln(AREA_t)} = 3.8988 + 0.8884\ln(PRICE_t) \qquad e_t = 0.4221 e_{t-1} + v_t$$
$$(\text{se}) \qquad (0.0922)\ \ (0.2593) \qquad\qquad (0.1660)$$

Exercise 9.15(d) (continued)

The *t*-value for testing whether the estimate for ρ is significantly different from zero is $t = 0.4221/0.1660 = 2.542$, with a *p*-value of 0.0164. We conclude that $\hat{\rho}$ is significantly different from zero at a 5% level. A 95% confidence interval for the elasticity of supply is

$$b_2 \pm t_{(0.975,30)} \times \text{se}(b_2) = 0.8884 \pm 2.0423 \times 0.2593 = (0.3588, 1.4179)$$

This confidence interval is narrower than the one from HAC standard errors in part (c), reflecting the increased precision from recognizing the AR(1) error. It is also slightly narrower than the one from least squares, although we cannot infer much from this difference because the least squares standard errors are incorrect.

(e) We write the ARDL(1,1) model as

$$\ln(AREA_t) = \delta + \theta_1 \ln(AREA_{t-1}) + \delta_0 \ln(PRICE_t) + \delta_1 \ln(PRICE_{t-1}) + e_t$$

The estimated model is

$$\widehat{\ln(AREA_t)} = \underset{(0.6557)}{2.3662} + \underset{(0.1666)}{0.4043}\ln(AREA_{t-1}) + \underset{(0.2798)}{0.7766}\ln(PRICE_t) - \underset{(0.2966)}{0.6109}\ln(PRICE_{t-1})$$

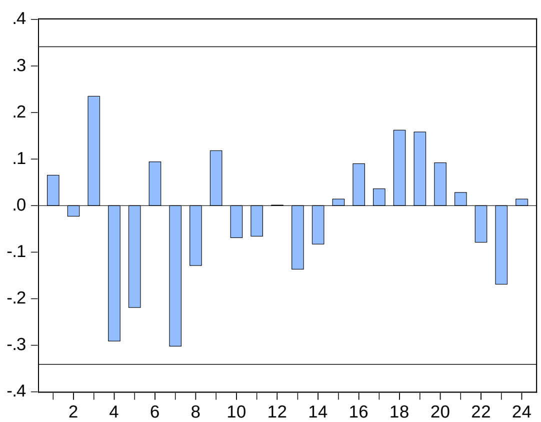
For this ARDL(1,1) model to be equal to the AR(1) model in part (d), we need to impose the restriction $\delta_1 = -\theta_1\delta_0$. Thus, we test $H_0: \delta_1 = -\theta_1\delta_0$ against $H_1: \delta_1 \neq -\theta_1\delta_0$.

The test value is

$$t = \frac{\hat{\delta}_1 - (-\hat{\theta}_1\hat{\delta}_0)}{\text{se}\left(\hat{\delta}_1 + \hat{\theta}_1\hat{\delta}_0\right)} = \frac{-0.6109 - (-0.4043 \times 0.7766)}{0.2812} = -1.0559$$

with *p*-value of 0.300. Thus, we fail to reject the null hypothesis and conclude that the two models are equivalent.

The correlogram presented below suggests the errors are not serially correlated. The significance bounds used are $\pm 1.96/\sqrt{33} = 0.3412$. The *LM* test with a *p*-value of 0.423 confirms this decision.



EXERCISE 9.16

(a) The forecast values for $\ln(AREA_t)$ in years $T+1$ and $T+2$ are 4.04899 and 3.82981, respectively. The corresponding forecasts for *AREA* using the natural predictor are

$$\widehat{AREA}_{T+1}^{n} = \exp(4.04899) = 57.34$$

$$\widehat{AREA}_{T+2}^{n} = \exp(3.82981) = 46.05$$

Using the corrected predictor, they are

$$\widehat{AREA}_{T+1}^{c} = \widehat{AREA}_{T+1}^{n} \exp\left(\hat{\sigma}^2/2\right) = 57.3395 \times \exp\left(0.284899^2/2\right) = 59.71$$

$$\widehat{AREA}_{T+2}^{c} = \widehat{AREA}_{T+2}^{n} \exp\left(\hat{\sigma}^2/2\right) = 46.0539 \times \exp\left(0.284899^2/2\right) = 47.96$$

(b) The standard errors of the forecast errors for $\ln(AREA)$ are

$$\text{se}(u_1) = \hat{\sigma} = 0.28490$$

$$\text{se}(u_2) = \hat{\sigma}\sqrt{1+\hat{\theta}_1^2} = 0.28490\sqrt{1+0.40428^2} = 0.3073$$

The 95% interval forecasts for $\ln(AREA)$ are:

$$\widehat{\ln(AREA)}_{T+1} \pm t_{(0.975,29)} \times \text{se}(u_1) = 4.04899 \pm 2.0452 \times 0.28490 = (3.4663, 4.63167)$$

$$\widehat{\ln(AREA)}_{T+2} \pm t_{(0.975,29)} \times \text{se}(u_2) = 3.82981 \pm 2.0452 \times 0.3073 = (3.20132, 4.45830)$$

The corresponding intervals for *AREA* obtained by taking the exponential of these results are:

For $T+1$: $(e^{3.46630}, e^{4.63167}) = (32.02, 102.69)$

For $T+2$: $(e^{3.20132}, e^{4.45830}) = (24.56, 86.34)$

(c) The lag and interim elasticities are reported in the table below:

Lag	β_s	Lag Elasticities	Interim Elasticities
0	$\beta_0 = \delta_0$	0.7766	0.7766
1	$\beta_1 = \delta_1 + \theta_1\beta_0$	–0.2969	0.4797
2	$\beta_2 = \theta_1\beta_1$	–0.1200	0.3597
3	$\beta_3 = \theta_1\beta_2$	–0.0485	0.3112
4	$\beta_4 = \theta_1\beta_3$	–0.0196	0.2916

The lag elasticities show the percentage change in area sown in the current and future periods when price increases by 1% and then returns to its original level. The interim elasticities show the percentage change in area sown in the current and future periods when price increases by 1% and is maintained at the new level.

Exercise 9.16 (continued)

(d) The total elasticity is given by

$$\sum_{j=0}^{\infty}\beta_j = \frac{\hat{\delta}_0+\hat{\delta}_1}{1-\hat{\theta}_1} = \frac{0.77663-0.61086}{1-0.40428} = 0.2783$$

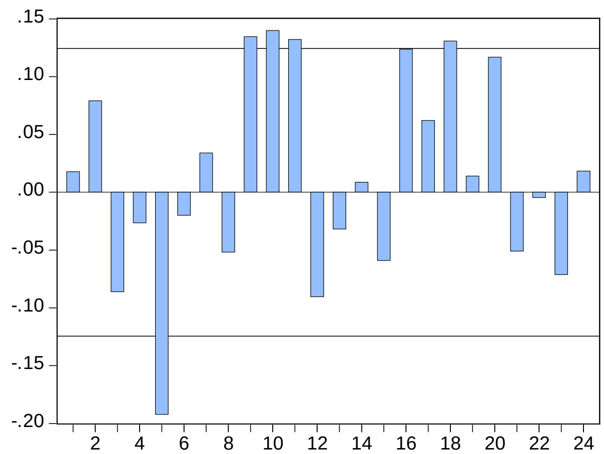
If price is increased by 1% and then maintained at its new level, then area sown will be 0.28% higher when the new equilibrium is reached.

EXERCISE 9.17

(a) The estimated model is

$$\hat{G}_t = 0.7316 + 0.4249G_{t-1} + 0.1332G_{t-2}$$
$$\text{(se)} \qquad (0.0633) \qquad (0.0636)$$

The correlogram of the residuals is shown below. The significance bounds are drawn at $\pm 1.96/\sqrt{248} = \pm 0.1245$. There are a few significant correlations at long lags (specifically at lag orders 5, 9, 10, 11 and 19), but apart from lag 5, they are relatively small.



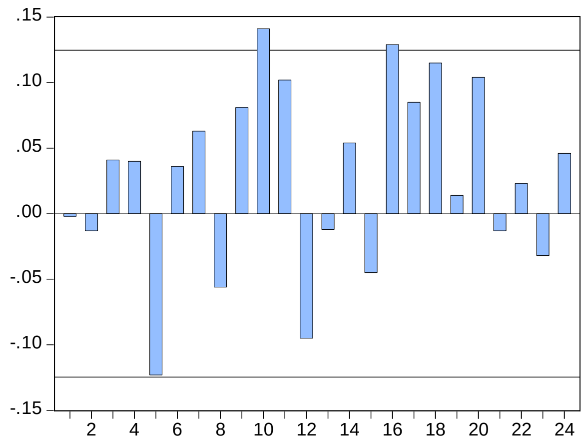
The test value for the *LM* test with two lags is $LM = 7.405$ and the corresponding *p*-value is 0.0247. Since the *p*-value is less than 0.05, we reject the null hypothesis that autocorrelation does not exist and conclude that there is evidence of autocorrelation at the 5% significance level.

(b) The estimated model is

$$\hat{G}_t = 0.8386 + 0.4432G_{t-1} + 0.1995G_{t-2} - 0.1533G_{t+3}$$
$$\text{(se)} \qquad (0.0627) \qquad (0.0676) \qquad (0.0635)$$

The correlogram of the residuals is shown below. The significance bounds are drawn at $\pm 1.96/\sqrt{247} = \pm 0.1247$. There are two significant correlations at the long lags of 10 and 16, but they are relatively small.

Exercise 9.17(b) (continued)



The test value for the *LM* test with two lags is $LM = 0.916$ and the corresponding *p*-value is 0.632. Since the *p*-value is greater than 0.05, we do not reject the null hypothesis of no autocorrelation; we conclude there is no evidence of autocorrelation at the 5% significance level.

(c) The results are presented in the table below. The *t*-value used to compute the forecast intervals was $t_{(0.975,247)} = 1.9696$.

Period	Forecasts	Standard Errors	Forecast Intervals	Actual Figures
2009Q4	1.3371	0.9899	(–0.613, 3.287)	1.15
2010Q1	1.6214	1.0827	(–0.511, 3.754)	1.18
2010Q2	1.7014	1.1515	(–0.567, 3.969)	0.914

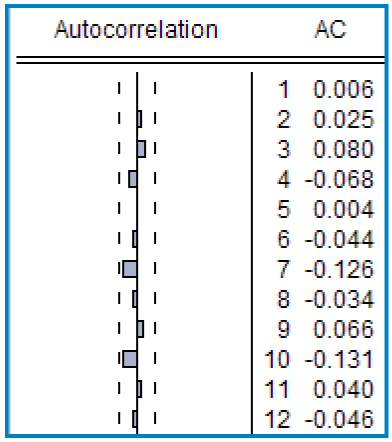
The actual figures fall within the intervals.

EXERCISE 9.18

(a) The estimated AR(2) model is

$$\widehat{SALES}_t = 11.614 + 0.3946 SALES_{t-1} + 0.1926 SALES_{t-2}$$

The correlogram below shows no evidence of serially correlated errors. *LM* tests at various lags similarly show no evidence of serial correlation.



(b) to (e) The following table contains the one-period ahead forecasts and forecast errors for both the AR(2) and exponential smoothing models after re-estimating both models for each period. Both methods tend to over or under forecast at the same time. In two periods the absolute value of the forecast error is lower for exponential smoothing and, in the other two periods, the forecast errors for the AR(2) model are smaller.

Forecast Period	Observed Value	AR(2) Forecast	Exp. Sm. Forecast	AR(2) Forecast Error	Exp. Sm. Forecast Error
154	28.963	28.2011	28.3925	–0.7619	–0.5705
155	26.430	28.5364	28.6896	2.1064	2.2596
156	25.900	27.6452	27.5187	1.7452	1.6187
157	28.020	26.9021	26.6542	–1.1179	–1.3658

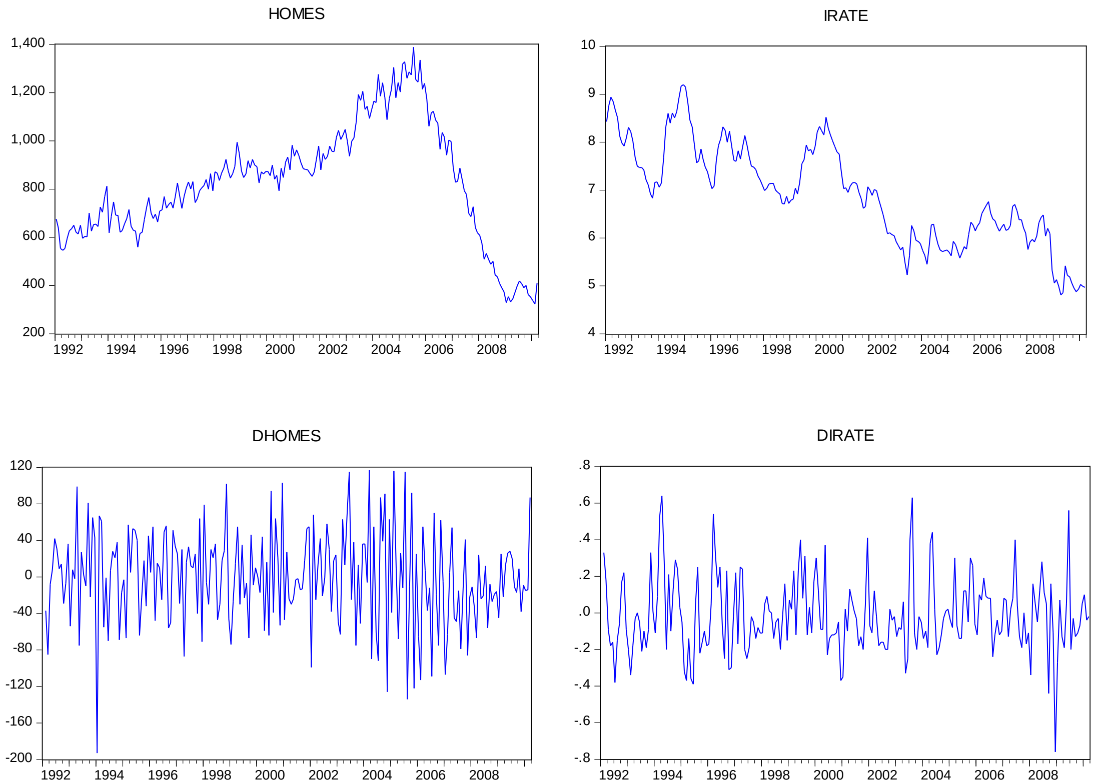
(f) The mean-square prediction errors for each set of forecasts is

$$\text{MSPE}\left[\text{AR}(2)\right] = 2.328 \qquad\qquad \text{MSPE}\left[\text{Exp. Sm.}\right] = 2.479$$

Using this criterion, the AR(2) model has led to the more accurate forecasts.

EXERCISE 9.19

(a) The four graphs are as follows



The series for *HOMES* and *IRATE* exhibit trends. *HOMES* trends upwards until 2005 and then trends downwards. *IRATE* wanders up and down, but, overall, trends downwards. On the other hand, the series for *DHOMES* and *DIRATE* do not appear to be trending but fluctuate around constant means.

(b) The estimated model is

$$\widehat{DHOMES}_t = -2.4912 - 0.3350 DHOMES_{t-1} - 50.7878 DIRATE_{t-1} - 28.8550 DIRATE_{t-2}$$
$$\text{(se)} \quad (3.3327) \quad (0.0649) \qquad (16.9283) \qquad (17.1278)$$

All estimates except for the intercept and $DIRATE_{t-2}$ are significantly different from zero at the 1% level.

(c) The test statistic for testing $H_0 : \theta_1\delta_1 = -\delta_2$ against the alternative $H_0 : \theta_1\delta_1 \neq -\delta_2$ is

$$t = \frac{\hat{\theta}_1\hat{\delta}_1 + \hat{\delta}_2}{\text{se}\left(\hat{\theta}_1\hat{\delta}_1 + \hat{\delta}_2\right)} = \frac{-11.8408}{19.2621} = -0.615$$

The 1% critical value is $t_{(0.995,212)} = \pm 2.599$, and the corresponding *p*-value is 0.5394. Since the *p*-value is greater than 0.05, we do not reject the null hypothesis, and conclude that the data are compatible with the hypothesis $H_0 : \theta_1\delta_1 = -\delta_2$.

Exercise 9.19(c) (continued)

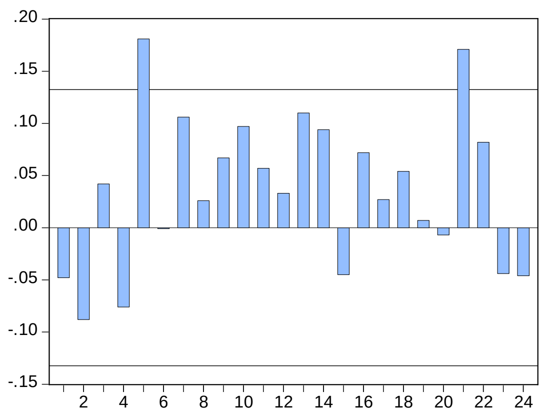
If H_0 is true, the model can be written as

$$DHOMES_t = \delta + \theta_1 DHOMES_{t-1} + \delta_1 DIRATE_{t-1} - \theta_1\delta_1 DIRATE_{t-2} + v_t$$

which is equivalent to the AR(1) error model

$$DHOMES_t = \delta + \delta_1 DIRATE_{t-1} + e_t \qquad e_t = \theta_1 e_{t-1} + v_t$$

(d) The correlogram of residuals is displayed below. Using the significance bounds $\pm 1.96/\sqrt{216} = \pm 0.133$ suggests that there are two significant correlations at lags at 5 and 21.



(e) The *LM* χ^2 test value with two lagged errors is 4.8536 with a corresponding *p*-value of 0.0883. At the 1% and 5% significance levels, we fail to reject the null hypothesis that the errors are serially uncorrelated. If we used a 10% significance level, we would conclude there is evidence of serial correlation.

(f) The estimated ARDL model is

$$\widehat{DHOMES}_t = -2.9215 - 0.3073 DHOMES_{t-1} + 0.2069 DHOMES_{t-5}$$
$$\text{(se)} \qquad (3.2841 \quad (0.0635) \qquad\qquad (0.0633)$$

$$- 64.324 DIRATE_{t-1} - 46.631 DIRATE_{t-3}$$
$$(15.974) \qquad\qquad (16.094)$$

Using the significance bounds $\pm 1.96/\sqrt{213} = \pm 0.1343$, the correlogram of residuals for this model does not suggest any autocorrelation except at lag 21 which is sufficiently distant to ignore. Also, the AIC and SC values for this model are slightly lower than those for the model in (9.92). And there are no coefficients (except the constant) that are not significantly different from zero. In (9.92) the coefficient of $DIRATE_{t-2}$ was not significant. These four things – the lack of serial correlation, the improved AIC and SC, the exclusion of a lag with an insignificant coefficient, and the inclusion of significant lags, lead us to conclude the new model is an improvement.

EXERCISE 9.20

(a) Recognizing that $DHOMES_t = HOMES_t - HOMES_{t-1}$, we can write the equation as

$$HOMES_t - HOMES_{t-1} = \delta + \theta_1(HOMES_{t-1} - HOMES_{t-2}) + \theta_5 DHOMES_{t-5} + \delta_0 DIRATE_{t-1} + \delta_3 DIRATE_{t-3} + v_t$$

Rearranging yields

$$\begin{aligned} HOMES_t &= \delta + \theta_1 HOMES_{t-1} - \theta_1 HOMES_{t-2} + HOMES_{t-1} + \theta_5 DHOMES_{t-5} \\ &\quad + \delta_0 DIRATE_{t-1} + \delta_3 DIRATE_{t-3} + v_t \\ &= \delta + (\theta_1 + 1)HOMES_{t-1} - \theta_1 HOMES_{t-2} + \theta_5 DHOMES_{t-5} \\ &\quad + \delta_0 DIRATE_{t-1} + \delta_3 DIRATE_{t-3} + v_t \end{aligned}$$

(b) The estimated equation is

$$\begin{aligned} \widehat{DHOMES}_t &= \underset{(3.2841}{-2.9215} - \underset{(0.0635)}{0.3073} DHOMES_{t-1} + \underset{(0.0633)}{0.2069} DHOMES_{t-5} \\ &\quad - \underset{(15.974)}{64.324} DIRATE_{t-1} - \underset{(16.094)}{46.631} DIRATE_{t-3} \end{aligned}$$

(se)

The equation to be used for forecasting is

$$\begin{aligned} \widehat{HOMES}_t &= -2.9215 + 0.6927 HOMES_{t-1} + 0.3073 HOMES_{t-2} + 0.2069 DHOMES_{t-5} \\ &\quad - 64.324 DIRATE_{t-1} - 46.631 DIRATE_{t-3} \end{aligned}$$

The forecasts for April, May and June 2010 are

$$\begin{aligned} \widehat{HOMES}_{APRIL} &= -2.9215 + 0.6927 \times 411 + 0.3073 \times 324 + 0.2069 \times (-38) \\ &\quad - 64.324 \times (-0.02) - 46.631 \times (0.1) \\ &= 370 \end{aligned}$$

$$\begin{aligned} \widehat{HOMES}_{MAY} &= -2.9215 + 0.6927 \times 370 + 0.3073 \times 411 + 0.2069 \times (-9) \\ &\quad - 64.324 \times (0.0) - 46.631 \times (-0.04) \\ &= 380 \end{aligned}$$

$$\begin{aligned} \widehat{HOMES}_{JUNE} &= -2.9215 + 0.6927 \times 380 + 0.3073 \times 370 + 0.2069 \times (-15) \\ &\quad - 64.324 \times (0.0) - 46.631 \times (-0.02) \\ &= 372 \end{aligned}$$

Exercise 9.20 (continued)

(c) The standard errors of the forecast errors are

$$\text{se}(u_1) = \hat{\sigma}_v = 47.502$$

$$\text{se}(u_2) = \hat{\sigma}_v\left(1+\left(\hat{\theta}_1+1\right)^2\right)^{1/2} = 47.502\left(1+0.6927^2\right)^{1/2} = 57.785$$

$$\text{se}(u_3) = \hat{\sigma}_v\left(\left(\left(\hat{\theta}_1+1\right)^2-\hat{\theta}_1\right)^2+\left(\hat{\theta}_1+1\right)^2+1\right)^{1/2}$$

$$= 47.502\left(\left(0.6927^2+0.3073\right)^2+0.6927^2+1\right)^{1/2} = 68.827$$

The three forecast intervals are

$$HOMES_{APRIL} \pm t_{(0.995,208)}\text{se}(u_1) = 370 \pm 2.600 \times 47.502 = (247{,}493)$$

$$HOMES_{MAY} \pm t_{(0.995,208)}\text{se}(u_2) = 380 \pm 2.600 \times 57.785 = (230{,}530)$$

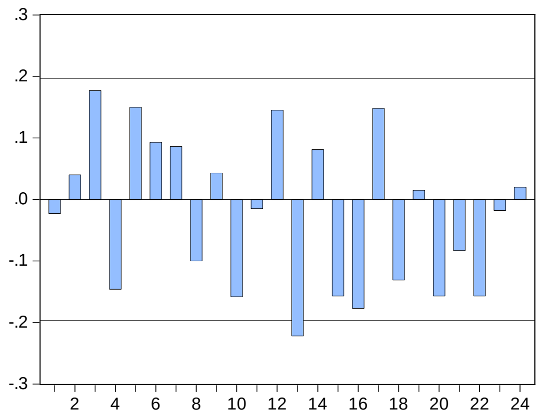
$$HOMES_{JUNE} \pm t_{(0.995,208)}\text{se}(u_3) = 372 \pm 2.600 \times 68.827 = (193{,}551)$$

EXERCISE 9.21

(a) The estimated equation is

$$\widehat{DU}_t = 0.3870 + 0.3501 DU_{t-1} - 0.1841 G_t - 0.0992 G_{t-1}$$
$$(\text{se}) \quad (0.0587) \; (0.0846) \qquad (0.0307) \quad (0.0368)$$

(b) The residual correlogram for lags up to 24 is presented below. No serious problems of error autocorrelation are apparent. The only slightly significant autocorrelation is at lag 13. The significance bounds used are $\pm 1.96 \big/ \sqrt{96} = \pm 0.2$.



(c) The following table gives the *LM* test results for lags up to 4. In all cases the *p*-values are greater than 0.1. Using any significance level up to 10%, we conclude there is no evidence of serial correlation in the errors.

Lags	χ^2 -value	*p*-value
1	0.170	0.680
2	0.271	0.873
3	3.896	0.273
4	6.141	0.189

(d) (i) The estimated model with DU_{t-2} added is

$$\widehat{DU}_t = 0.3742 + 0.3230 DU_{t-1} + 0.0458 DU_{t-2} - 0.1823 G_t - 0.0971 G_{t-1}$$
$$(\text{se}) \quad (0.0586) \; (0.1060) \qquad (0.0990) \qquad (0.0314) \quad (0.0374)$$

(ii) The estimated model with G_{t-2} added is

$$\widehat{DU}_t = 0.3876 + 0.3391 DU_{t-1} - 0.1832 G_t - 0.0991 G_{t-1} - 0.0082 G_{t-2}$$
$$(\text{se}) \quad (0.0720) \; (0.0979) \qquad (0.0311) \quad (0.0370) \quad (0.0360)$$

Exercise 9.21(d) (continued)

(iii) The estimated model with both DU_{t-2} and G_{t-2} added is

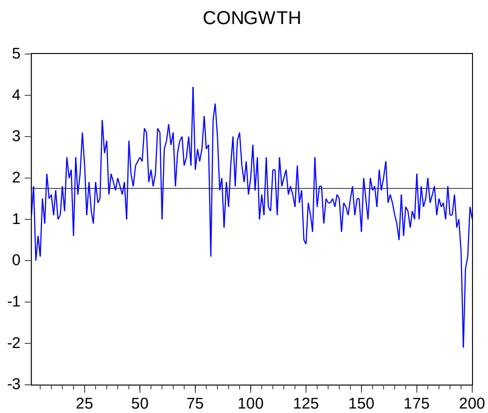
$$\widehat{DU}_t = 0.3778 + 0.3208DU_{t-1} + 0.0429DU_{t-2} - 0.1821G_t - 0.0970G_{t-1} - 0.0030G_{t-2}$$
$$\text{(se)} \quad (0.0758)\ (0.1103) \qquad (0.1065) \qquad (0.0316) \quad (0.0376) \quad (0.0389)$$

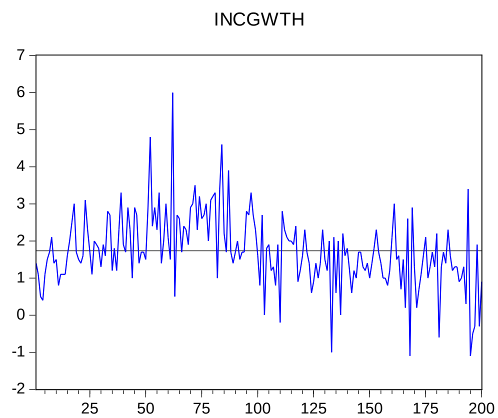
For all three estimated equations, the coefficient estimates found to be significant at the 5% percent level were those for DU_{t-1}, G_t and G_{t-1}. Whenever DU_{t-2} or G_{t-2} or both were added to the original equation, their estimated coefficients were insignificant.

(e) In parts (b) and (c), we concluded that error autocorrelation is not significant. Both the correlogram and the *LM* tests supported such a conclusion. Also, in part (d), adding DU_{t-2} and/or G_{t-2} did not improve the model. Their coefficients were not significantly different from zero. For these reasons, we conclude that the Okun's law specification given in (9.59) is satisfactory.

EXERCISE 9.22

(a) The times series graphs for *CONGWTH* and *INCGWTH* follow. While both exhibit considerable serial correlation, they do appear to fluctuate around their respective constant means.



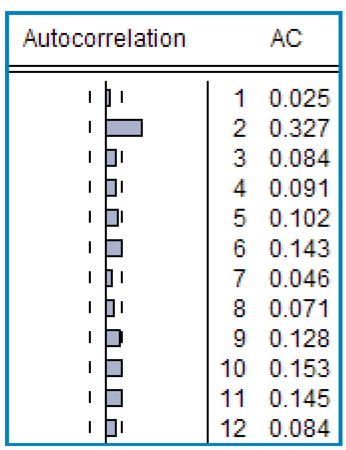


(b) The estimated model is

$$\widehat{CONGWTH}_t = 0.9738 + 0.4496\, INCGWTH_t$$
$$\text{(se)} \qquad (0.0996) \quad (0.0497)$$

The estimate $\hat{\delta}_0 = 0.4496$ suggests that a 1% increase in the income growth rate increases the consumption growth rate by 0.46%.

The correlogram below shows significant serial correlation in the errors at lag 2. There is also some slight evidence of serially correlated errors at some longer lags (6, 10 and 11). For the *LM* test, we find $\chi^2_{(2)} = 21.93$, with a *p*-value less than 0.00005 – a strong indication of serially correlated errors.



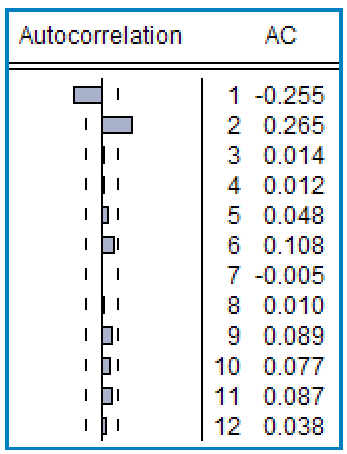
Exercise 9.22 (continued)

(c) The estimated model after adding $CONGWTH_{t-1}$ is

$$\widehat{CONGWTH}_t = 0.6716 + 0.2714\,CONGWTH_{t-1} + 0.3501\,INCGWTH_t$$
$$\text{(se)} \quad (0.1188) \quad (0.0635) \qquad\qquad (0.0530)$$

The estimate $\hat{\theta}_1 = 0.2714$ is significantly different from zero at the 5% significance level $(t = 4.27)$. The AIC and SC values for this model are -0.1250 and -0.0750, respectively, compared to -0.0452 and -0.0119 for the model discussed in part (b); the lower values suggest this model is an improvement. (The corresponding EViews AIC and SC values are 2.0197 and 2.0697 for the above model, and 2.0995 and 2.1328 for the model in part (b). See footnote 12 on page 366 of *POE4*.)

However, the correlogram of the residuals displayed below suggests there is still significant serial correlation in the errors at lags 1 and 2. The *LM* test also rejects the null hypothesis that the errors are not serially correlated $\left[\chi^2_{(2)} = 34.45,\ p\text{-value} = 0.0000\right]$.



We conclude that the model is an improvement over that in part (b), but it is still not satisfactory.

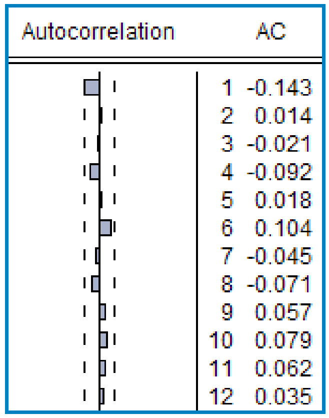
(d) The estimated model after adding $CONGWTH_{t-2}$ is

$$\widehat{CONGWTH}_t = 0.4249 + 0.1594\,CONGWTH_{t-1} + 0.2806\,CONGWTH_{t-2} + 0.3216\,INCGWTH_t$$
$$\text{(se)} \quad (0.1254) \quad (0.0653) \qquad\qquad (0.0615) \qquad\qquad (0.0509)$$

The estimate $\hat{\theta}_2 = 0.2806$ is significantly different from zero at the 5% significance level $(t = 4.57)$. The AIC and SC values for this model are -0.2174 and -0.1508, respectively, compared to -0.1250 and -0.0750 for the model discussed in part (c); the lower values suggest this model is an improvement. (The corresponding EViews AIC and SC values are 1.9273 and 1.9940 for the above model, and 2.0197 and 2.0697 for the model in part (c).)

Exercise 9.22(d) (continued)

In the correlogram of the residuals given below, the first autocorrelation is significantly different from zero, although its magnitude, $r_1 = -0.143$, is not large. The *LM* test gives a $\chi^2_{(2)}$ value of 15.46, with corresponding *p*-value = 0.0004, suggesting that serially correlated errors are still a problem.

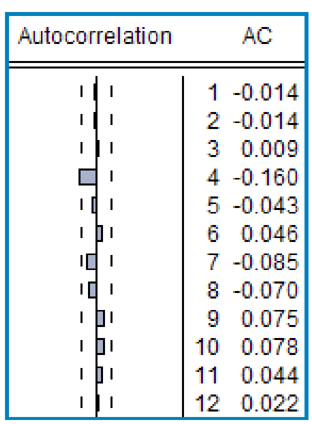


We conclude that adding $CONGWTH_{t-2}$ has improved the model, but the existence of serially correlated errors means that it is still not satisfactory.

(e) The estimated model after adding $INCGWTH_{t-1}$ is

$$\begin{aligned} \widehat{CONGWTH}_t &= \underset{(0.1219)}{0.3320} + \underset{(0.0699)}{0.0233}CONGWTH_{t-1} + \underset{(0.0610)}{0.2101}CONGWTH_{t-2} \quad \text{(se)} \\ &\quad + \underset{(0.0491)}{0.3493}INCGWTH_t + \underset{(0.0539)}{0.2334}INCGWTH_{t-1} \end{aligned}$$

The estimate $\hat{\delta}_1 = 0.2334$ is significantly different from zero at the 5% significance level $(t = 4.33)$. The AIC and SC values for this model are -0.3004 -0.2170, respectively, lower than that for the model discussed in part (d). (The EViews values are 1.8444 and 1.9277.)



Exercise 9.22(e) (continued)

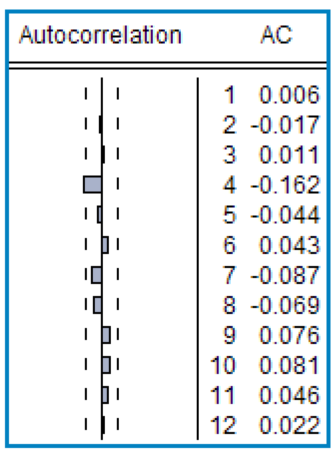
The correlogram above shows a significant but not large autocorrelation at lag 4. However, performing the *LM* test with 2 and 4 lags gives $\chi^2_{(2)} = 0.220$ (p-value $= 0.8957$) and $\chi^2_{(4)} = 7.204$ (p-value $= 0.1255$) suggesting serial correlation is no longer a problem. We conclude that this model is an improvement over that in part (d).

(f) Adding $CONGWTH_{t-3}$ or $INCGWTH_{t-2}$ did not improve the model in part (e). In both cases, the extra coefficient was not significantly different from zero, and the AIC and SC values increased. Furthermore, the correlgrams and *LM* statistics led to the same conclusion about serially correlated errors as was reached in part (e).

(g) Dropping $CONGWTH_{t-1}$ from the model in part (e) and re-estimating gives

$$\begin{aligned}
\widehat{CONGWTH}_t &= \underset{(0.1188)}{0.3407} + \underset{(0.0596)}{0.2143}\,CONGWTH_{t-2} + \underset{(0.0454)}{0.3555}\,INCGWTH_t \quad \text{(se)} \\
&\quad + \underset{(0.0480)}{0.2414}\,INCGWTH_{t-1}
\end{aligned}$$

The AIC and SC values are -0.3099 and -0.2433, respectively – values that are lower than those for the model estimated in part (e). (EViews values are 1.8348 and 1.9015.) The correlogram below shows some evidence of serially correlated errors at lag 4, but the *LM* test values, $\chi^2_{(2)} = 0.145$ (p-value $= 0.9301$), and $\chi^2_{(4)} = 6.593$ (p-value $= 0.1591$) do not suggest serial correlation is a problem.



EXERCISE 9.23

The estimated equation is

$$\widehat{CONGWTH}_t = \underset{(0.1188)}{0.3407} + \underset{(0.0596)}{0.2143} CONGWTH_{t-2} + \underset{(0.0454)}{0.3555} INCGWTH_t + \underset{(0.0480)}{0.2414} INCGWTH_{t-1}$$
$$\text{(se)}$$

The forecasts, the standard errors of the forecasts and the forecast intervals are given in the following table. The intervals are relatively wide, showing that there is a great deal of uncertainty about future consumption growth.

Period	Forecasts	Standard Errors	Forecast Intervals
2010Q1	1.0499	0.5995	(0.059, 2.041)
2010Q2	0.9842	0.5995	(–0.007, 1.975)
2010Q3	1.0077	0.6132	(–0.006, 2.021)

Using C as an abbreviation for *CONGWTH* and I as an abbreviation for *INCGWTH*, the forecasts are obtained as follows

$$\begin{aligned}\hat{C}_{2010Q1} &= 0.34074 + 0.21428 C_{2009Q3} + 0.35545 I_{2010Q1} + 0.24144 I_{2009Q4} \\ &= 0.34074 + 0.21428 \times 1.3 + 0.35545 \times 0.6 + 0.24144 \times 0.9 \\ &= 1.04987\end{aligned}$$

$$\begin{aligned}\hat{C}_{2010Q2} &= 0.34074 + 0.21428 C_{2009Q4} + 0.35545 I_{2010Q2} + 0.24144 I_{2010Q1} \\ &= 0.34074 + 0.21428 \times 1.0 + 0.35545 \times 0.8 + 0.24144 \times 0.6 \\ &= 0.98424\end{aligned}$$

$$\begin{aligned}\hat{C}_{2010Q3} &= 0.34074 + 0.21428 \hat{C}_{2010Q1} + 0.35545 I_{2010Q3} + 0.24144 I_{2010Q2} \\ &= 0.34074 + 0.21428 \times 1.04987 + 0.35545 \times 0.7 + 0.24144 \times 0.8 \\ &= 1.00767\end{aligned}$$

The standard errors of the forecast errors are

$$\hat{\sigma}_1 = \hat{\sigma}_v = 0.59954$$

$$\hat{\sigma}_2 = \hat{\sigma}_v = 0.59954$$

$$\hat{\sigma}_3 = \hat{\sigma}_v\left(1+\theta_2^2\right) = 0.59954 \times \sqrt{1+0.214277^2} = 0.61315$$

The forecast intervals are given by $\hat{C}_j \pm t_{(0.95,193)} \hat{\sigma}_j$ where $t_{(0.95,193)} = 1.6528$.

EXERCISE 9.24

(a) The model in (9.94), without the error term, is given by

$$CONGWTH_t = \delta + \theta_2 CONGWTH_{t-2} + \delta_0 INCGWTH_t + \delta_1 INCGWTH_{t-1}$$

It can be written in lag operator notation as

$$(1-\theta_2 L^2)\, CONGWTH_t = \delta + (\delta_0 + \delta_1 L)\, INCGWTH_t$$

or

$$CONGWTH_t = (1-\theta_2 L^2)^{-1}\delta + (1-\theta_2 L^2)^{-1}(\delta_0 + \delta_1 L)\, INCGWTH_t$$

Equating this equation with the infinite lag representation

$$CONGWTH_t = \alpha + (\beta_0 + \beta_1 L + \beta_2 L^2 + \beta_3 L^3 + \beta_4 L^4 + \cdots + \beta_s L^s) INCGWTH_t$$

implies

$$(1-\theta_2 L^2)^{-1}(\delta_0 + \delta_1 L) = \beta_0 + \beta_1 L + \beta_2 L^2 + \beta_3 L^3 + \beta_4 L^4 + \cdots$$

Thus,

$$\begin{aligned}\delta_0 + \delta_1 L &= (1-\theta_2 L^2)(\beta_0 + \beta_1 L + \beta_2 L^2 + \beta_3 L^3 + \beta_4 L^4 + \cdots \\ &= \beta_0 + \beta_1 L + \beta_2 L^2 + \beta_3 L^3 + \beta_4 L^4 + \cdots \\ &\qquad - \theta_2\beta_0 L^2 - \theta_2\beta_1 L^3 - \theta_2\beta_2 L^4 - \cdots\end{aligned}$$

giving

$$\beta_0 = \delta_0 \qquad \beta_1 = \delta_1 \qquad \beta_2 = \theta_2\beta_0 \qquad \beta_3 = \theta_2\beta_1 \qquad \beta_s = \theta_2\beta_{s-2} \qquad s \ge 2$$

(b) The estimated multipliers are presented in the table below.

Lag	Delay Multiplier	Interim Multiplier
1	0.3555	0.3555
2	0.2414	0.5969
3	0.0762	0.6731

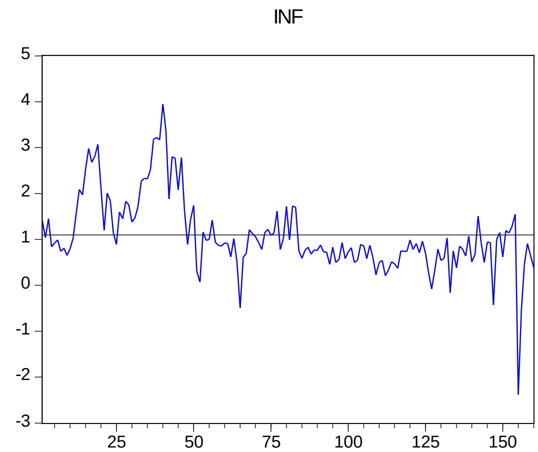
The total multiplier estimate is

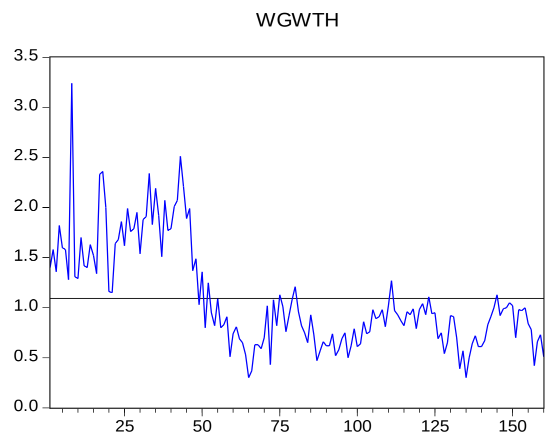
$$\sum_{j=0}^{\infty}\hat{\beta}_j = \frac{\hat{\delta}_0 + \hat{\delta}_1}{1-\hat{\theta}_2} = \frac{0.35545 + 0.24144}{1-0.21428} = 0.7597$$

The delay multipliers show that if the growth rate of income is increased by 1% and then returned to its original level, then the growth rate of consumption will increase by 0.36% in the current quarter, by 0.24% in the next quarter and by 0.08% in the quarter after that. The interim multipliers show that if the growth rate of income is increased by 1% and then maintained at this new level, then the growth rate of consumption will increase by 0.36% in the current quarter, by 0.60% in the next quarter and by 0.67% in the quarter after that. When a new equilibrium is reached consumption growth will have increased by the total multiplier, namely 0.86%.

EXERCISE 9.25

(a)





Neither of the series appears to be trending over the given time period. However, an assumption of a constant mean over the whole period could be questioned for both series. Both appear to have a higher mean for the earlier period, up to about observation 50 (1982Q3), and a lower mean after that.

(b) The estimated equation is

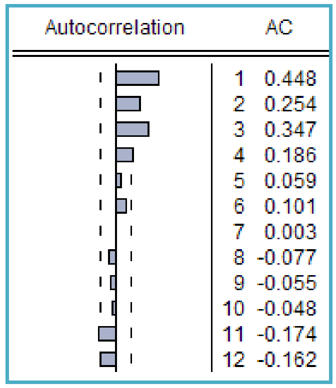
$$\begin{array}{lll} \widehat{INF}_t = & -0.0215 + & 1.0254\,WGWTH_t \\ \text{(se)} & & (0.0942) \end{array}$$

The coefficient of *WGWTH* suggests that an increase in wage growth of 1% results in a 1.025 percent increase in the inflation rate.

The residual correlogram that follows shows significant autocorrelations at lags 1, 2, 3 and 4. The significant bounds are $\pm 2/\sqrt{160} = \pm 0.158$.

The *LM* test for AR(2) errors yields a test value of $LM = 33.56$, with corresponding *p*-value of 0.0000. Thus, we conclude that the errors are autocorrelated.

Exercise 9.25(b) (continued)



(c) The estimated equation is

$$\widehat{INF}_t = -0.0352 + 0.5405 INF_{t-1} + 0.4914 WGWTH_t$$
$$\text{(se)} \qquad\qquad (0.0652) \qquad (0.1021)$$

To find the impact and total multipliers, we need to rewrite the model in terms of the infinite distributed lag representation

$$INF_t = \alpha + \sum_{s=0}^{\infty} \beta_s WGWTH_{t-s} + e_t$$

Working in this direction, we have

$$INF_t = (1-\theta_1)^{-1}\delta + (1-\theta_1 L)^{-1}\delta_0 WGWTH_t + e_t$$
$$= \alpha + (\beta_0 + \beta_1 L + \beta_2 L^2 + \beta_3 L^3 + \cdots) WGWTH_t + e_t$$

and

$$(1-\theta_1 L)^{-1}\delta_0 = (\beta_0 + \beta_1 L + \beta_2 L^2 + \beta_3 L^3 + \cdots)$$

or,

$$\delta_0 = (1-\theta_1 L)(\beta_0 + \beta_1 L + \beta_2 L^2 + \beta_3 L^3 + \cdots)$$
$$= \left(\beta_0 + \beta_1 L + \beta_2 L^2 + \beta_3 L^3 + \cdots\right) + \left(-\beta_0\theta_1 L - \beta_1\theta_1 L^2 - \beta_2\theta_1 L^3 - \cdots\right)$$
$$= \beta_0 + \left(\beta_1 - \beta_0\theta_1\right)L + \left(\beta_2 - \beta_1\theta_1\right)L^2 + \left(\beta_3 - \beta_2\theta_1\right)L^3 + \cdots$$

Equating coefficients of equal powers in the lag operator gives

$$\delta_0 = \beta_0 \qquad \beta_j - \theta_1\beta_{j-1} = 0 \ \text{ for } \ j \ge 1$$

Thus, the impact multiplier is given by $\hat{\beta}_0 = \hat{\delta}_0 = 0.4914$.

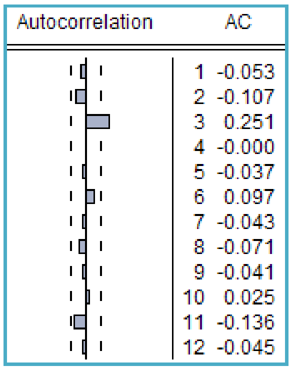
And the total multiplier is given by

$$\sum_{j=0}^{\infty} \beta_j = \beta_0 + \beta_0\theta_1 + \beta_0\theta_1^2 + \beta_0\theta_1^3 + \cdots = \frac{\beta_0}{1-\theta_1} = \frac{0.4914}{1-0.5405} = 1.069$$

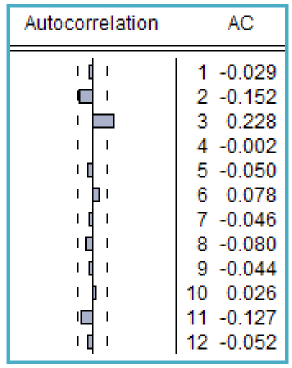
In part (b) the total multiplier and the impact multiplier were both equal to 1.0254. Introducing a lagged value of INF has led to an impact multiplier that is much less, but a total multiplier that is approximately the same.

Exercise 9.25 (continued)

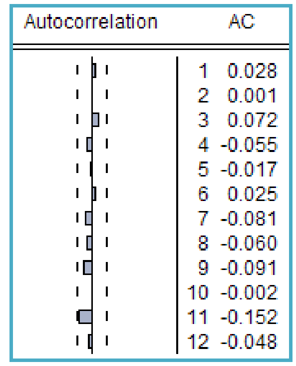
(d)&(e) The residual correlograms for models with INF_{t-1} added, and then INF_{t-2}, and then INF_{t-3}, and the results of the various *LM* tests, are given below.



(INF_{t-1})



(INF_{t-1}, INF_{t-2})



$(INF_{t-1}, INF_{t-2}, INF_{t-3})$

	LM Test and *p* Values			
	Lags included in test			
Lags included in equation	2		3	
	LM value	*p* value	*LM* value	*p* value
(INF_{t-1})	6.439	0.040	12.246	0.007
(INF_{t-1}, INF_{t-2})	8.137	0.017	12.064	0.007
$(INF_{t-1}, INF_{t-2}, INF_{t-3})$	1.143	0.565	2.342	0.505

After adding INF_{t-1}, a significant autocorrelation remains at lag 3, but those at lags 1, 2 and 4 are no longer significant. The *LM* tests confirm that serial correlation remains, with χ^2 values that are significant at the 5% level for error processes involving 2 and 3 lags.

Adding INF_{t-2} does nothing to improve the situation. The significant autocorrelation at lag 3 remains and the *LM* test values do not improve.

Adding INF_{t-3} eliminates the serial correlation at all lags. There are no significant autocorrelations at the 5% level and the *p*-values for the *LM* test for processes involving 2 and 3 lags are 0.565 and 0.505, respectively.

(f) The estimated equation is

$$\widehat{INF}_t = -0.0504 + 0.4537 INF_{t-1} + 0.2174 INF_{t-3} + 0.3728 WGWTH_t$$
$$\text{(se)} \qquad (0.0691) \qquad (0.0676) \qquad (0.1068)$$

In the model $INF_t = \delta + \theta_1 INF_{t-1} + \theta_2 INF_{t-2} + \theta_3 INF_{t-3} + \delta_0 WGWTH_t + v_t$, the coefficient $\hat{\theta}_2$ was not significantly different from zero (*p*-value = 0.4497), and so it was worth considering dropping it. Omitting it led to a fall in the SC of 0.028 and a fall in the AIC of 0.009, and did not introduce any serial correlation in the errors. Adding $WGWTH_{t-1}$ did not improve the equation. Its coefficient was not significantly different from zero and the AIC and SC both increased.

EXERCISE 9.26

The estimated equation used for forecasting is given by:

$$\widehat{INF}_t = -0.0504 + 0.4537 INF_{t-1} + 0.2174 INF_{t-3} + 0.3728 WGWTH_t$$

The forecast values are

$$\begin{aligned}
\widehat{INF}_{2010Q2} &= -0.0504 + 0.4537 INF_{2010Q1} + 0.2174 INF_{2009Q3} + 0.3728 WGWTH_{2010Q2} \\
&= -0.0504 + 0.4537 \times 0.38 + 0.2174 \times 0.91 + 0.3728 \times 0.6 \\
&= 0.5435
\end{aligned}$$

$$\begin{aligned}
\widehat{INF}_{2010Q3} &= -0.0504 + 0.4537 INF_{2010Q2} + 0.2174 INF_{2009Q4} + 0.3728 WGWTH_{2010Q3} \\
&= -0.0504 + 0.4537 \times 0.5435 + 0.2174 \times 0.65 + 0.3728 \times 0.5 \\
&= 0.5239
\end{aligned}$$

$$\begin{aligned}
\widehat{INF}_{2010Q4} &= -0.0504 + 0.4537 INF_{2010Q3} + 0.2174 INF_{2010Q1} + 0.3728 WGWTH_{2010Q4} \\
&= -0.0504 + 0.4537 \times 0.5239 + 0.2174 \times 0.38 + 0.3728 \times 0.7 \\
&= 0.5309
\end{aligned}$$

$$\begin{aligned}
INF_{2011Q1} &= -0.0504 + 0.4537 INF_{2010Q4} + 0.2174 INF_{2010Q2} + 0.3728 WGWTH_{2011Q1} \\
&= -0.0504 + 0.4537 \times 0.5309 + 0.2174 \times 0.5435 + 0.3728 \times 0.4 \\
&= 0.4578
\end{aligned}$$

The standard errors of the forecast errors are

$$\text{se}(u_1) = \hat{\sigma}_v = 0.5111$$

$$\text{se}(u_2) = \hat{\sigma}_v \left(1 + \hat{\theta}_1^2\right)^{1/2} = 0.51115\left(1 + 0.45369^2\right)^{1/2} = 0.5613$$

$$\text{se}(u_3) = \hat{\sigma}_v \left(1 + \hat{\theta}_1^2 + \hat{\theta}_1^4\right)^{1/2} = 0.51115\left(1 + 0.45369^2 + 0.45369^4\right)^{1/2} = 0.5711$$

$$\text{se}(u_4) = \hat{\sigma}_v \left(1 + \hat{\theta}_1^2 + \hat{\theta}_1^4 + \left(\hat{\theta}_1^3 + \hat{\theta}_3\right)^2\right)^{1/2} = 0.5928$$

The 95% forecast intervals are

$$\widehat{INF}_{2010Q2} \pm t_{(0.975,153)} \times \text{se}(u_1) = 0.5435 \pm 1.976 \times 0.5111 = (-0.466, 1.553)$$

$$\widehat{INF}_{2010Q3} \pm t_{(0.975,153)} \times \text{se}(u_2) = 0.5239 \pm 1.976 \times 0.5613 = (-0.585, 1.633)$$

$$\widehat{INF}_{2010Q4} \pm t_{(0.975,153)} \times \text{se}(u_3) = 0.5309 \pm 1.976 \times 0.5711 = (-0.598, 1.659)$$

$$\widehat{INF}_{2011Q1} \pm t_{(0.975,153)} \times \text{se}(u_4) = 0.4578 \pm 1.976 \times 0.5928 = (-0.714, 1.629)$$

These forecast intervals are very wide, containing both positive and negative values, and hence do not contain much information about likely values of future inflation. Knowing wage growth might help predict inflation, but it still leaves a great deal of uncertainty.

EXERCISE 9.27

(a) The equation is

$$INF_t = \delta + \theta_1 INF_{t-1} + \theta_3 INF_{t-3} + \delta_0 WGWTH_t + v_t$$

Applying the lag operator to this equation, we have,

$$(1-\theta_1 L-\theta_3 L^3)\, INF_t = \delta + \delta_0 WGWTH_t$$

and

$$\begin{aligned} INF_t &= (1-\theta_1 L-\theta_3 L^3)^{-1}\delta + (1-\theta_1 L-\theta_3 L^3)^{-1}\delta_0 WGWTH_t \\ &= \alpha + (\beta_0 + \beta_1 L + \beta_2 L^2 + \beta_3 L^3 + \cdots) WGWTH_t + e_t \end{aligned}$$

Thus,

$$\alpha = (1-\theta_1 L-\theta_3 L^3)^{-1}\delta = \frac{\delta}{1-\theta_1-\theta_3}$$

and

$$(1-\theta_1 L-\theta_3 L^3)^{-1}\delta_0 = (\beta_0 + \beta_1 L + \beta_2 L^2 + \beta_3 L^3 + \cdots)$$

or,

$$\begin{aligned} \delta_0 &= (1-\theta_1 L-\theta_3 L^3)(\beta_0 + \beta_1 L + \beta_2 L^2 + \beta_3 L^3 + \beta_4 L^4 + \cdots) \\ &= \left(\beta_0 + \beta_1 L + \beta_2 L^2 + \beta_3 L^3 + \beta_4 L^4 + \cdots\right) + \left(-\beta_0\theta_1 L - \beta_1\theta_1 L^2 - \beta_2\theta_1 L^3 - \beta_3\theta_1 L^4 - \cdots\right) \\ &\qquad + \left(-\beta_0\theta_3 L^3 - \beta_1\theta_3 L^4 - \cdots\right) \\ &= \beta_0 + \left(\beta_1 - \beta_0\theta_1\right)L + \left(\beta_2 - \beta_1\theta_1\right)L^2 + \left(\beta_3 - \beta_2\theta_1 - \beta_0\theta_3\right)L^3 + \left(\beta_4 - \beta_3\theta_1 - \beta_1\theta_3\right)L^3 + \cdots \end{aligned}$$

Equating coefficients of equal powers in the lag operator gives

$$\delta_0 = \beta_0 \qquad \beta_1 - \theta_1\beta_0 = 0 \qquad \beta_2 - \theta_1\beta_1 = 0$$

$$\beta_j - \theta_1\beta_{j-1} - \theta_3\beta_{j-3} = 0 \ \text{ for } \ j \ge 3$$

Thus, expressions that can be used to calculate α and the β_s are

$$\delta_0 = \beta_0 \qquad \beta_1 = \theta_1\beta_0 \qquad \beta_2 = \theta_1\beta_1$$

$$\beta_j = \theta_1\beta_{j-1} + \theta_3\beta_{j-3} \ \text{ for } \ j \ge 3$$

(b) When *WGWTH* remains constant at zero, estimated inflation is

$$\hat{\alpha} = \frac{\hat{\delta}}{1-\hat{\theta}_1-\hat{\theta}_3} = \frac{-0.0504}{1-0.45369-0.21743} = -0.1532$$

To test $H_0: \alpha = 0$, we can use $t = \hat{\alpha}/\text{se}(\hat{\alpha})$, or, alternatively, since $\alpha = 0$ when $\delta = 0$, we can use $t = \hat{\delta}/\text{se}(\hat{\delta})$. The test values from these two alternatives are

$$t = \hat{\alpha}/\text{se}(\hat{\alpha}) = -0.153247/0.28758 = -0.533$$

$$t = \hat{\delta}/\text{se}(\hat{\delta}) = -0.0504/0.09345 = -0.539$$

At $\alpha = 0.05$, the critical values are $\pm t_{(0.975,153)} = \pm 1.976$. Thus, we do not reject H_0. There is no evidence to suggest that inflation will be nonzero when wage growth is zero.

Exercise 9.27 (continued)

(c) The rate of inflation when wage growth is constant at 0.25 is

$$\widehat{INF} = \hat{\alpha} + 0.25\sum_{i=0}^{\infty}\hat{\beta}_i$$

Computing the total multiplier $\sum_{i=0}^{\infty}\hat{\beta}_i$ numerically, we find $\sum_{i=0}^{\infty}\hat{\beta}_i = 1.1335$. Thus an estimate of the inflation rate is

$$\widehat{INF} = -0.1532 + 0.25\times 1.1335 = 0.1301$$

An EViews program that can be used to compute the total multiplier is

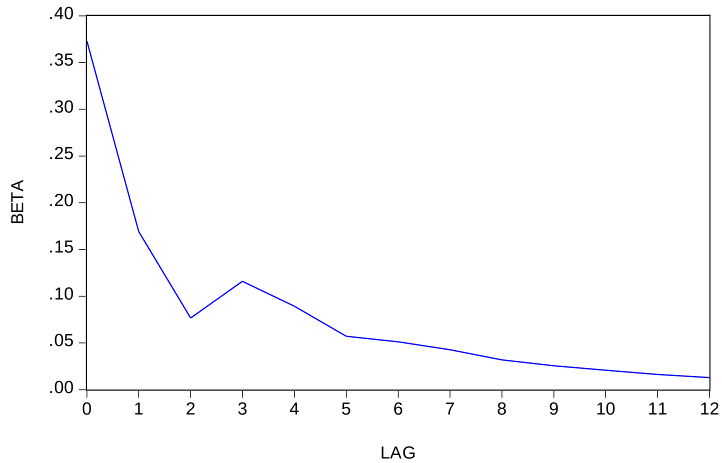
```
vector(200) b
b(1)=c(4)
b(2)=c(2)*b(1)
b(3)=c(2)*b(2)
for !i=4 to 200
b(!i)=c(2)*b(!i-1)+c(3)*b(!i-3)
next
scalar tot_mul=@sum(b)
```

(d) The delay and interim multipliers for up to 12 quarters are

Delay multiplier	Estimate	Interim multiplier
$\beta_0 = \delta_0$	0.3728	0.3728
$\beta_1 = \theta_1\beta_0$	0.1691	0.5419
$\beta_2 = \theta_1\beta_1$	0.0767	0.6187
$\beta_3 = \theta_1\beta_2 + \theta_3\beta_0$	0.1159	0.7345
$\beta_4 = \theta_1\beta_3 + \theta_3\beta_1$	0.0893	0.8239
$\beta_5 = \theta_1\beta_4 + \theta_3\beta_2$	0.0572	0.8811
$\beta_6 = \theta_1\beta_5 + \theta_3\beta_3$	0.0511	0.9322
$\beta_7 = \theta_1\beta_6 + \theta_3\beta_4$	0.0426	0.9749
$\beta_8 = \theta_1\beta_7 + \theta_3\beta_5$	0.0318	1.0067
$\beta_9 = \theta_1\beta_8 + \theta_3\beta_6$	0.0255	1.0322
$\beta_{10} = \theta_1\beta_9 + \theta_3\beta_7$	0.0209	1.0531
$\beta_{11} = \theta_1\beta_{10} + \theta_3\beta_8$	0.0164	1.0694
$\beta_{12} = \theta_1\beta_{11} + \theta_3\beta_9$	0.0130	1.0824

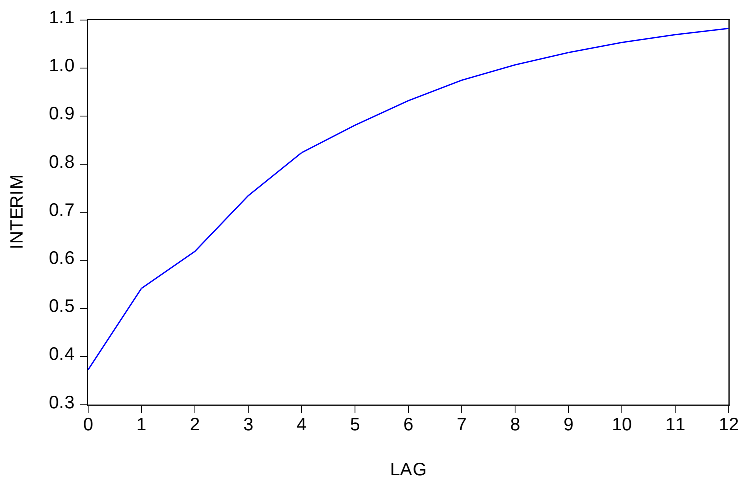
Exercise 9.27(d) (continued)

The graph for the delay multipliers for up to 12 quarters follows



An increase in wage growth increases the inflation rate. However, the effect decreases as the lag increases, with the exception of a spike at lag 3. After 12 quarters, the effect is nearly zero.

(e) The graph for interim multipliers for up to 12 quarters is



If wage growth increases to a new level and then is held constant at that new level, inflation increases at a diminishing rate, approaching the total multiplier which is approximately 1.1.

(f) The estimated changes in inflation are given in the following table.

Quarter	$T+1$	$T+2$	$T+3$	$T+4$	$T+5$
Change in Inflation	$0.1\beta_0$	$0.1\beta_1 + 0.2\beta_0$	$0.1\beta_2 + 0.2\beta_1$	$0.1\beta_3 + 0.2\beta_2$	$0.1\beta_4 + 0.2\beta_3$
Estimate	0.0373	0.0915	0.0415	0.0269	0.0321