ECON 3150/4150. Lecture 9.

The multiple regression model (III)

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References to Lecture 9

- SW: Ch 7.1 and Appendix 7.1
- BN: Kap 7.4-7.10

- For the case of $k = 2$, we derived expressions and the properties of OLS estimators for the coefficients β_j $(j = 0, 1, ..., k)$ in

$$Y_i = \beta_0 + \sum_{j=1}^{k} \beta_j X_{ji} + \varepsilon_i \quad (1)$$

  in the case of

$$E(\varepsilon_i \mid X_{1i}, X_{2i}, ..., X_{ki}) = 0 \ \forall i \quad (2)$$

  and with classical properties for the disturbances ε_i $(i = 1, 2, \ldots, n)$
- The result shown for $k = 2$ holds for any k:
  1. $E(\hat{\beta}_j) = \beta_j \ \forall j$
  2. $\text{plim}(\hat{\beta}_j) = \beta_j \ \forall j$

3. The $\hat{\beta}_k$ estimators are BLUE
4. $t_j = \frac{\hat{\beta}_j - E(\hat{\beta}_j)}{\widehat{se}(\hat{\beta}_j)}$ is distributed $t(n-k-1)$ when the disturbances are $IIN(0, \sigma^2)$, and asymptotically $N(0,1)$ under the classical assumptions.
   Memo: $\widehat{se}(\hat{\beta}_j) = \sqrt{var(\hat{\beta}_j)}$ makes use of $\hat{\sigma}^2 = \frac{\sum_{i=1}^n \hat{\varepsilon}_i^2}{n-k-1}$.
   $\text{plim}(\hat{\sigma}^2) = \sigma^2$.

t-tests I

- The t-test in the regression output is for the test situation H_0: $\beta_j = 0$ against $H_1 : \beta_j \neq 0$
- The only difference from the simple regression case is the formula for $\widehat{se}(\hat{\beta}_j)$ (see Lecture 8) and the degrees of freedom for the *t-distribution* which is $n - k - 1$ in general.
- If the questions is about including a regressor or not, these tests can be used instead of the *Adj* R^2, or other information criteria (AIC, SC, see separate slide).
  - It can be shown that $|t| > 1$ is enough to increase *Adj* R^2
  - But note the problem of controlling the overall significance level when doing repeated *t-tests*, see Bonferroni Test appendix 7.1.

t-tests II

- Often, the economic problem that we work with leads to other test-situations that also can be tackled by *t-tests*
- Example from *Andy's*: Log-linear model for $sales_i$. Could be interesting to test $H_0 : \beta_2 = 1$ against $H_1 : \beta_2 < 0$. Why?
- If H_0 is rejected would then have formal evidence that, for a given price level, advertisement expenditure is taking a bigger share of sale revenues.

$$\widehat{\ln(sales)}_i = 5.31 - \underset{(0.079)}{0.5} \ln(price_i) + \underset{(0.0137)}{0.0454} \ln(advert_i)$$

- The relevant statistic, which is $t(72)$ distributed under H_0 is

$$t = \frac{0.0454 - 1}{0.0137}$$

t-tests III

- Calculate the one-sided *p-value* and conclude!

Information criteria (short reference) I

In modern econometrics two information criteria are often cited alongside, or instead of Adj R^2:

- *AIC*: Akaike information criterion

$$AIC = \ln\left(\frac{\sum_{i=1}^{n} \hat{\varepsilon}_i^2}{n}\right) + \frac{2(k-1)}{n}$$

- *SC*: Schwarz criterion

$$SC = \ln\left(\frac{\sum_{i=1}^{n} \hat{\varepsilon}_i^2}{n}\right) + \frac{(k-1)\ln(n)}{n}$$

- Like *Adj* R^2,they penalize extra regressors
- For $n \geq 8$ *SC* is stricter than *AIC* (not shown)

Testing with the use of the delta method I

- Lecture 1: A non-linear function of two random variables X and Y:

$$g(X,Y) = \frac{X}{Y}$$

- Since E and *var* are linear operators, we must first find a linear approximation to $g(X,Y)$.
- This is done by Taylor expansion (Sydsæter 2003, Kap 7).

Testing with the use of the delta method II

- BN page 72-73 it is show that the following holds

$$E\left(\frac{X}{Y}\right) \approx \frac{\mu_X}{\mu_Y}, \tag{3}$$

$$var\left(\frac{X}{Y}\right) \approx \left(\frac{1}{\mu_Y}\right)^2 \left[\sigma_X^2 + \left(\frac{\mu_X}{\mu_Y}\right)^2 \sigma_Y^2 - 2\left(\frac{\mu_X}{\mu_Y}\right)\sigma_{X,Y}\right] \tag{4}$$

Delta method example I

- Consider the sales-advertisement data form Andy's and the
- We can consider the multivariate model:

$$sales_i = \beta_0 + \beta_1 price_i + \beta_2 advert_i + \beta_3 advert_i^2 + \varepsilon_i$$

- Let $advert_0$ be the optimal level of advertisement defined by the 1oc:

$$\beta_2 + 2\beta_3 advert_o = 1$$

- $advert_0$ is a derived parameter and a non-linear function of the regression parameter β_2 and β_3.

$$advert_o = \frac{1}{2} \frac{1 - \beta_2}{\beta_3}$$

Delta method example II

- We consider

$$\widehat{advert_o} = \frac{1}{2}\frac{1-\hat{\beta}_2}{\hat{\beta}_3}$$

as an estimator of the parameter $advert_o$. If we want to test an hypothesis like

$$H_0 : advert_o = 0$$

we need to approximate $Var(\widehat{advert_o})$ by the *delta method*:

Use $cov(\hat{\beta}_2, \hat{\beta}_3)$ form the estimation output, and use it in the formula (4):

$$var(\widehat{advert_o}) = \left(\frac{1}{2}\right)^2 var(\frac{1-\hat{\beta}_2}{\hat{\beta}_3}) \approx \left(\frac{1}{2}\right)^2 \left(\frac{1}{2.768}\right)^2 \times$$

$$\left[(3.556)^2 + \left(\frac{1-12.151}{-2.768}\right)^2 \cdot (0.941)^2 - 2 \cdot \left(\frac{(1-12.151)}{-2.768}\right) \cdot 3.2887\right]$$

$$= \frac{1}{4} * 0.130\,52 * 0.518\,41 = 0.016916.$$

- An approximate t-value for $H_0 : advert_o = 0$ is therefore:

$$t = \frac{\frac{1}{2}\frac{1-12.1512}{-2.768} - 0}{\sqrt{0.016916}} = \frac{2.014}{\sqrt{0.016916}} = 15.0$$

- Reject H_0 at very singificance level (low p-value)!
- Lecture 10: The F-test for so called joint hypothesis testing.