Supplement to Lecture 9: Confidence interval for the "natural rate of unemployment"

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PCM natural rate of unemployment I

- The **natural rate of unemployment** is a central concept in macro economics
- In order to estimate the natural rate, we must specify a model where it is a parameter (explicitly or implicitly)
- There are many such models, but we consider a **linear Phillips curve model** (PCM)

$$\pi_t = \beta_0 + \beta_1 U_t + \varepsilon_t \tag{1}$$

where π_t is Norwegian inflation in year t and U_t is the unemployment percentage U_t.

PCM natural rate of unemployment II

- The natural rate can defined in such a way that it becomes a parameter in (1). Re-write the PCM as

$$\begin{aligned} \pi_t &= \beta_1(U_t - \frac{-\beta_0}{\beta_1}) + \varepsilon_t \\ &= \beta_1(U_t - U^{nat}) + \varepsilon_t \end{aligned} \tag{2}$$

  and define

$$U^{nat} := \frac{-\beta_0}{\beta_1}$$

  as the natural rate of unemployment.
- U^{nat} is at parameter in both (1) and (2), though implicit in (1).

PCM natural rate of unemployment III

- (2) is however NOT linear in parameters. To estimate U^{nat} from (2) requires Non-linear Least Squares, (NLS).
- However, with the use of the **delta method** we can make inference about U^{nat} by estimating the linear-in parameter model (1)

With annual data from 1975 to 2005 **(**$T = 25$**) we estimate:**

$$\hat{\pi}_t = \underset{(1.453)}{10.5} - \underset{(0.423)}{1.83}\, U_t$$

Nat-rate $(\hat{U}^{nat})$	5.73 %
IT-rate $(\hat{U}^{it})$	4.36 %

Note:

- U^{it} is the" inflation target rate of unemployment": the "natural rate for $\pi_t = 2.5$, instead of 0

Use the **delta-method formula** from Lecture 1:

$$var(\hat{U}^{nat}) = var(\frac{-\hat{\beta}_0}{\hat{\beta}_1}) \approx \left(\frac{1}{\hat{\beta}_1}\right)^2 \left[var(-\hat{\beta}_0) + \left(\hat{U}^{nat}\right)^2 var(\hat{\beta}_1) - 2\left(\hat{U}^{nat}\right) cov(-\hat{\beta}_0, \hat{\beta}_1)\right]$$

From the estimation: $cov(\hat{\beta}_0, \hat{\beta}_1) = -0.57401$

$$\begin{aligned} var(\hat{U}^{nat}) = var(\frac{-\hat{\beta}_0}{\hat{\beta}_1}) &\approx \left(\frac{1}{-1.83}\right)^2 \cdot \left[1.453^2 + (5.73)^2 \cdot 0.423^2 - 2 \cdot (5.73) \cdot 0.57401\right] \\ &= 0.42038 \end{aligned}$$

Approximate 95 % confidence interval for U^{nat} is therefore

$$5.73 \pm 2 \cdot \sqrt{0.42038} = 5.73 \pm 2 \cdot 0.64\,37$$

or

$$[4.43\% \ ; \ 7.03\%]$$

Memo: Direct estimation using the mthod of Non Linear Least Squares (NLS) gives:
$var(\hat{U}^{nat}) = 0.6479^2 = 0.41977$