ECON 3150/4150: Repeated sampling and Monte Carlo simulation

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1 Repeated sampling

In Lecture 4 and 5 we showed the result that if the assumptions of the classical regression model (either with deterministic or random X) hold, the OLS estimator β_1 is unbiased and consistent (among other important properties).

Unbiasedness could be sought demonstrated in practice by *repeated sampling.* The point is that by drawing repeated samples of, for example size $n = 40$, from a large population of individuals, the regression could be done several times: for example 10 times, which would be referred to as 10 replications of the estimation. The average of the 10 OLS estimates of β_1 is an estimate of the expectation of the OLS estimator $E(\hat{\beta}_1)$. If the OLS based estimation theory holds what it promises, we would expect that the average of the 10 OLS estimates comes close to the true value of the parameter β_1.

2 Monte Carlo simulation and analysis

Repeated sampling may often be costly in practice, but we can do something similar at a very low cost: We can use the computer to generate a number of data sets (call it M), each of sample length n, and then estimate the regression model on all M data sets and average all of the M estimates of the slope parameter β_1.

The result of the experiment is that we have M OLS estimates $\hat{\beta}_{1j}$ ($j = 1, 2, \ldots, M$). Since the data sets are independent (the computer takes care of that), the average

$$\tilde{\beta}_1 = \frac{\sum_{j=1}^{M} \hat{\beta}_{1j}}{M}.$$

will come close to the true $E(\hat{\beta}_1)$ when M is large. Note that we can make a reference to the *Law of large numbers* (Lecture 6) here: Because the experiment we perform on the computer satisfies the assumptions of the *Law of large numbers* we can use that

$$plim\ \tilde{\beta}_1 = E(\hat{\beta}_1),\ M \ \longrightarrow \infty$$

This means that by repeated sampling and estimation, we obtain a sequence of estimates $\tilde{\beta}_1$ that has the true expectation of the OLS estimator $\hat{\beta}_1$ as its probability limit. This is the main principle of a Monte Carlo analysis.

3 "Checking" theoretical results by Monte Carlo analysis

Monte Carlo experiment is an (often eye-opening) way of "checking" and getting intuition about theoretical statistical results.

As an example, the theory of the regression model states that *if* the classical assumptions of the regression model hold, the OLS estimator $\hat{\beta}_1$ is *unbiased*:

$$E(\hat{\beta}_1 - \beta_1) = 0$$

We can use Monte Carlo analysis to check this theoretical result. To do that, we instruct the computer to generate data in accordance with the regression model with classical assumptions.

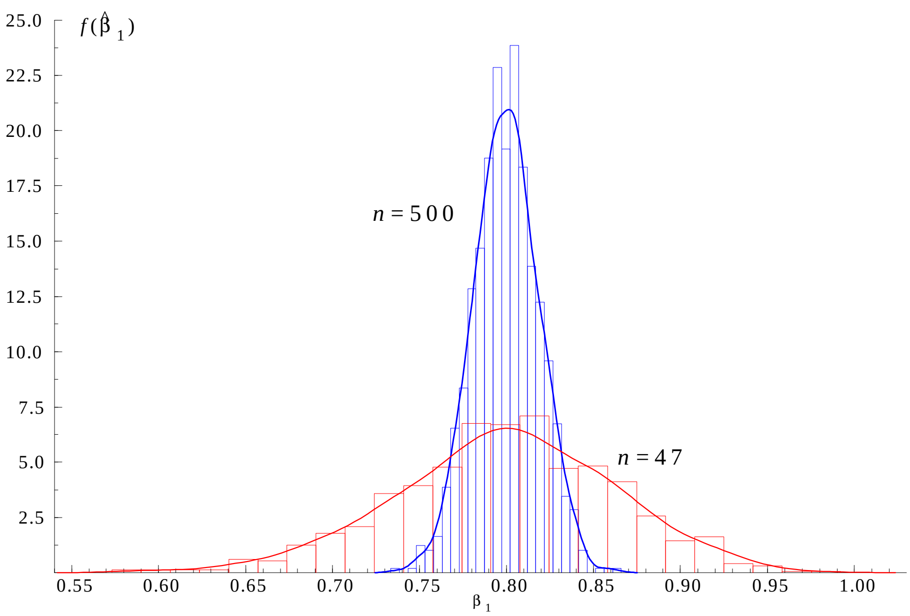
Since the our regression model then is the data generating process ("we have the true model"), we expect to find that $\widetilde{\beta}_1 \approx \beta_1$ for a reasonably large M, (e.g., 20) and that the approximation only gets better if M becomes really large.

In so called *recursive* Monte Carlo analysis we keep the number of replications M fixed and calculate the whole sequence of $\tilde{\beta}_{1(n)}$ estimates (from small n to large n (e.g., $n = 100$), to check that the bias $\tilde{\beta}_{1(n)} - \beta_1$ disappears as the sample size grows larger. A plot of the bias $\tilde{\beta}_{1(n)} - \beta_1$ as a function of n will illustrate the consistency property:

$$plim\ \tilde{\beta}_{1(n)} = \beta_1\ n\ \longrightarrow \infty$$

of the OLS estimator under the classical assumptions of the regression model.

When the number of replications is large, it is also instructive to plot the distributions for different sample sizes:



Monte Carlo analysis: Two distributions (for $n = 47$and $n = 500$) of the OLS estimator $\hat{\beta}_1$of the slope coefficient in the case where the classical assumptions of the regression model hold, and $X$is non-random. The true value of $\beta_1$is 0.80.