ECON3150/4150 Spring 2016

Lecture 7
Hypothesis testing

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Overview

- Hypothesis testing in SRLM
- Hypothesis testing in MLRM

Hypothesis testing

- In the revision part we used the sample average as an estimator for the unknown population mean
- Now we are dealing with unknown parameters (β_0 and β_1) and we have derived estimators for them.
- As with sample average we formulate a null hypothesis that β_1 is equal to some specific value $\beta_{1,0}$ and an alternative hypothesis.

Hypothesis testing

Given the null and alternative hypothesis:

- Compute the test statistic
- Choose your desired significance level
  - Compare test statistic to critical test statistic
  - Compare significance level to the p-value of the test-statistic
- Two outcomes:
  - Reject the null in favor of an alternatives.
  - Fail to reject the null.

Finding the critical value and the p-value require knowing the distribution of the statistic. The distribution of the OLS estimators depend on the distribution of the error term.

Compute test statistic

The test statistic for the regression coefficient is the t statistic

$$t = \frac{\text{estimator} - \text{hypothesised value}}{\text{standard error of the estimator}}$$

- Since the standard error is always positive the t-statistic has the same sign as the difference between the estimator and the hypothesized value.
- For a given standard error the larger value of the estimator the larger value of the t-statistic.
- If the null hypothesis is that the true parameter is zero, a large estimator provides evidence against the null.
- t-values sufficiently far from the hypothesized value result in rejection of the null.

Compute t-statistic

In the single regression model: $Y = \beta_0 + \beta_1 X + u$

- Compute the t-statistics:

$$t = \frac{\hat{\beta}_1 - \beta_{1,0}}{SE(\hat{\beta}_1)}$$

- If the null hypothesis is that $\beta_1 = 1$

$$t = \frac{\hat{\beta}_1 - 1}{SE(\hat{\beta}_1)}$$

  The standard error is given by:

$$SE(\hat{\beta}_1) = \sqrt{\hat{\sigma}^2_{\beta_1}}$$

Hypothesis testing

Example where the standard error is reported below in parenthesis:

$$wage = \beta_0 + \beta_1 educ + u$$

$$wage = \underset{(0.104)}{0.284} + \underset{(0.007)}{0.092} educ$$

If $H_0 : \beta_1 = 0$:

$$t = 0.092/0.007 = 13.14$$

Distribution of estimators

We can use the t-statistic only if the regression coefficient is normally distributed. Under the OLS assumptions including the normality assumption, sampling distribution of the OLS estimators is normal. Because:

- A random variable which is a linear function of a normally distributed variable is itself normally distributed.
- If we assume that $u \sim N(0, \sigma^2)$ then Y_i is normally distributed.
- Since the estimators $\hat{\beta}_0$ and $\hat{\beta}_1$ is linear functions of the Y_i's then the estimators are normally distributed.
- Thus $\hat{\beta}_1 \sim N[\beta_1, Var(\hat{\beta}_1)]$

Normality assumption

- In large samples we can invoke the CLT to conclude that the OLS satisfy asymptotic normality.
- Whenever Y takes on just a few values and we have few observations it cannot have anything close to a normal distribution.
- If the $\hat{\beta}_1$ is not normally distributed the t-statistic does not have t distribution.
- The normal distribution of u is the same as the distribution of Y given X.

Distribution of estimators

- If $\hat{\beta}_1$ is either normally distributed or approximately normally distributed it can be standardized.

$$t = \frac{\hat{\beta}_j - \beta_j}{se(\hat{\beta}_j)}$$

- t statistic for $\hat{\beta}_1$ is t-distributed
- Thus: $(\hat{\beta}_1 - \beta_1)/se(\hat{\beta}_1) \sim t_{n-k-1}$
- t_{n-k-1} represents the "t-distribution" with $n-k-1$ degrees of freedom.
- As the degrees of freedom in the t-distribution gets large, the t distribution approaches the standard normal distribution.

Hypothesis testing in MLRM

Rather than testing β_1 you can test any β_j

- If $H_0 : \beta_j = \beta_{j,0}$

$$t = \frac{\hat{\beta}_j - \beta_{j,0}}{se(\hat{\beta}_j)}$$

Hypothesized value

- In practice it is unusual to have a feeling for the actual value of the coefficients.
- Very often the objective of the analysis is to demonstrate that Y is influenced by X, without having any specific prior notion of the actual coefficients of the relationships.
- In this case it is usual to define $\beta_j = 0$ as the null, which means that the null hypothesis is that X has no partial effect on Y.
- If $H_0 : \beta_j = 0$ then the t-statistic is just the coefficient divided by the standard error.

$$t = \frac{\hat{\beta}_j}{se(\hat{\beta}_j)}$$

- This statistic is automatically printed as part of the regression results.

Example

```
1 . reg wage educ, robust

  Linear regression                                      Number of obs =        935
                                                            F(  1,   933) =      95.65
                                                            Prob > F      =     0.0000
                                                            R-squared     =     0.1070
                                                            Root MSE      =     382.32
```

wage	Coef.	Robust Std. Err.	t	P>|t|	[95% Conf.	Interval]
educ	60.21428	6.156956	9.78	0.000	48.1312	72.29737
_cons	146.9524	80.26953	1.83	0.067	-10.57731	304.4822

Is the t-statistic for the coefficient of education large? What about the statistic of the intercept?

The t-statistic

- The further the estimated coefficient is from the hypothesized value the more likely the hypothesis is incorrect.
- If $H_0 : \beta_j = 0$ then if H_0 is true β_j should be close to 0

$$\text{small}|\frac{\hat{\beta}_j}{se(\hat{\beta}_j)}|$$

- If H_0 is false $\hat{\beta}_j$ should be far away from 0.

$$\text{large}|\frac{\hat{\beta}_j}{se(\hat{\beta}_j)}|$$

But what is small and what is large?

Critical value

The critical value of the t-distribution determines what is small and large.

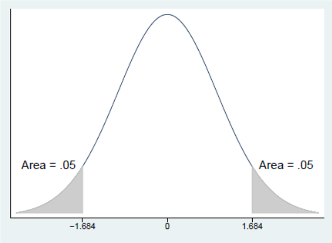
- The critical value depends on your desired level of accuracy.
- The accuracy is determined by the significance level - the probability of rejecting H_0 when it is in fact true. (type I error)
- A 5% significance level means that we mistakenly reject H_0 5% of the time.
- The critical value increases as significance level falls, thus a null hypothesis that is rejected at a 5% level is automatically rejected at the 10% level as well.
- The lower the possible significance level the lower is the risk that we mistakenly reject the null.

Make conclusion

- When $H_0 : \beta_j = 0$ is rejected at the 5% level we usually say
  - X_j is statistically significant at the 5% level
  - X_j is statistically different from zero at the 5% level.
- If H_0 is not rejected we say that X is statistically insignificant at the 5% level.
- If we fail to reject H_0 we never say that we accept H_0 because there are many other values for β_1 which cannot be rejected and they cannot all be true.

Make conclusions

We could be wrong as we do not measure β_1. For a 10% significance level we falsely reject the null 10% of the time.



The graphs illustrates $f(\hat{\beta}_1/(se(\hat{\beta}_1))$. 10% of the distribution is in the grey area.

Finding critical values

TABLE 2 Critical Values for Two-Sided and One-Sided Tests Using the Student *t* Distribution

Degrees of Freedom	Significance Level				
	20% (2-Sided) 10% (1-Sided)	10% (2-Sided) 5% (1-Sided)	5% (2-Sided) 2.5% (1-Sided)	2% (2-Sided) 1% (1-Sided)	1% (2-Sided) 0.5% (1-Sided)
1	3.08	6.31	12.71	31.82	63.66
2	1.89	2.92	4.30	6.96	9.92
3	1.64	2.35	3.18	4.54	5.84
4	1.53	2.13	2.78	3.75	4.60
5	1.48	2.02	2.57	3.36	4.03
6	1.44	1.94	2.45	3.14	3.71
7	1.41	1.89	2.36	3.00	3.50
8	1.40	1.86	2.31	2.90	3.36
9	1.38	1.83	2.26	2.82	3.25
10	1.37	1.81	2.23	2.76	3.17
11	1.36	1.80	2.20	2.72	3.11
12	1.36	1.78	2.18	2.68	3.05
13	1.35	1.77	2.16	2.65	3.01
14	1.35	1.76	2.14	2.62	2.98
15	1.34	1.75	2.13	2.60	2.95
16	1.34	1.75	2.12	2.58	2.92
17	1.33	1.74	2.11	2.57	2.90
18	1.33	1.73	2.10	2.55	2.88
19	1.33	1.73	2.09	2.54	2.86
20	1.33	1.72	2.09	2.53	2.85
21	1.32	1.72	2.08	2.52	2.83
22	1.32	1.72	2.07	2.51	2.82
23	1.32	1.71	2.07	2.50	2.81
24	1.32	1.71	2.06	2.49	2.80
25	1.32	1.71	2.06	2.49	2.79
26	1.32	1.71	2.06	2.48	2.78
27	1.31	1.70	2.05	2.47	2.77
28	1.31	1.70	2.05	2.47	2.76
29	1.31	1.70	2.05	2.46	2.76
30	1.31	1.70	2.04	2.46	2.75
60	1.30	1.67	2.00	2.39	2.66
90	1.29	1.66	1.99	2.37	2.63
120	1.29	1.66	1.98	2.36	2.62
∞	1.28	1.64	1.96	2.33	2.58

Values are shown for the critical values for two-sided ($\neq$) and one-sided ($>$) alternative hypotheses. The critical value for the

P-value

- An alternative to comparing the t-statistic to the critical t-statistic is to compute the p-value for the t-statistic.
- The p-value is more informative as it gives you the smallest significance level at which the null hypothesis would have been rejected.
- A null that is rejected at a 5% level must have a p-value smaller than 5%.

P-value

- We remember that the p-value is the probability of obtaining the observed t statistic (or one more extreme) as a matter of chance if the null hypothesis $H_0 : \beta_j = 0$ is true.
- It therefore gives the lowest significance level at which the null hypothesis could be rejected.
- It is not a direct probability of rejecting H_0 when it is actually true.

P-value stata

```
1 . reg wage educ, robust

  Linear regression                                    Number of obs =       935
                                                          F(  1,   933) =     95.65
                                                          Prob > F      =    0.0000
                                                          R-squared     =    0.1070
                                                          Root MSE      =    382.32

  ------------------------------------------------------------------------------
               |               Robust
          wage |      Coef.   Std. Err.      t    P>|t|     [95% Conf. Interval]
  -------------+----------------------------------------------------------------
          educ |   60.21428    6.156956     9.78   0.000      48.1312    72.29737
         _cons |   146.9524    80.26953     1.83   0.067    -10.57731    304.4822
  ------------------------------------------------------------------------------
```

Computing p-values for t-tests

- The p-value (in SLRM) is calculated by computing the probability that a t random variable with (n-2) degrees of freedom is larger than t^{act} in absolute value.
- Thus the p-value is the significance level of the test when we use the value of the test statistic as the critical value for the test.

For the two sided test:

$$\begin{aligned} \text{p-value} &= Pr_{H_0}(|t| > |t^{act}|) = 2P(t > t^{act}) \\ &= Pr(|Z| > |t^{act}|) = 2\phi(-|t^{act}|) \text{ in large samples} \end{aligned}$$

Finding p-value

Standard Normal Probabilities

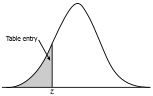


Table entry for z is the area under the standard normal curve to the left of z.

z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
−3.4	.0003	.0003	.0003	.0003	.0003	.0003	.0003	.0003	.0003	.0002
−3.3	.0005	.0005	.0005	.0004	.0004	.0004	.0004	.0004	.0004	.0003
−3.2	.0007	.0007	.0006	.0006	.0006	.0006	.0006	.0005	.0005	.0005
−3.1	.0010	.0009	.0009	.0009	.0008	.0008	.0008	.0008	.0007	.0007
−3.0	.0013	.0013	.0013	.0012	.0012	.0011	.0011	.0011	.0010	.0010
−2.9	.0019	.0018	.0018	.0017	.0016	.0016	.0015	.0015	.0014	.0014
−2.8	.0026	.0025	.0024	.0023	.0023	.0022	.0021	.0021	.0020	.0019
−2.7	.0035	.0034	.0033	.0032	.0031	.0030	.0029	.0028	.0027	.0026
−2.6	.0047	.0045	.0044	.0043	.0041	.0040	.0039	.0038	.0037	.0036
−2.5	.0062	.0060	.0059	.0057	.0055	.0054	.0052	.0051	.0049	.0048
−2.4	.0082	.0080	.0078	.0075	.0073	.0071	.0069	.0068	.0066	.0064
−2.3	.0107	.0104	.0102	.0099	.0096	.0094	.0091	.0089	.0087	.0084
−2.2	.0139	.0136	.0132	.0129	.0125	.0122	.0119	.0116	.0113	.0110
−2.1	.0179	.0174	.0170	.0166	.0162	.0158	.0154	.0150	.0146	.0143
−2.0	.0228	.0222	.0217	.0212	.0207	.0202	.0197	.0192	.0188	.0183
−1.9	.0287	.0281	.0274	.0268	.0262	.0256	.0250	.0244	.0239	.0233
−1.8	.0359	.0351	.0344	.0336	.0329	.0322	.0314	.0307	.0301	.0294
−1.7	.0446	.0436	.0427	.0418	.0409	.0401	.0392	.0384	.0375	.0367
−1.6	.0548	.0537	.0526	.0516	.0505	.0495	.0485	.0475	.0465	.0455
−1.5	.0668	.0655	.0643	.0630	.0618	.0606	.0594	.0582	.0571	.0559
−1.4	.0808	.0793	.0778	.0764	.0749	.0735	.0721	.0708	.0694	.0681
−1.3	.0968	.0951	.0934	.0918	.0901	.0885	.0869	.0853	.0838	.0823
−1.2	.1151	.1131	.1112	.1093	.1075	.1056	.1038	.1020	.1003	.0985
−1.1	.1357	.1335	.1314	.1292	.1271	.1251	.1230	.1210	.1190	.1170
−1.0	.1587	.1562	.1539	.1515	.1492	.1469	.1446	.1423	.1401	.1379
−0.9	.1841	.1814	.1788	.1762	.1736	.1711	.1685	.1660	.1635	.1611
−0.8	.2119	.2090	.2061	.2033	.2005	.1977	.1949	.1922	.1894	.1867
−0.7	.2420	.2389	.2358	.2327	.2296	.2266	.2236	.2206	.2177	.2148
−0.6	.2743	.2709	.2676	.2643	.2611	.2578	.2546	.2514	.2483	.2451
−0.5	.3085	.3050	.3015	.2981	.2946	.2912	.2877	.2843	.2810	.2776
−0.4	.3446	.3409	.3372	.3336	.3300	.3264	.3228	.3192	.3156	.3121
−0.3	.3821	.3783	.3745	.3707	.3669	.3632	.3594	.3557	.3520	.3483
−0.2	.4207	.4168	.4129	.4090	.4052	.4013	.3974	.3936	.3897	.3859
−0.1	.4602	.4562	.4522	.4483	.4443	.4404	.4364	.4325	.4286	.4247
−0.0	.5000	.4960	.4920	.4880	.4840	.4801	.4761	.4721	.4681	.4641

Finding p-value

From the example last week:

```
1 . reg wage educ, robust

Linear regression                                      Number of obs =       935
                                                          F(  1,   933) =     95.65
                                                          Prob > F      =    0.0000
                                                          R-squared     =    0.1070
                                                          Root MSE      =    382.32
```

wage	Coef.	Robust Std. Err.	t	P>|t|	[95% Conf.	Interval]
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_cons	146.9524	80.26953	1.83	0.067	-10.57731	304.4822

The constant has a computed t value of 1.83. Since n is large we can use the z-table. The p-value is $2\phi(-1.83)$.

Finding p-value

Standard Normal Probabilities

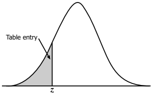

Table entry for z is the area under the standard normal curve to the left of z.

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−3.2	.0007	.0007	.0006	.0006	.0006	.0006	.0006	.0005	.0005	.0005
−3.1	.0010	.0009	.0009	.0009	.0008	.0008	.0008	.0008	.0007	.0007
−3.0	.0013	.0013	.0013	.0012	.0012	.0011	.0011	.0011	.0010	.0010
−2.9	.0019	.0018	.0018	.0017	.0016	.0016	.0015	.0015	.0014	.0014
−2.8	.0026	.0025	.0024	.0023	.0023	.0022	.0021	.0021	.0020	.0019
−2.7	.0035	.0034	.0033	.0032	.0031	.0030	.0029	.0028	.0027	.0026
−2.6	.0047	.0045	.0044	.0043	.0041	.0040	.0039	.0038	.0037	.0036
−2.5	.0062	.0060	.0059	.0057	.0055	.0054	.0052	.0051	.0049	.0048
−2.4	.0082	.0080	.0078	.0075	.0073	.0071	.0069	.0068	.0066	.0064
−2.3	.0107	.0104	.0102	.0099	.0096	.0094	.0091	.0089	.0087	.0084
−2.2	.0139	.0136	.0132	.0129	.0125	.0122	.0119	.0116	.0113	.0110
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−1.9	.0287	.0281	.0274	.0268	.0262	.0256	.0250	.0244	.0239	.0233
−1.8	.0359	.0351	.0344	.0336	.0329	.0322	.0314	.0307	.0301	.0294
−1.7	.0446	.0436	.0427	.0418	.0409	.0401	.0392	.0384	.0375	.0367
−1.6	.0548	.0537	.0526	.0516	.0505	.0495	.0485	.0475	.0465	.0455
−1.5	.0668	.0655	.0643	.0630	.0618	.0606	.0594	.0582	.0571	.0559
−1.4	.0808	.0793	.0778	.0764	.0749	.0735	.0721	.0708	.0694	.0681
−1.3	.0968	.0951	.0934	.0918	.0901	.0885	.0869	.0853	.0838	.0823
−1.2	.1151	.1131	.1112	.1093	.1075	.1056	.1038	.1020	.1003	.0985
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−0.1	.4602	.4562	.4522	.4483	.4443	.4404	.4364	.4325	.4286	.4247
−0.0	.5000	.4960	.4920	.4880	.4840	.4801	.4761	.4721	.4681	.4641

$$p = 2 * 0.036$$
$$= 0.0672$$

The Z-table

Remember that:

$$\phi(-Z) = 1 - \phi(Z)$$

Thus the table does not have to show the probability distribution for both positive and negative Z.

Interpreting p-values

- Small p-values are evidence against the null, large p-values provide little evidence against the null.
- If for example the p-value $=0.5$ then we would observe a value of the t statistic as extreme as we did in 50% of all random samples if the null hypothesis is true.
- If α denotes the significance level then H_0 is rejected if p-value$< \alpha$.

Interpreting p-values

p-value

Correct interpretation: Assuming that the null is true you would obtain the observed difference or more in p% of studies due to random sampling error.

Wrong interpretation: P-value is the probability of making a mistake by rejecting a true null hypothesis.

- The p-value is calculated based on the assumption that the null is true for the population, thus it cannot tell you the probability that the null is true or false.
- A low p-value indicates that your data are unlikely assuming a true null, but it cannot evaluate whether it is more likely that the low p-value comes from the null being true but having an unlikely sample or the null being false.

Collective illusions

Think that you have made a t-test and get a p-value of 0.01

- You have *not* absolutely disproved the null hypothesis.
- You have *not* found the probability of the null hypothesis being true.
- You *do not* know if you decide to reject the null hypothesis the probability that you are making the wrong decision.

An example

The MEAP93 data contains observations on 408 school districts on average teacher salary in thousands of dollars (sal) and the percentage of students passing the MEAP math. A regression gives the following output:

$$\widehat{math10} = \underset{(3.22)}{8.28} + \underset{(0.10)}{0.498} sal$$

- The constant term is 8.28 with a standard error of 3.22
- The slope parameter is 0.498 with a standard error of 0.1

An example

$$\hat{math10} = \underset{(3.22)}{8.28} + \underset{(0.10)}{0.498} sal$$

$$t = \frac{0.498 - 0}{0.10} = 4.98$$

$$p - value = 2\phi(-4.98) < 0.00001$$

- The 5% critical value is given by: $t^c_{406} = 1.96$
- We can reject the null that salary does not affect the percentage of students passing the math10.

An example

The stata output for the same regression shows the same conclusion.

```
1 . reg math10 sal

      Source |       SS           df       MS      Number of obs   =       408
-------------+----------------------------------   F(  1,    406)  =     24.62
       Model |  2562.57022         1  2562.57022   Prob > F        =    0.0000
    Residual |  42254.6103       406  104.075395   R-squared       =    0.0572
-------------+----------------------------------   Adj R-squared   =    0.0549
       Total |  44817.1805       407  110.115923   Root MSE        =    10.202

------------------------------------------------------------------------------
      math10 |      Coef.   Std. Err.      t    P>|t|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
         sal |   .4980309   .1003674     4.96   0.000     .3007264    .6953355
       _cons |   8.282175   3.228869     2.57   0.011     1.934787    14.62956
------------------------------------------------------------------------------
```

One-sided vs two-sided test

- Two-sided tests, as described so far, are appropriate when we have no information about the alternative hypothesis
- When we are in a position to say that if the null hypothesis is not true the coefficient cannot be lower (greater) than that specified by it we can do a one sided test.
- Note: there is a trade-off between the significance level and the power of the test that comes into play when choosing a one-sided test.

T-statistic one sided test

We remember that for the two sided test it was the absolute value of the t-statistic that was important. For the one sided test the sign of the t-statistic is also important.

Rejection rules for one sided tests:

- One sided: $H_1 : \beta_1 > 0 : t^{act} > t^c$
- One sided: $H_1 : \beta_1 < 0 : t^{act} < -t^c$

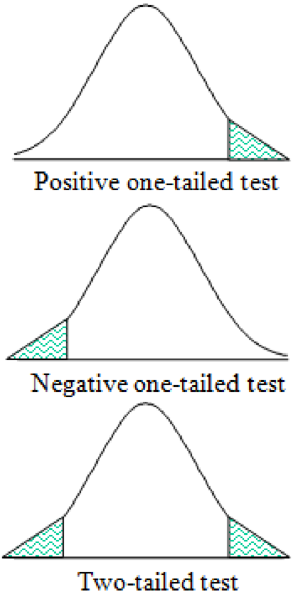
Note: The degrees of freedom of the test statistic is given by $n - k - 1$ where k is the number of independent variables.

One sided p-value

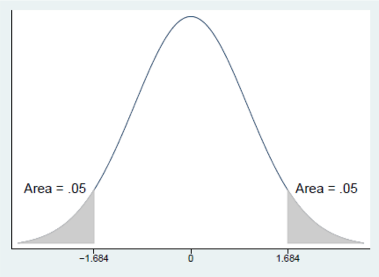
- One sided test: $H_1 : \beta_1 > 0$
- If $\hat{\beta}_1 < 0$ we know that the p-value is greater than 0.5 and there is no need to calculate it.
- If $\hat{\beta}_1 > 0$ then $t > 0$ and the p-value is half of the two-sided p-value.
- Since the t-distribution is symmetric around zero the reversed applied to the one sided test that $\beta_1 < 0$

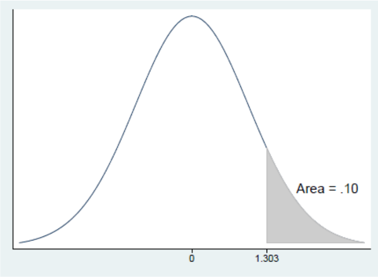
$$p - value = Pr_{H_0}(t < t^{act}) = Pr_{H_0}(t > |t^{act}|)$$

One-sided vs two-sided test



One-sided vs two-sided test





An illustration of the difference between two-sided and one sided test with 40 degrees of freedom.

Example one-sided test

Consider the following regression:

$$\hat{price} = \underset{(0.05)}{1.21} + \underset{(0.10)}{0.82}\, wage$$

$$t = \frac{0.82 - 1}{0.10} = -1.8$$

With n=20 and $t_{c,18,5\%} = 2.101$) (two-sided) we cannot reject the null hypothesis that $\beta_1 = 1$ at the 5% level.

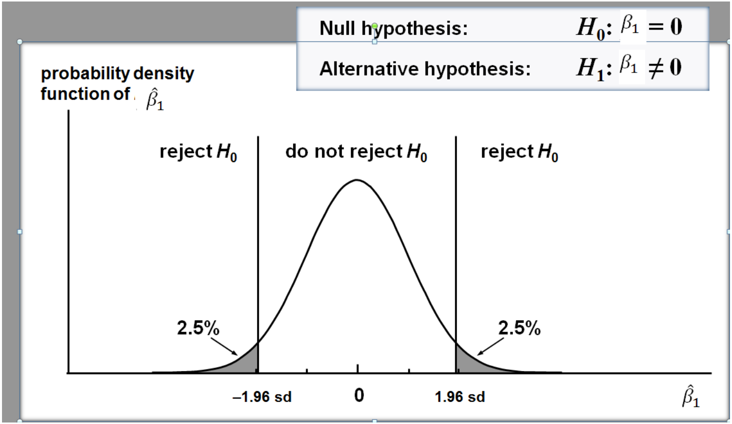
Example one-sided test

$H_0 : \beta_2 = 1$ vs $H_1 : \beta_1 < 1$
The t-statistic is the same as long as the null is the same. But the critical t-value changes.

$$t_{c,18,5\%} = 1.734 \text{ one-sided test}$$

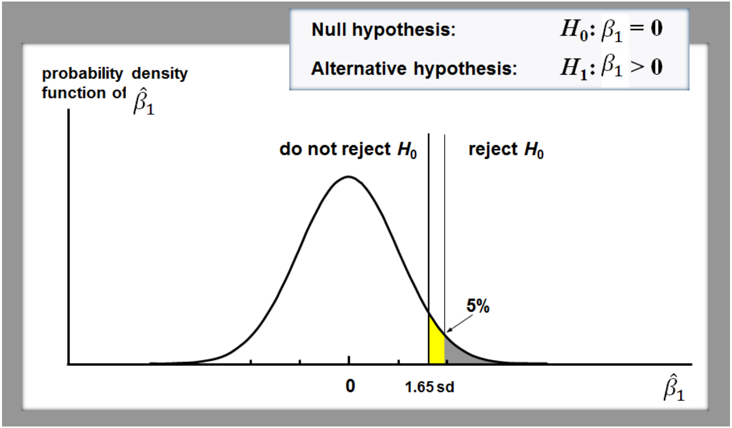
Now we can reject the null hypothesis and conclude that price inflation is significantly lower than wage inflation at the 5% level.

One sided vs two sided



If you use a two-sided 5% significance test your estimate must be 1.96 standard deviations above or below 0 if you are to reject the null hypothesis.

One sided vs two sided



If you can justify the use of a one-sided test, for example with $H_0 : \beta_1 > 0$, the estimate has to be only 1.65 standard deviations above 0.

Economic versus statistical significance

- The statistical significance of a variable is determined entirely by the size of the computed t-statistic.
- A coefficient can be statistically significant either because the coefficient is large, or because the standard error is small.
- With large samples parameters can be estimated very precisely which usually results in statistical significance.
- The economic significance is related to the size (and sign) of $\hat{\beta}_1$.
- Thus you should also discuss whether the coefficient is economically important (i.e. the magnitude of the coefficient)

Example

```
1 . reg math10 sal
```

Source	SS	df	MS
Model	2562.57022	1	2562.57022
Residual	42254.6103	406	104.075395
Total	44817.1805	407	110.115923

Number of obs	408
F(1, 406)	24.62
Prob > F	0.0000
R-squared	0.0572
Adj R-squared	0.0549
Root MSE	10.202

math10	Coef.	Std. Err.	t	P>|t|	[95% Conf.	Interval]
sal	.4980309	.1003674	4.96	0.000	.3007264	.6953355
_cons	8.282175	3.228869	2.57	0.011	1.934787	14.62956

If you increase salary by $1000 this is predicted to increase the percentage of students passing the math test by 0.5 percentage point. Is this an economically significant effect?

Homoskedasticity

Homoskedasticity assumption:

```
1 . reg ahe female
```

Source	SS	df	MS
Model	13091.0876	1	13091.0876
Residual	779560.368	7709	101.12341
Total	792651.456	7710	102.80823

Number of obs	=	7711
F(1, 7709)	=	129.46
Prob > F	=	0.0000
R-squared	=	0.0165
Adj R-squared	=	0.0164
Root MSE	=	10.056

ahe	Coef.	Std. Err.	t	P>|t|	[95% Conf.	Interval]
female	-2.629912	.2311422	-11.38	0.000	-3.083013	-2.17681
_cons	20.11387	.1520326	132.30	0.000	19.81584	20.41189

Heteroskedasticity robust:

```
1 . reg ahe female, robust
```

Linear regression

Number of obs	=	7711
F(1, 7709)	=	134.80
Prob > F	=	0.0000
R-squared	=	0.0165
Root MSE	=	10.056

ahe	Coef.	Robust Std. Err.	t	P>|t|	[95% Conf.	Interval]
female	-2.629912	.2265122	-11.61	0.000	-3.073937	-2.185886
_cons	20.11387	.1614226	124.60	0.000	19.79744	20.4303

Implication of heteroskedasticity

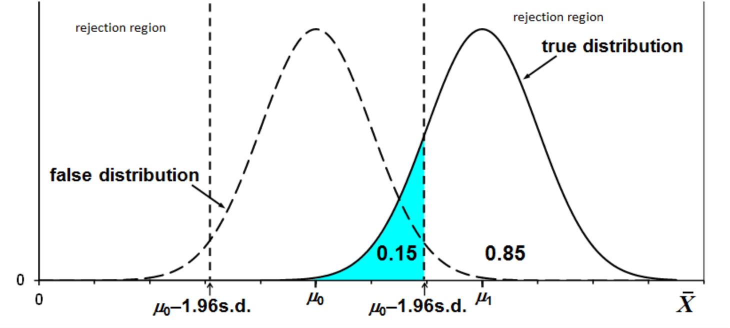
- If the regression errors are homoskedastic and normally distributed and if the homoskedasticity-only t-statistics is used, then critical values should be taken from the Student t distribution.
- In econometric applications the errors are rarely homoskedastic and normally distributed, but as long as n is large and we compute heteroskedasticity robust standard errors we can compute t-statistics and hence p-values and confidence intervals as normal.

Note of caution:

- The test statistic: The t-value and hence the p-value and confidence interval is only as good as the underlying assumptions used to construct it.
- If any of the underlying assumptions are violated the test statistic is not reliable.
- Most often the violated assumption is the zero conditional mean assumption, X is often correlated with the error term.
- More about this in the next lecture when we talk about omitted variable bias.

Power

A lot of emphasis is put on the significance of a test. It is important to remember that there is also another side to the story: power.



Power is the probability of rejecting the null hypothesis when it is false.