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Fall 2009

Some exercises for ECON4160

1. Consider the simple linear regression $y_i = \beta_0 + \beta_1 x_i + \varepsilon_i$

   (a) Obtain formulae for $\hat{\beta}_0$ and $\hat{\beta}_1$ from the general expression $\hat{\beta} = (X'X)^{-1} X'y$

   (b) Obtain formulae for $var\left(\hat{\beta}_0\right)$, $var\left(\hat{\beta}_1\right)$, and $cov\left(\hat{\beta}_0, \hat{\beta}_1\right)$

2. In the regression model $y = X\beta + \varepsilon$, where X in a $N \times K$ matrix, what is the effect on the OLS estimates if

   (a) explanatory variable k is multiplied by a constant λ. What does this tell us of the effect of measuring a variable in NOK or 1000 NOK.

   (b) a constant α is added to explanatory variable k.

3. Consider the formulation $y_t = \beta_0 + \beta_1 x_{1t} + \ldots + \beta_k x_{Kt} + \varepsilon_t = \beta' x_t + \varepsilon_t$ where x_t is a $K \times 1$ vector for each time period t.

   (a) Show that the first difference formulation $\Delta y_t = \Delta \beta_1 x_{1t} + \ldots + \Delta \beta_k x_{Kt} + \Delta \varepsilon_t = \beta' \Delta x_t + \Delta \varepsilon_t$ where $\Delta y_t = y_t - y_{t-1}$ and so on, can be written in the form $Ay = AX\beta + A\varepsilon$ where A is a $(T-1) \times T$ matrix (i.e. find A). Show that $Ai = 0$ where i is a vector of ones.

   (b) Show using the Gauss-Markov theorem that estimation based on $Ay = AX\beta + A\varepsilon$ is less efficient than extimation based on $y = X\beta + \varepsilon$. Find the variance of the estimated parameters using the two approaches and compare.

4. † Consider the regression mode $y = S\beta_1 + X_2\beta_2 + \varepsilon$ with T quarterly obervations (so T is a multiple of 4) and S a $T \times 4$ matrix of quarterly dummies $(d_{1t}, d_{2t}, d_{3t}, d_{4t})$ where d_{1t} is 1 in quarter 1 and zero otherwise and so on.

(a) Show that S can be written as

$$S = \begin{pmatrix} I_4 \\ I_4 \\ \vdots \\ I_4 \end{pmatrix}$$

where I_4 is the 4×4 identity matrix. (Extremists can also try to show that $S = i_{T/4} \otimes I_4$ where $\otimes$ is the Kronecker product and $i_{T/4}$ is a $T/4$ vector of ones[1], but this is very optional). Hence show that

$$S'S = \frac{T}{4} I_4$$

(b) Consider a (not related) $T \times 1$ vector z. Define $z_1^* = (z_1\ z_2\ x_3\ z_4)'$, $z_2^* = (z_5\ z_6\ z_7\ z_8)'$, and so on. Show that

$$S'z = z_1^* + z_2^* + \ldots + x_{T/4}^* = \frac{T}{4} \begin{pmatrix} \bar{z}_1 \\ \bar{z}_2 \\ \bar{z}_3 \\ \bar{z}_4 \end{pmatrix}$$

where $\bar{z}_i$ is the average for the i'th quarter

(c) Construct the "deseasonalizing" transformation $M_S = I_T - S\,(S'S)^{-1}\,S'$, and show that the effect of pre-multiplying a vector z is to change z into a vector where each elemtn is the deviation from the quarterly average $\bar{z}_i$.

(d) Use this result to show that the vector β_2 can equivalently be estimated from a regression of quarterly deseasonalized y on quarterly deseasonalized X (Hint: Frisch-Waugh)

In Greene:

Ch. 3, Ex. 3, 4, 10;

Ch 4, Ex. 1, 7, 12†, 14†;

Ch. 5, Ex. 1, 2;

Ch. 8, Ex. 1, 9, 10, 12

† denotes more difficult exercises

[1] Notice that there was a typo here in an earlier version