ECON4160 ECONOMETRICS – MODELLING AND SYSTEMS ESTIMATION

Forty exercises.

For seminar discussion, individual training, exam preparation etc.

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EXERCISE 1. Specify the regression equation

$$y_i = \beta_0 + \beta_1 x_{1i} + \beta_2 x_{2i} + u_i, \qquad i = 1, \dots, n,$$

as a complete econometric model when the x's are considered as stochastic. Give an interpretation of the assumptions you have specified. Express

$$m_{yk} = M[y, x_k] = \frac{1}{n} \sum_{i=1}^{n} (y_i - \bar{y})(x_{ki} - \bar{x}_k) \ , \qquad k = 1, 2,$$

by means of β_0, β_1, β_2, the (empirical) variances and the covariances of the x's as well as the covariances between the x's and the u's. Let $\widehat{\beta}_1$ and $\widehat{\beta}_2$ be the ordinary least squares estimators of β_1 and β_2. Derive expressions for $\widehat{\beta}_1 - \beta_1$ and $\widehat{\beta}_2 - \beta_2$ and use the result to show that $\widehat{\beta}_1$ and $\widehat{\beta}_2$ are unbiased and consistent when the standard assumptions of the regression model are satisfied. Which of these assumptions are *necessary* to ensure that the estimators are

(a) Gauss-Markov-estimators (MVLUE)?
(b) unbiased?
(c) consistent?

EXERCISE 2.
(a) Explain precisely, in relation to a simple regression model with stochastic right hand side variables, what is meant by saying that the right hand side variables are exogenous. If you can give alternative definitions of exogeneity, it would be fine.

(b) Discuss critically the following statement: "In order to use ordinary least squares as a method for estimating a regression model we have to assume that the regression equation is linear and that the disturbances are mutually uncorrelated and have zero expectations and constant variances."

EXERCISE 3. Explain the properties of the ordinary least squares estimators of the coefficients of linear regression models whose disturbances are (i) heteroskedastic, (ii) autocorrelated. Which estimation methods would you recommend in such situations? Give the reason for your answer.

EXERCISE 4. Explain what is meant by multicollinearity. In which ways can multicollinearity frequently be an obstacle in estimating the coefficients of a regression equation? It is often asserted that multicollinearity mainly occurs in analyzing time series data. Do you agree, and if so, why? You may well use examples to illustrate your points. Which characteristics would you use to detect multicollinearity in a specific case?

EXERCISE 5. Consider the regression equation

$$(i) \qquad Y = \alpha + \beta X + \gamma Z + u.$$

In addition we assume that a linear relationship exists between X and Z, of the form

$$(ii) \qquad Z = \lambda + \delta X + v,$$

where u and v are disturbances. We assume that $E(u|X) = E(v|X) = 0$, $E(u^2|X) = \sigma_u^2$, $E(v^2|X) = \sigma_v^2$, $E(uv|X) = \sigma_{uv}$. Show that these assumptions imply that $E(u) = E(v) = 0$, $cov(u, X) = cov(v, X) = 0$, $E(u^2) = \sigma_u^2$, $E(v^2) = \sigma_v^2$, $E(uv) = \sigma_{uv}$. Derive expressions for $cov(u, Z)$ and $cov(v, Z)$. Which of the assumptions with respect to the disturbances above should be satisfied for ordinary least squares estimation of (i) to give consistent estimators of β and γ?

EXERCISE 6. We are interested in estimating an Engel function for a consumption commodity from data from a survey of households' consumption expenditures. We have, however, only observations in the form of group means for K groups of households (e.g., from a table in a publication). Assume that we formulate our regression equation as follows:

$$\bar{y}_k = \alpha + \beta \bar{x}_k + u_k, \qquad k = 1, \dots, K,$$

where $\bar{y}_k$ is the mean expenditure of the relevant commodity in group k and $\bar{x}_k$ is the corresponding mean value of total expenditure. The number of households in group k is n_k $(k = 1, \dots, K)$, which is assumed to be known. Do you have comments on this model formulation? How would you proceed to estimate the parameters of this Engel function? Try to compare your proposed estimator of β with the one you would have applied if you had had access to the observations from all the $n = \sum n_k$ individual households participating in the survey of households' consumption expenditures.

EXERCISE 7. We want to estimate a (Keynesian) consumption function on the basis of aggregate time series data. The function can be formulated as a linear regression equation on three different ways:

$$(a) \qquad \frac{C_t}{P_t} = \alpha + \beta \frac{R_t}{P_t} + u_t,$$

$$(b) \qquad C_t = \alpha P_t + \beta R_t + v_t,$$

$$(c) \qquad \frac{C_t}{R_t} = \alpha \frac{P_t}{R_t} + \beta + w_t,$$

where C_t and R_t are, respectively, consumption expenditure and income at current values and P_t is a consumer price index. Which assumptions would you make about the disturbances u_t, v_t and w_t? Explain how you would estimate the marginal propensity to consume, β, (i) by OLS and (ii) by GLS, in the three cases. What can you say about the properties of the estimators? In the consumption functions above we have assumed that the consumers do not have money illusion, since a proportional change in P_t and R_t is assumed to leave the real value of consumption, C_t/P_t, unchanged. Could you, by modifying the specifications (a), (b) or (c) in suitable ways, use them as a starting point for investigating econometrically whether the consumers have money illusion?

EXERCISE 8. You have been given the task of estimating a linear regression equation between the total capital stock of a production sector and the output of the sector from annual data. You strongly suspects that the disturbances of the equation in any two successive years are positively correlated. Assume that you represent this by means of a first order autoregressive process (AR(1) process).

(i) Which could be the reasons for having such a suspicion?
(ii) How would you formulate the regression model?
(iii) How would you estimate the coefficients of the regression equation?
(iv) Which are the properties of the OLS in this case?
(v) Could there be arguments for estimating the model after the variables have been transformed to first-differences?

EXERCISE 9.
(a) Describe one of more methods which you would find useful in investigating whether the disturbances u_t or v_t or w_t in models (a)–(c) in Exercise 7 exhibit heteroskedasticity.
(b) Describe one or more methods that you would find useful in order to investigate whether the disturbance of the model you have chosen in Exercise 7, exhibits autocorrelation.

EXERCISE 10. Explain the differences between the Generalized Least Squares (GLS) and the Feasible Generalized Least Squares (FGLS) methods. Explain how you would proceed to apply the latter method in a specific situation. What can you say about the properties of the FGLS method?

EXERCISE 11. Give examples illustrating the use of the GLS in estimating the coefficients of a single regression equation. Can it be convenient to apply this estimation method in situations where two or more regression equations are combined into one equation (SUR)? Explain.

EXERCISE 12. Consider a static model of household consumer demand specified as the linear expenditure systems (LES).

(a) Derive the corresponding cost function and indirect utility function. Which are the relationships between them?

(b) Show that the demand system satisfies Roy's identity.

(c) Some econometricians have taken a parametric specification of the indirect utility function as their point of departure when developing empirical models of consumer demand (f.ex. models based on additive indirect utility functions).

(d) Which arguments may be given for and against such a modelling strategy?

EXERCISE 13. A researcher wants to analyze the substitution between the production factors labour and capital in a sector. He/she chooses for this purpose a CES production function. The model contains, inter alia, the following variables:

K = Capital input.
L = Labour input.
W = Wage rate.
C = Price of capital services (user cost of capital).

Assume that the producers are price takers and minimize the cost of the two factors for given production. It can be shown that the following relationship holds (derivation is not required):

$$\ln\left(\frac{K}{L}\right) = a - \sigma \ln\left(\frac{C}{W}\right),$$

where σ is the elasticity of substitution between labour and capital and a is a constant. The researcher wants to analyze the substitution properties econometrically from annual data, but thinks that this static equation is too simple because the factor ratio must be assumed to respond to changes in the price ratio with some sluggishness. He/she therefore will use a dynamic specification, represented by a lag distribution.

The researcher sets $y = \ln(K/L)$ and $x = \ln(C/W)$ and will attempt using three different dynamic versions:

(1) $$y_t = a + b_0 x_t + b_1 x_{t-1} + b_2 x_{t-2} + \varepsilon_t,$$

where a, b_0, b_1, b_2 are free parameters,

(2) $$y_t = a + b_0 x_t + b_1 x_{t-1} + b_2 x_{t-2} + \varepsilon_t, \text{ where } b_i = b_0\left(1 - \frac{i}{3}\right), \; i = 0, 1, 2,$$

and a and b_0 are free parameters,

(3) $$y_t = \alpha + \beta x_t + \lambda y_{t-1} + \varepsilon_t,$$

where α, β, λ are free parameters and $0 \leq \lambda < 1$. [Hint: It may be convenient to consider (3) as a first order linear difference equation in y.]

(a) Explain briefly what we, in general, mean by the term lag distribution. Next, explain what sort of lag distributions between x and y which are represented by (1)–(3).

(b) Give econometric specifications of (1), (2) and (3) and explain how you would estimate (b_0, b_1, b_2) in (1) and (2) and (β, λ) in (3). The researcher has obtained the following estimates of the coefficients in the three equations:

For (1): $(\widehat{b}_0, \widehat{b}_1, \widehat{b}_2) = (-0.40,\ -0.25,\ -0.20)$.
For (2): $\widehat{b}_0 = -0.45$.
For (3): $\widehat{\beta} = -0.21,\ \ \widehat{\lambda} = 0.75$.

Estimate the effect on $\ln(K/L)$ of a change in $\ln(C/W)$ which, according to the three models and results above, are realized

(i) in the current year,
(ii) after one year,
(iii) after two years,
(iv) after three years,
(v) in the long run.

Explain briefly what these results say about the substitution between labour and capital in the sector analyzed.

EXERCISE 14. Using a Logit model for qualitative (discrete) response, we want to analyze factors which can determine whether a household uses tobacco or not. Formulate a simple Logit model, and explain briefly its interpretation. Let x_i denote the vector of explanatory variables for household no. i, and let β denote the vector of the corresponding coefficients, which occur in the form $x_i\beta$ in the exponential function in the Logit expression. The explanatory variables include, inter alia, the income, the relative price of tobacco (price of tobacco divided by the consumer price index), the number of persons in four age groups (given below), the age and year of birth of the head person. Estimation based on Norwegian data for 25 180 households observed over the period 1975-1994 by means of the maximum likelihood method gave the following estimates of the corresponding coefficients in the β-vector and their standard deviations (in parenthesis):

income	0.1716	(0.0355)
price	-0.1053	(0.1552)
no. of persons 0-15 years	-0.1082	(0.0164)
no. of persons 16-30 years	0.2361	(0.0215)
no. of persons 31-60 years	0.3543	(0.0286)
no. of persons 61-99 years	0.2613	(0.0405)
age divided by 10	-0.4875	(0.0605)
year of birth divided by 10	-0.2056	(0.0558)

Interpret these results, and explain what they say about the effect of the income, the tobacco price, the number of persons in the four age classes, and the age and year of birth of the head person on the propensity to use tobacco in Norwegian households.

EXERCISE 15. One is frequently interested in analyzing factors which determine qualitative variables. Often such variables are represented by variables which only can take the values zero or one. Give reasons why using a linear regression model may be inconvenient when the endogenous variable is such a binary variable. Describe, preferably by means of an example, a simple Logit model, and explain briefly why it may be more suitable than a classical linear regression model in analyzing the effect of the specified explanatory variables.

EXERCISE 16. Explain precisely the terms structural form and reduced form of a simultaneous, linear equation system. Explain precisely the difference between a simultaneous linear equation system and a system of linear regressions equations.

EXERCISE 17. Consider a market model of a consumer commodity:

$$(S) \qquad y_t = \alpha_1 + \beta_1 p_t + \gamma_{11} z_{1t} + u_t,$$
$$(D) \qquad y_t = \alpha_2 + \beta_2 p_t + \gamma_{22} z_{2t} + \gamma_{23} z_{3t} + v_t,$$

where (S) is the supply function, (D) is the demand function, y is the quantity traded, p is the market price, the z's are three exogenous variables and u and v are disturbances. If you can propose specific interpretations of the z's it will be fine.

(i) Formulate the market model as a complete econometric model.

(ii) Derive the model's reduced form.

(iii) What are the conditions for the supply function and the demand functions, respectively, to be identifiable?

(iv) A proposal has been made to estimate the model's parameters by first applying OLS on the reduced form equations. Discuss this procedure.

(v) Assume that the quantity supplied is price inelastic. How would you reformulate the model in order to take this into account?

(vi) Can it be convenient to estimate the demand function by means of OLS in a situation as described under (v), and if so, how?

EXERCISE 18. Consider the following simple model of a market for a consumer commodity:

$$(D) \qquad y_t = a + b\,p_t + u_t \qquad \text{(demand function)},$$
$$(S) \qquad y_t = c + d\,p_t + v_t \qquad \text{(supply function)},$$

where y_t is the quantity traded of the commodity in year t, p_t is the market price in year t $(t = 1, \ldots, T)$, a, b, c, d are unknown constants and where the disturbances u_t and v_t are uncorrelated, have zero expectations and variances equal to σ_u^2 and σ_v^2, respectively. It is assumed not to be correlation between the disturbances from different years. We want to estimate the price sensitivity of the demand, b, from the T observations of the market price and the traded quantity.

(i) Show that the OLS estimator of the price coefficient in (D), denoted as $\hat{b}$, has probability limit

$$\text{plim}(\hat{b}) = \frac{\sigma_v^2\, b + \sigma_u^2\, d}{\sigma_v^2 + \sigma_u^2}.$$

(ii) How would you interpret this result? Explain briefly what you understand by *simultaneity bias.*

A statistician shows the model to three economists, A, B and C, who all assert that it is erroneously specified. They add the following:

A: "In this market not only the commodity price, but also the oil price (which is an important factor price) and the interest rate affects the supply of the commodity."

B: "In this market, the supply is approximately price inelastic, i.e., d is approximately zero."

C: "From my experiences, inter alia based on the situation in other countries, the supply is approximately proportional to the market price, i.e., in (S) $c = 0$ holds as a good approximation."

How would you formulate the model and estimate the coefficient b in order to take into account the comments from, respectively,

(iii) economist A, (iv) economist B, (v) economist C?

State the properties of the estimators in cases (iii) and (iv). In point (v) a sketch of the argument is sufficient.

(vi) It may be some reason for calling the assumption that u_t and v_t are uncorrelated into doubt. Would any of your conclusions in points (iii)–(v) be changed if you omitted this assumption?

EXERCISE 19. Consider the following partial market model for a commodity:

(1) $\quad p_t = a_0 + a_1 x_t + u_t,$

(2) $\quad x_t = b_0 + b_1 p_{t-1} + v_t,$

where p_t and x_t denote the price of and the traded quantity of the commodity in period t and the disturbances u_t and v_t are assumed to be stochastically independent for all t.

(i) Discuss briefly this econometric model.

(ii) Examine whether the structural coefficients are identified.

(iii) Discuss methods for estimating those structural coefficients which you find identifiable.

EXERCISE 20. We want to estimate the marginal propensity to consume of income from aggregate time series data from Norwegian households. Discuss the choice of model, estimation problems and estimation methods for the following cases:

(i) The consumption function belongs to a macro model where both consumption and income are endogenous variables.

(ii) Income is considered as exogenous, but we have to take into consideration that our measurements of this variable contain random errors. Explain in this connection what you understand by random measurement errors.

(iii) Income is considered as exogenous, and both the observations on consumption and income are affected by measurement errors.

EXERCISE 21. We return to the model in Exercise 5. Consider now (i) and (ii) as a simultaneous model with two stochastic equations. What are the conditions for equation (i) being identifiable? Can a connection be said to exist between the multicollinearity problem and the identification problem?

EXERCISE 22. We want to estimate the marginal propensity to consume of income in a consumption function on the basis of cross-section data from a sample of households. We find it permissible to consider the household income as exogenous, but we suspect our income variable to contain a random measurement error.

(a) Compare the following estimation methods:

(i) regress observed consumption (Y) on observed income (X),
(ii) regress observed income (X) on observed consumption (Y),

and examine the asymptotic bias (inconsistency) in the (derived) estimators of the marginal propensity to consume in the two cases.

(b) Assume that you know, from other investigations, that the variance of the measurement error in income, by experience, amounts to 10 per cent of the variance of the observed income. How could you utilize this information to form a consistent estimator of the marginal propensity to consume? Perform the estimation by means of the following data for empirical variances and covariance from the consumption and income surveys of Statistics Norway for 1973:

$$M_{YY} = 660859, \qquad M_{XX} = 399334, \qquad M_{YX} = 289837.$$

EXERCISE 23. Consider again the market model in Exercise 17.

(a) Explain the indirect least squares (ILS) method in general and how you could use this method to estimate the coefficients of the supply and the demand functions, (S) and (D).

(b) Another possibility is to use the two-stage least squares method. Explain this method in general and how it can be applied in this specific example. Which conditions should be satisfied for the two-stage least squares method to be applicable?

(c) Explain precisely why an equation which does not satisfy the order condition for identification, cannot be estimated by the two-stage least squares method.

EXERCISE 24. Consider a model with the structural equations

$$y_{1t} = \alpha_1 + \beta_1 y_{2t} + \gamma_{11} x_{1t} + \gamma_{12} x_{2t} + \varepsilon_{1t},$$
$$y_{2t} = \alpha_2 + \beta_2 y_{1t} + \gamma_{13} x_{3t} + \gamma_{24} x_{4t} + \varepsilon_{2t},$$

where the y's are endogenous and the x's are exogenous variables, all of which are observable. Show that this model specification imposes two restrictions on its reduced form coefficients. Describe an estimation method for the model's coefficients which takes these conditions into account.

EXERCISE 25.
(a) Explain briefly, but precisely, the *general* meaning of the term *the identification problem* in an econometric context. Explain the order condition, and explain precisely which requirements the model must satisfy in order to use the order condition to investigate whether an equation in a simultaneous linear system is identifiable. What is required for all equations in such a system to be identifiable?

(b) Discuss reasons why it may often be necessary to formulate and use a simultaneous systems of structural equation as the basis for an econometric analysis, even if the primary purpose of the analysis is to estimate the coefficients of one of the structural equations.

EXERCISE 26. It is reasonable to assume, as an approximation, that the production structure in a sector can be described by a Cobb-Douglas production function in labour and capital, with constant returns to scale (scale elasticity equal to 1). Specify econometric models for the following cases:

(a) the producers act as profit maximizers with capital as exogenously determined input,
(b) the producers act as cost minimizers with exogenously determined output.

How would you estimate the input elasticity of labour from cross section data from a sample of firms in the two cases? Discuss problems of identification, data problems, and measurement problems that may arise.

EXERCISE 27. Specify an econometric model based on a CES production function in labour and capital

(a) when the producers are profit maximizers,
(b) when the producers are cost minimizers with exogenously given output.

Explain how you, in case (b), would (i) estimate the elasticity of substitution between the two factors and (ii) investigate whether the Cobb-Douglas production function is an acceptable simplification of the production structure.

EXERCISE 28.
(a) Explain precisely the meaning of the term *instrumental variable* in an econometric context.

(b) We want to estimate the marginal propensity to consume of income in a simple keynesian consumption function. The function is part of a simple macro model of an open economy together with, for instance, an investment relation, an import relation and a general budget equation. Specify such a model econometrically. Indicate precisely which variables are exogenous and endogenous. Examine whether the consumption function of your proposed model can be identified. Assume that you have access to mean values and empirical variances and covariances of all the exogenous and endogenous variables in the model. How would you, on the basis of this data set, estimate the marginal propensity to consume by means of a procedure based on instrumental variables? Will all sample means and variances/covariances have to be known for this purpose? What can you say about the properties of the proposed estimators?

EXERCISE 29. Estimating structural coefficients within the framework of econometric models based on cross section data from micro units, may often give results departing widely from those obtained by using models based on aggregated time series data. Discuss, preferably by means of examples, possible explanations of such discrepancies. Describe also ways of combining cross-section data and time-series data in estimating the coefficients of a structural equation.

EXERCISE 30. Assume that you have been given the task to utilize a simultaneous econometric model in formulating predictions of the model's endogenous variables for given values of its exogenous variables. For this purpose it is sufficient to have estimated the model's reduced form. An assertion has been made that in such a situation it is unnecessary to be concerned with estimating the model's structural form and with the problem of identification, since one can formulate the model's reduced form and estimate its coefficients by ordinary least squares directly. Discuss this assertion.

EXERCISE 31. We want to examine how the volume of imports of a good, M, is changed when the real income in the importing country, X, and the ratio of the prices of the import of the good and a related good produced in the importing country, P, is changed. We transform the variables into logarithms and specify the following import function:

$$m_t = \alpha + \beta x_t + \gamma p_t + u_t, \tag{1}$$

where α, β and γ are unknown coefficients, the subscript t $(t = 1, \ldots, T)$, denotes year, $m = \ln(M)$, $x = \ln(X)$, $p = \ln(P)$ and u_t is a disturbance.

(a) Specify a complete econometric model version for (1) when x and p are considered as exogenous and u_t has zero expectation, is serially uncorrelated and

$$\text{var}(u_t|x_t) = \theta^2 x_t,$$

where θ is an unknown constant. Explain how you would proceed to estimate β and γ as well as θ in an optimal way. State the reasons for your choice of method.

(b) We are not satisfied with this model description, however. We consider the same import function, (1), but change the specification of the distribution of the disturbances into: u_t has zero expectation,

$$u_t = \rho u_{t-1} + \varepsilon_t,$$

where ρ is a known constant, $\rho \in (-1, +1)$. We further assume that ε_t has zero expectation, is serially uncorrelated and

$$\text{var}(\varepsilon_t|x_t) = \lambda^2 x_t,$$

where λ is an unknown constant. Give reasons why we may want to make this change. How should we proceed to estimate β and γ as well as λ in an optimal way? Assuming ρ known may seem unreasonable. How would you carry out the estimation if it is unknown?

(c) An analysis of two import functions of the type (1), one for good 1 and one for good 2, shall be performed. We specify them as

$$m_{1t} = \alpha_1 + \beta_1 x_t + \gamma_1 p_{1t} + u_{1t}, \tag{2}$$

$$m_{2t} = \alpha_2 + \beta_2 x_t + \gamma_2 p_{2t} + u_{2t}, \tag{3}$$

where $\alpha_1, \alpha_2, \beta_1, \beta_2, \gamma_1, \gamma_2$ are unknown coefficients, (m_{1t}, m_{2t}) are the logarithms of the two import volumes and (p_{1t}, p_{2t}) the logarithms of their relative import/home market prices. For simplicity, we now assume that the disturbances, (u_{1t}, u_{2t}), have zero expectations, constant variances and are serially uncorrelated, but we allow for $\text{cov}(u_{1t}, u_{2t}) \neq 0$. State your additional assumptions and explain how you would estimate β_1, β_2 and γ_1, γ_2 in an optimal way.

(d) From the theory leading to (2)–(3) there are reasons to believe that $\beta_1 = \beta_2$, that is, the two import commodities have the same income elasticity. Why would it be unwise not to take this restriction into account? How would you proceed in order to exploit it in your estimation?

(e) Another critique raised against the model in **(c)** is that the linear functional form is too simple. Choose a model version and explain briefly how you could perform a test to decide whether assuming that the logarithm of the import volumes are linear in the logarithms of the right hand side variables is an acceptable simplification.

EXERCISE 32. The following model for the market for loans from banks to business firms in the USA is specified:

(1) $$Q_t = \alpha_0 + \alpha_1 RR_t + \alpha_2 RS_t + \alpha_3 Y_t + v_t,$$

(2) $$Q_t = \beta_0 + \beta_1 RR_t + \beta_2 RD_t + \beta_3 X_t + u_t,$$

where (1) is the supply function for loans, (2) is the demand function for loans, t ($t = 1, \dots, T$) represents the calendar month, u_t and v_t are disturbances and

Q_t = total commercial loans (in billions of dollars),
RR_t = average interest rate charged by banks,
RS_t = Treasury bill rate (considered an alternative rate of returns for banks),
RD_t = interest rate of bonds issued by firms (considered as the price of alternative financing to firms),
X_t = a production index for manufacturing (considered an indicator of the activity level of the economy),
Y_t = total bank deposits (in billions of dollars).

The model's endogenous variables are assumed to be Q_t and RR_t.

(a) Give an interpretation of the model, complete the econometric model description, and examine whether the demand and supply functions are identified. Explain briefly, but precisely what we mean by saying that an equation in an econometric model is identified.

(b) Equations (1) and (2) have been estimated by ordinary least squares (OLS) as well as two-stage least squares (2SLS) from monthly data from the years 1979–1984 ($T = 72$) for the USA. Explain briefly the 2SLS procedure in the present case, and state the most important properties of the two estimation methods. The coefficient estimates are given below, the $\widehat{\alpha}_j, \widehat{\beta}_j$'s denoting OLS estimates and the $\widetilde{\alpha}_j, \widetilde{\beta}_j$'s denoting 2SLS estimates (t-values in parentheses. For 2SLS they are the ratios between the coefficient estimates and their asymptotic standard errors.)

A. For the supply function:

$\widehat{\alpha}_1 =$	2.41	(2.9)	$\widetilde{\alpha}_1 =$	6.90	(3.6)
$\widehat{\alpha}_2 =$	−1.89	(−1.8)	$\widetilde{\alpha}_2 =$	−7.08	(−3.1)
$\widehat{\alpha}_3 =$	0.33	(51.3)	$\widetilde{\alpha}_3 =$	0.33	(42.9)
$R^2 =$	0.7804	(OLS)	$R^2 =$	0.7485	$(2SLS)$

B. For the demand function:

$\widehat{\beta}_1 =$	−15.99	(−12.0)	$\widetilde{\beta}_1 =$	−20.19	(−12.6)
$\widehat{\beta}_2 =$	2.29	(5.4)	$\widetilde{\beta}_2 =$	2.34	(5.2)
$\widehat{\beta}_3 =$	36.07	(14.2)	$\widetilde{\beta}_3 =$	46.76	(14.4)
$R^2 =$	0.9768	(OLS)	$R^2 =$	0.9666	$(2SLS)$

Here R^2 denotes the squared coefficient of multiple correlation, computed as the squared coefficient of correlation between Q_t and the computed (predicted) values of Q_t from the respective estimated equations. Do you find the signs of the coefficient estimates reasonable?

(c) Two economists, A and B, who have taken a course in elementary regression analysis, consider the table presenting the above results. Economist A states that "with the data set of 72 observations used, the OLS estimates must be preferable to the 2SLS estimates since the R^2 values of the former exceed those of the latter in the equations". Economist B argues that "we should look at the t-values to decide which of the two methods is the best one; R^2 is not a good quality measure in this case." State whether you agree or disagree with these statements, and give the reasons for your answer. Also indicate whether your would have liked additional calculations to be performed in order to give a better foundation for your conclusions.

(d) As a part of estimation by the 2SLS, the model's reduced form is estimated by OLS. Not infrequently, the R^2 values from this kind of estimation of the reduced form equations are fairly large. Explain briefly why we in such cases often will find that the 2SLS estimates are close to the OLS estimates.

EXERCISE 33. A researcher engaged as an econometric consultant for a firm in a particular industry wants to estimate the input elasticities (marginal elasticities) of labour and capital, α and β, in a Cobb-Douglas production function for the firm. Her data are annual data for T years for the output volume X_t, its price P_t, the labour input L_t, its price W_t, and the input of capital services K_t and its price Q_t from this firm. The production function is specified as:

$$X_t = AL_t^{\alpha}K_t^{\beta}e^{v_t}, \qquad t = 1, \dots, T, \tag{1}$$

where A is a constant and v_t is a disturbance. The subscript t denotes year.

Provided that the firm is a price taker and has profit maximization as its objective, a fairly common estimation method for the two input elasticities is the so-called factor-share method or factor-cost-share method. It consists in estimating α and β by, respectively, (proof not required)

$$\widehat{\alpha} = \prod_{t=1}^{T} \left(\frac{W_t L_t}{P_t X_t}\right)^{1/T}, \qquad \widehat{\beta} = \prod_{t=1}^{T} \left(\frac{Q_t K_t}{P_t X_t}\right)^{1/T}, \tag{2}$$

where, for an arbitrary time series Z_t, we let $\prod_{t=1}^{T} Z_t = Z_1 \cdot Z_2 \cdots Z_T$.

(a) Interpret these estimators. Are $\widehat{\alpha}$ and $\widehat{\beta}$ unbiased and consistent for α and β, respectively, under the assumptions stated and possible additional assumptions you find it convenient to make? State the reason for your answer. What would be your conclusion if the researcher found that $\widehat{\alpha} + \widehat{\beta} > 1$?

(b) Now, the firm is not a price taker, but a dominating producer in the market in which it sells its output. We assume that the time series we observe are the results of the firm having behaved as a profit maximizing monopolist and it has been confronted with a demand function with price elasticity d. From this theory it can be shown that the firm will obtain maximal profit by letting

$$\text{marginal productivity of } L = \frac{W}{P^*}, \quad \text{marginal productivity of } K = \frac{C}{P^*},$$

where $P^* = P[1 + (1/d)]$. Assume that the firm's management knows the value of d. Would you still recommend using the factor-share method (2) in estimating α and β? If not, how would you modify this method to obtain consistent estimators for these parameters when we from our observations find

$$\prod_{t=1}^{T} \left(\frac{W_t L_t}{P_t X_t} \right)^{1/T} = 0.48, \qquad \prod_{t=1}^{T} \left(\frac{Q_t K_t}{P_t X_t} \right)^{1/T} = 0.20$$

in the following two cases and point out possible identification problems.:

(i) when you, from previous studies of the output market, know that $d = -1.8$,
(ii) when you as an econometrician do not know d.

(c) The observation period is rather long, and it therefore does not seem reasonable to assume, as in (1), that the production technology has been the same in the entire period. We therefore allow for neutral technical progress and replace (1) by

$$\text{(3)} \qquad X_t = A_0 e^{\gamma t} L_t^{\alpha} K_t^{\beta} e^{v_t}, \quad t = 1, \ldots, T,$$

where A_0 and γ are positive constants. Would you then modify your conclusions about the estimation procedures for α and β in **(a)** and **(b)**? Could you propose a consistent method for estimating γ?

EXERCISE 34. We want to investigate how the capital stock of two kinds in a production sector, buildings (B) and machinery (M), depend on the production capacity of the sector. The production capacity is difficult to measure, and we have to prepare ourselves that our observations contain measurement errors. We assume that the measurement errors are random. The following model has been proposed:

$$\text{(1)} \qquad X = X^* + u,$$

$$\text{(2)} \qquad K_B = \alpha_B + \beta_B X^* + \varepsilon_B,$$

$$\text{(3)} \qquad K_M = \alpha_M + \beta_M X^* + \varepsilon_M,$$

where K_B and K_M are the observed stocks of the two kinds of capital, X is the observed production capacity, X^* is the actual, unobserved magnitude of this variables, u is a random measurement error, and ε_B and ε_M are disturbances. The data are sampled from a cross-section of firms in a certain year. We assume that X^*, u, ε_B and ε_M are mutually uncorrelated and that u, ε_B and ε_M have zero expectations and variances σ_u^2, σ_B^2 and σ_M^2, respectively.

(a) Comment briefly on the model, and examine whether $\alpha_B, \beta_B, \alpha_M$ and β_M are identifiable, possibly under which additional assumptions.

(b) Propose estimators of these four parameters.

(c) Assume that there exists a third kind of capital, transport equipment, used by the firms in the sector, with observed value K_T, and assume that we extend the model (1)–(3) by adding the equation

$$K_T = \alpha_T + \beta_T X^* + \varepsilon_T. \tag{4}$$

Would this influence the estimation method you would propose in question **(b)**?

EXERCISE 35. An econometric version of a very simple keynesian macro model consists of two equations:

$$C_t = \alpha + \beta Y_t + u_t, \tag{1}$$
$$Y_t = C_t + Z_t. \tag{2}$$

where C_t, Y_t, Z_t are, respectively, private consumption, GNP, and a variable representing the sum of gross investment, public consumption, and export surplus, all measured at constant prices, and u_t is a disturbance with zero expectation. Subscript t denotes year, and the observations on C, Y and Z cover the years $t = 1, \dots, T$. All variables are considered as stochastic and we assume

$$\text{cov}(u_t, Z_s) = 0; \tag{3}$$
$$\text{var}(u_t) = \sigma^2; \quad \text{cov}(u_t, u_s) = 0, \quad t \neq s; \quad t, s = 1, \dots, T. \tag{4}$$

We denote the model (1)–(4) as *Model **A***.

(a) Interpret assumption (3), and explain briefly why estimating the consumption function (1) by Ordinary Least Squares gives inconsistent estimators of α and β. Explain verbally what we mean by saying that an estimator is inconsistent. Describe a consistent method for estimating the marginal propensity to consume, β and the consumption multiplier of Z, $\Delta C/\Delta Z = \beta/(1-\beta)$, and explain why the method in both cases is consistent.

(b) We are in doubt that the disturbance in Model **A** satisfies assumptions (4), but we still believe that (3) holds. How would you estimate β if we replace (4) with, respectively,

$$\text{var}(u_t|Z_t) = \tau^2 Z_t, \quad \text{cov}(u_t, u_s) = 0, \quad t \neq s, \tag{5}$$

and

$$u_t = \rho u_{t-1} + \varepsilon_t \quad (|\rho| < 1), \quad \text{var}(\varepsilon_t) = \theta^2, \quad \text{cov}(\varepsilon_t, \varepsilon_s) = 0, \ t \neq s, \tag{6}$$

where τ and θ are positive constants. You may, if you want, assume that ρ is known, but if you could indicate briefly how it could be estimated, it would be fine.

The institution which constructs the national accounts from which the time series C_t, Y_t and Z_t have been taken – let us call it SSB – finds it difficult, from the primary statistics available, based on records from the specific agents in the economy, to construct these time series exactly as the keynesian macro theory prescribes. As alternatives to Model **A** in order to allow for measurement errors in different ways, three models have been proposed:

*Model **B**: Random measurement errors in all variables:*

$$(7) \qquad C_t^* = \alpha + \beta Y_t^* + u_t,$$
$$(8) \qquad Y_t^* = C_t^* + Z_t^*.$$

where C_t^*, Y_t^* and Z_t^* are the unobservable theory variables private consumption, GNP, and investment etc. What SSB calculates, and is observed by us are C_t, Y_t and Z_t determined by, respectively,

$$(9) \qquad \begin{aligned} C_t &= C_t^* + v_{Ct}, \\ Y_t &= Y_t^* + v_{Yt}, \\ Z_t &= Z_t^* + v_{Zt}. \end{aligned}$$

where v_{Ct}, v_{Yt} and v_{Zt} er random measurement errors.

*Model **C**: Random measurement errors in (C, Y), systematic error in Z:*
In this model we assume that (7) and (8) still hold, but that SSB's calculations implies that we observe C_t, Y_t and Z_t determined from, respectively,

$$(10) \qquad \begin{aligned} C_t &= C_t^* + v_{Ct}, \\ Y_t &= Y_t^* + v_{Yt}, \\ Z_t &= a_Z + b_Z Z_t^* + v_{Zt}, \end{aligned}$$

where a_Z and b_Z are unknown constants and v_{Ct}, v_{Yt} and v_{Zt} are random error terms.

*Model **D**: Systematic measurement errors in all variables:*
Also in this model we assume that (7) and (8) still hold, whereas SSB's calculations now give us observations on C_t, Y_t and Z_t determined from, respectively,

$$(11) \qquad \begin{aligned} C_t &= a_C + b_C C_t^* + v_{Ct}, \\ Y_t &= a_Y + b_Y Y_t^* + v_{Yt}, \\ Z_t &= a_Z + b_Z Z_t^* + v_{Zt}, \end{aligned}$$

where a_C, b_C, a_Y, b_Y, a_Z and b_Z are unknown constants and v_{Ct}, v_{Yt} and v_{Zt} are random error terms.

In models **B**, **C** and **D** we assume that u_t, v_{Ct}, v_{Yt}, v_{Zt} and Z_t^* are mutually uncorrelated and that u_t, v_{Ct}, v_{Yt} and v_{Zt} have zero expectations, variances equal to σ_u^2, σ_{vC}^2, σ_{vY}^2 and σ_{vZ}^2, respectively, and non-autocorrelated.

(c) Would it be possible to estimate the marginal propensity to consume of income β consistently from SSB's time series for C_t, Y_t, Z_t in models **B**, **C** and **D**, and if so, how?

(d) Going more deeply into SSB's data base you have been able to split the time series for Z_t into I_t = gross investment, G_t = public consumption, and A_t = export surplus, such that $I_t + G_t + A_t = Z_t$. Would you then modify your answer regarding estimation procedures in problem **(c)**, and if so, how?

EXERCISE 36. The following simple model for determination of the quarterly development of wages and prices in a country is specified:

$$\dot{p}_t = a_0 + a_1 \dot{w}_t + a_2 \dot{p}_{It} + a_3 \dot{q}_t + \epsilon_t, \quad (1)$$

$$\dot{w}_t = b_0 + b_1 \dot{p}_t + b_2 u_t + b_3 n_t + \delta_t, \quad (2)$$

where subscript t denotes quarter, the a's and b's are constants, ϵ_t and δ_t disturbances and

$\dot{p}_t$ = Rate of increase of the consumption price index, pro anno.
$\dot{w}_t$ = Rate of increase of the mean wage rate, pro anno.
$\dot{p}_{It}$ = Rate of increase of the import price index, pro anno.
$\dot{q}_t$ = Rate of increase of the labour productivity, pro anno.
u_t = The unemployment rate at the beginning of the quarter.
n_t = Share of the labour force unionized at the beginning of the quarter.

In view of the purpose that the model is intended to serve, we consider $\dot{p}$ and $\dot{w}$ as endogenous variables and consider $\dot{p}_I$, $\dot{q}$, u and n as exogenous.

(a) Discuss the model briefly, and specify it stochastically. Examine whether the two equations are identifiable.

Two authors presented in an article published in the journal *Economica* in 1970 estimation results for the price equation (1) and the wage equation (2) based on ordinary least squares and quarterly data for Great Britain for the period 1948:3-1968:2 (80 observations). In parts of the period, the central government had introduced a wage and price control. The authors found, inter alia:

A. For the price equation (1):
(i) Using the complete data set:

$$\begin{aligned} \widehat{a}_1 &= 0.562 \quad (5.53) \\ \widehat{a}_2 &= 0.085 \quad (4.60) \\ \widehat{a}_3 &= -0.145 \quad (-3.48) \\ R^2 &= 0.697 \quad DW = 0.946 \end{aligned}$$

(ii) Using only data from quarters where wage and price control was not in effect:

$$\begin{aligned} \widehat{a}_1 &= 0.851 \quad (5.52) \\ \widehat{a}_2 &= 0.073 \quad (2.93) \\ \widehat{a}_3 &= -0.092 \quad (-1.90) \\ R^2 &= 0.843 \quad DW = 1.274 \end{aligned}$$

(t-values in parenthesis. DW = Durbin-Watson-statistic)

B. For the wage equation (2):
(i) Using the complete data set:

$$\begin{aligned} \widehat{b}_1 &= 0.482 \quad (5.76) \\ \widehat{b}_2 &= -0.891 \quad (-1.77) \\ \widehat{b}_3 &= 3.315 \quad (2.09) \\ R^2 &= 0.616 \quad DW = 0.742 \end{aligned}$$

(ii) Using only data from quarters where wage and price control was not in effect:

$$\begin{aligned} \widehat{b}_1 &= 0.457 \quad (6.25) \\ \widehat{b}_2 &= -2.372 \quad (-3.64) \\ \widehat{b}_3 &= 0.136 \quad (0.07) \\ R^2 &= 0.856 \quad DW = 1.231 \end{aligned}$$

(t-values in parenthesis. DW = Durbin-Watson-statistic)

(b) Do you find the sign of the coefficient estimates reasonable? Describe briefly how you could perform a test to investigate whether this active price and wage policy had a significant effects on the formation of wages and prices in Great Britain in the actual period, and indicate, without going into details, additional information you would have to possess and supplementary calculations that would be required.

(c) In the article the authors state, inter alia,

(i) "We have used ordinary least squares. In order to be able to use two-stage least squares for estimating (1) and (2) it would be necessary to treat all variables except $\dot{p}$ and $\dot{w}$ as exogenous."

(ii) "If use of two-stage least squares on (1)–(2) should have been possible and we still treated $\dot{q}$, u and n as endogenous variable, we would have had to specify completely the more comprehensive model to which these two equations belong."

Discuss these two statements.

(d) Assume that you, from the estimation results above or by using some other method that you would prefer, should give the minister of finance in the country an estimate of the effect on the wage increase of an increase in, respectively, the unemployment rate and the rate of increase of the labour productivity. How would you proceed? State the reason for you answer.

EXERCISE 37. We are interested in investigating the relationship between the households' consumption and the gross national product (GNP), i.e., a kind of a macro consumption function, on the basis of time series data for Norway for the period 1865-1939 from the data base of historical data from Statistics Norway. We use the following symbols:

Y: Gross national product, at constant prices.
C: Household consumption, at constant prices.

Eight static and three dynamic specifications of the consumption function have been estimated by means of ordinary least squares (OLS):

(1) $$C_t = \alpha + \beta Y_t + u_{1t},$$

(2) $$\ln C_t = \delta + \epsilon \ln Y_t + u_{2t},$$

(3) $$\Delta C_t = \beta \Delta Y_t + u_{3t},$$

(4) $$\Delta C_t = \gamma + \beta \Delta Y_t + u_{4t},$$

(5) $$\Delta \ln C_t = \epsilon \Delta \ln Y_t + u_{5t},$$

(6) $$\Delta \ln C_t = \phi + \epsilon \Delta \ln Y_t + u_{6t},$$

(7) $$\frac{C_t}{Y_t} = \alpha \frac{1}{Y_t} + \beta + u_{7t},$$

(8) $$\frac{C_t}{Y_t} = \alpha \frac{1}{Y_t} + \beta + \beta' Y_t + u_{8t},$$

(9) $$C_t = a + bC_{t-1} + cY_t + u_{9t},$$

(10) $$C_t = \alpha + \beta_0 Y_t + \beta_1 Y_{t-1} + \beta_2 Y_{t-2} + \beta_3 Y_{t-3} + u_{10t},$$

(11) $$C_t = \alpha + \beta_0 Y_t + \beta_1 Y_{t-1} + u_{11t},$$

where $\alpha, \beta, \dots, \phi$ and a, b, c are unknown constants and $u_{1t}, \dots, u_{11t}$ are disturbances. The result, in the form of edited printouts from the computer programme package Pc-Give, denoted as EQ(1)–EQ(11), is given at the end of the set of questions. The estimate of the standard deviation of the disturbance is denoted as *"sigma"*.

(a) Explain briefly the relationship between the disturbances u_{1t}, u_{3t}, u_{7t} and between the disturbances u_{2t}, u_{5t}, and which properties u_{1t} would have to possess if OLS estimation based on (7) should give Gauss-Markov-estimators of α and β. Which interpretation would you give of the intercept term γ in (4) and the intercept term ϕ in (6)?

(b) Over such a long time span as 75 years, containing, inter alia, periods with good and bad business prospects and with a varying degree of uncertainty in the economy, it is not unlikely that the disturbance variance in (1) has varied. Explain how you would proceed to investigate this and which additional calculations such an investigation would require.

(c) From the estimation results for the consumption functions (1)–(6) it can be argued that autocorrelation in the disturbances is a markedly more prevalent phenomenon when the consumption function is estimated in level form than when it is transformed to first differences before OLS estimation is performed. Do you agree, and how would you explain this? Would you from this recommend estimation on difference form rather than on level form?

(d) Would you, from the estimation results for the consumption functions (1)–(6), support or reject (i) a hypothesis that the level of the household consumption has been an approximately constant share of GNP and (ii) a hypothesis that the increase in consumption has been a constant share of the increase in GNP over the long observation period? Perform a corresponding investigation for public consumption, denoted as G_t, on the basis of the results reported as EQ(1*)–EQ(6*) after Exercise 38.

(e) Do the estimation results under EQ(9)–EQ(11) indicate that there is a sluggishness in the adjustment of the consumption to the income? State the reason for your answer, and explain what is meant my the terms short-run and long-run propensity to consume of income. Explain what kind of lag-distribution (9) describes. Estimate these two parameters from the results under EQ(9)–EQ(11) after Exercise 38. Are the estimators you propose, consistent?

EXERCISE 38 (continuation of **EXERCISE 37**). The data set also contains:

G: Public consumption, at constant prices.
I: Investment, at constant prices.
X: Export, at constant prices.
M: Import, at constant prices.

We now consider G, I, X as exogenous variables.

(a) In equation EQ(12) at the end the consumption function (1) is estimated by two-stage least squares, with G and I used as instruments for GNP. In equation EQ(13), (4) is estimated by two-stage least squares, with ΔG and ΔI as instruments for the increase in GNP. Explain this procedure and give your opinion on whether it is sensible.

(b) In equation EQ(14) at the end an attempt is made to estimate, by OLS, an import function in the form of first differences (i.e. with ΔM as left hand side variable and ΔC, ΔG and ΔI as right hand side variables). In equation EQ(15), the same function is estimated by two-stage least squares, with ΔC considered as endogenous and ΔG and ΔI as exogenous. In the last case, ΔG, ΔI, and ΔX are used as instruments. Explain the procedure in the last case, and give your conclusions from the estimation results about the three marginal propensities to import.

```
EQ( 1) Modelling c by OLS
       The estimation sample is: 1865 to 1939

                Coefficient  Std.Error  t-value  t-prob Part.R^2
Constant            6885.14      646.0     10.7   0.000   0.6088
y                  0.685457    0.01159     59.1   0.000   0.9795

sigma        2659.99     RSS       516516181 R^2 0.979544
F(1,73) = 3496 [0.000]** log-likelihood -696.863 DW = 0.228
no. of observations 75   no. of parameters     2
mean(c) 40482.8 var(c)           3.36669e+008
------------------------------------------------------------------

EQ( 2) Modelling lnc by OLS
       The estimation sample is: 1865 to 1939
```

```
                 Coefficient  Std.Error  t-value  t-prob Part.R^2
Constant             1.06799     0.1214     8.80   0.000   0.5147
lny                 0.884891    0.01136     77.9   0.000   0.9881

sigma      0.0496391     RSS     0.179875153 R^2 0.988103
F(1,73) = 6063 [0.000]** log-likelihood 119.816 DW = 0.228
no. of observations 75   no. of parameters 2
mean(lnc) 10.5079 var(lnc)            0.201593
-------------------------------------------------------------------

EQ( 3) Modelling dc by OLS
       The estimation sample is: 1866 to 1939

                 Coefficient  Std.Error  t-value  t-prob Part.R^2
dy                  0.719745    0.04998     14.4   0.000   0.7396

sigma         1265.1     RSS       116835137
log-likelihood -633.073  DW =            2.32
no. of observations 74   no. of parameters 1
mean(dc) 922.176  var(dc) 5.21326e+006
--------------------------------------------------------------------

EQ( 4) Modelling dc by OLS
       The estimation sample is: 1866 to 1939

                 Coefficient  Std.Error  t-value  t-prob Part.R^2
Constant            -72.7657      166.8   -0.436   0.664   0.0026
dy                  0.731182    0.05669     12.9   0.000   0.6979

sigma        1272.18  RSS       116527118 R^2 0.697945
F(1,72) = 166.4 [0.000]** log-likelihood -632.976 DW = 2.31
no. of observations 74  no. of parameters          2
mean(dc) 922.176 var(dc)          5.21326e+006
--------------------------------------------------------------------

EQ( 5) Modelling dlnc by OLS
       The estimation sample is: 1866 to 1939

                 Coefficient  Std.Error  t-value  t-prob Part.R^2
dlny                0.951038    0.06180     15.4   0.000   0.7644

sigma      0.0235392      RSS    0.0404489229
log-likelihood 172.934    DW =            2.23
no. of observations 74 no. of parameters 1
mean(dlnc)          0.021408 var(dlnc) 0.00186157
--------------------------------------------------------------------

EQ( 6) Modelling dlnc by OLS
       The estimation sample is: 1866 to 1939

                 Coefficient  Std.Error  t-value  t-prob Part.R^2
Constant         -0.00235126   0.003281   -0.717   0.476   0.0071
dlny                0.980110    0.07410     13.2   0.000   0.7085

sigma       0.023618    RSS    0.0401624424 R^2 0.708452
F(1,72) = 175 [0.000]** log-likelihood 173.197 DW = 2.23
no. of observations 74  no. of parameters          2
mean(dlnc) 0.021408 var(dlnc)          0.00186157
--------------------------------------------------------------------

EQ( 7) Modelling c/y by OLS
       The estimation sample is: 1865 to 1939

                 Coefficient  Std.Error  t-value  t-prob Part.R^2
Constant            0.769203    0.01380     55.7   0.000   0.9770
1/y                  3261.26      480.4     6.79   0.000   0.3870

sigma      0.0492623      RSS     0.177154859 R^2 0.386971
F(1,73) = 46.08 [0.000]** log-likelihood 120.388 DW = 0.169
no. of observations 75    no. of parameters          2
mean(cdy) 0.85456 var(cdy)           0.0038531
------------------------------------------------------------------

EQ( 8) Modelling c/y by OLS
       The estimation sample is: 1865 to 1939

                 Coefficient  Std.Error  t-value  t-prob Part.R^2
Constant             1.14272    0.02657     43.0   0.000   0.9625
1/y                 -4140.27      563.2    -7.35   0.000   0.4288
```

```
y              -3.66814e-006 2.517e-007    -14.6   0.000   0.7468

sigma      0.0249588  RSS    0.0448517325 R^2 0.844794
F(2,72) = 196 [0.000]** log-likelihood 171.9
DW = 0.72 no. of observations 75
mean(cdy)            0.85456 var(cdy) 0.0038531
------------------------------------------------------------------

EQ( 9) Modelling c by OLS
       The estimation sample is: 1866 to 1939

                  Coefficient  Std.Error  t-value  t-prob Part.R^2
Constant              2570.59      647.2     3.97   0.000   0.1818
c_1                  0.601241    0.06423     9.36   0.000   0.5524
y                    0.288427    0.04295     6.71   0.000   0.3884

sigma       1787.69     RSS       226904926 R^2 0.990821
F(2,71) = 3832 [0.000]** log-likelihood -657.633 DW = 1.78
no. of observations 74    no. of parameters          3
mean(c) 40791.9 var(c)          3.34053e+008
--------------------------------------------------------------------

EQ(10) Modelling c by OLS
       The estimation sample is: 1868 to 1939

                  Coefficient  Std.Error  t-value  t-prob Part.R^2
Constant              6627.58      712.5     9.30   0.000   0.5636
y                    0.453792     0.1417     3.20   0.002   0.1328
y_1                 0.0697683     0.1689    0.413   0.681   0.0025
y_2                 -0.159472     0.1688   -0.945   0.348   0.0131
y_3                  0.350236     0.1487     2.35   0.021   0.0764

sigma       2573.55    RSS       443750169 R^2 0.981237
F(4,67) = 876 [0.000]** log-likelihood -664.991 DW = 0.329
no. of observations 72  no. of parameters          5
mean(c) 41425.6 var(c)          3.28475e+008
-------------------------------------------------------------------

EQ(11) Modelling c by OLS
       The estimation sample is: 1868 to 1939

                  Coefficient  Std.Error  t-value  t-prob Part.R^2
Constant              7223.78      683.9     10.6   0.000   0.6179
y                    0.616561     0.1284     4.80   0.000   0.2504
y_1                 0.0665599     0.1343    0.496   0.622   0.0035

sigma       2646.38     RSS       483230117 R^2 0.979568
F(2,69) = 1654 [0.000]** log-likelihood -668.06 DW = 0.264
no. of observations 72   no. of parameters          3
mean(c) 41425.6 var(c)          3.28475e+008
--------------------------------------------------------------------

EQ(1*) Modelling g by OLS
       The estimation sample is: 1865 to 1939

                  Coefficient  Std.Error  t-value  t-prob Part.R^2
Constant             -1583.25      198.6    -7.97   0.000   0.4653
y                    0.139897   0.003565     39.2   0.000   0.9547

sigma       818.012     RSS        48847498 R^2 0.954733
F(1,73) = 1540 [0.000]** log-likelihood -608.423 DW = 0.507
no. of observations 75   no. of parameters          2
mean(g) 5273.79 var(g)             1.4388e+007
------------------------------------------------------------------

EQ(2*) Modelling lng by OLS
       The estimation sample is: 1865 to 1939

                  Coefficient  Std.Error  t-value  t-prob Part.R^2
Constant             -7.32874     0.3653    -20.1   0.000   0.8465
lny                   1.46491    0.03420     42.8   0.000   0.9617

sigma      0.149388     RSS      1.62912252 R^2 0.961733
F(1,73) = 1835 [0.000]** log-likelihood 37.1839 DW = 0.32
no. of observations 75   no. of parameters          2
mean(lng)            8.2987 var(lng)            0.56763
-------------------------------------------------------------------

EQ(3*) Modelling dg by OLS
```

```
       The estimation sample is: 1866 to 1939

                  Coefficient  Std.Error  t-value  t-prob Part.R^2
dy                  0.0494036    0.02042     2.42   0.018   0.0742

sigma 516.926           RSS 19506545.8
log-likelihood -566.843 DW =  1.35
no. of observations 74  no. of parameters 1
mean(dg) 193.716  var(dg) 247206
-------------------------------------------------------------------

EQ(4*) Modelling dg by OLS
       The estimation sample is: 1866 to 1939

                  Coefficient  Std.Error  t-value  t-prob Part.R^2
Constant              160.904      65.56     2.45   0.017   0.0772
dy                  0.0241134    0.02228     1.08   0.283   0.0160

sigma        500.006    RSS      18000423.1 R^2 0.016008
F(1,72) = 1.171 [0.283] log-likelihood -563.87 DW = 1.36
no. of observations 74  no. of parameters          2
mean(dg)              193.716 var(dg)               247206
-------------------------------------------------------------------

EQ(5*) Modelling dlng by OLS
       The estimation sample is: 1866 to 1939

                  Coefficient  Std.Error  t-value  t-prob Part.R^2
dlny                 0.422283     0.1851     2.28   0.025   0.0665

sigma   0.0705215  RSS 0.363049645 log-likelihood 91.7379
DW =   1.77 no. of observations 74 no. of parameters 1
mean(dlng) 0.0336292              var(dlng) 0.00412476
------------------------------------------------------------------

EQ(6*) Modelling dlng by OLS
       The estimation sample is: 1866 to 1939

                  Coefficient  Std.Error  t-value  t-prob Part.R^2
Constant            0.0334052   0.009045     3.69   0.000   0.1593
dlny               0.00924111     0.2043   0.0452   0.964   0.0000

sigma      0.0651093          RSS     0.305223916 R^2 2.84243e-005
F(1,72) =    0.002047 [0.964] log-likelihood 98.1572 DW = 1.98
no. of observations 74        no. of parameters           2
mean(dlng)          0.0336292 var(dlng) 0.00412476
-------------------------------------------------------------------

EQ(12) Modelling c by 2SLS
       The estimation sample is: 1865 to 1939
       Instruments: g, i

                  Coefficient  Std.Error  t-value  t-prob
Constant              6511.29      651.7     9.99   0.000 y
Y     0.693084    0.01172     59.2   0.000

sigma        2667.87  RSS       519578581 Reduced form sigma 2103.6
no. of observations    75     no. of parameters       2
no. endogenous variables    2 no. of instruments      3
mean(c) 40482.8  var(c) 3.36669e+008
------------------------------------------------------------------

EQ(13) Modelling dc by 2SLS
       The estimation sample is: 1866 to 1939
       Instruments: dg, di

                  Coefficient  Std.Error  t-value  t-prob
Constant             -258.170      185.3    -1.39   0.168 dy
Y     0.867436    0.07610     11.4   0.000

sigma        1322.23  RSS       125877046 Reduced form sigma 1492.7
no. of observations        74  no. of parameters 2
no. endogenous variables    2  no. of instruments 3
mean(dc) 922.176  var(dc) 5.21326e+006
-------------------------------------------------------------------

EQ(14) Modelling dm by OLS
       The estimation sample is: 1866 to 1939
```

```
                 Coefficient  Std.Error  t-value  t-prob Part.R^2
Constant            -444.811      132.6    -3.35   0.001   0.1385
dc                  0.805837    0.07938     10.2   0.000   0.5955
dg                 0.0783570     0.2657    0.295   0.769   0.0012
di                  0.231318     0.1194     1.94   0.057   0.0509

sigma        998.364    RSS      69771216.5 R^2 0.825003
F(3,70) = 110 [0.000]** log-likelihood -613.998 DW = 2.57
no. of observations 74  no. of parameters         4
mean(dm)             417.081 var(dm)          5.38783e+006
--------------------------------------------------------------------

EQ(15) Modelling dm by 2SLS
       The estimation sample is: 1866 to 1939
       Instruments: dg, di, dx

                 Coefficient  Std.Error  t-value  t-prob
Constant            -393.067      159.3    -2.47   0.016 dc
Y     0.391022     0.2613     1.50   0.139 dg
0.687570     0.4759     1.44   0.153 di                    0.706447
0.3129     2.26   0.027

sigma        1177.11  RSS      96991653.6 Reduced form sigma 1555.6
no. of observations        74  no. of parameters   4
no. endogenous variables    2  no. of instruments  4
mean(dm) 417.081  var(dm) 5.38783e+006
```

EXERCISE 39. We are interesting in analyzing the capital-labour substitution in two manufacturing industries (2-digit SIC industries) from aggregate input data from the USA for the years 1958-1996 ($T = 39$ observations). The input quantities are in millions of dollars at constant 1992 prices, the price indexes are normalized to 1992 = 1, and subscript t denotes year. At the end of the set of questions are a slightly edited printout from PcGive, and a comprehensive table of critical values for the Durbin-Watson test (DW). The variables which occur in the econometric models to be considered, are [ln = the natural logarithm]:

$y_{1t} = \ln\left(\frac{K_{1t}}{L_{1t}}\right)$ = log of (capital input quantity/labour input quantity) in Industry 1,

$y_{2t} = \ln\left(\frac{K_{2t}}{L_{2t}}\right)$ = log of (capital input quantity/labour input quantity) in Industry 2,

$x_{1t} = \ln\left(\frac{P_{K1t}}{P_{L1t}}\right)$ = log of (index for capital price/index for labour price) in Industry 1,

$x_{2t} = \ln\left(\frac{P_{K2t}}{P_{L2t}}\right)$ = log of (index for capital price/index for labour price) in Industry 2,

We assume that the typical firms in the two industries have a production technology of the CES (Constant Elasticity of Substitution) form and minimizes input costs for given output. It can then be shown from the optimizing conditions that

$$\ln\left(\frac{K_{1t}}{L_{1t}}\right) = \alpha_1 + \beta_1 \ln\left(\frac{P_{K1t}}{P_{L1t}}\right) + u_{1t},$$
$$\ln\left(\frac{K_{2t}}{L_{2t}}\right) = \alpha_2 + \beta_2 \ln\left(\frac{P_{K2t}}{P_{L2t}}\right) + u_{2t},$$

where $(-\beta_1)$ and $(-\beta_2)$ are the elasticity of substitution between capital and labour in the two industries, and u_{1t} and u_{2t} are disturbances. (Proof not required.)

Using the simplified notation, the system of factor input equations can be written as

$$(12) \qquad y_{1t} = \alpha_1 + \beta_1 x_{1t} + u_{1t},$$
$$(13) \qquad y_{2t} = \alpha_2 + \beta_2 x_{2t} + u_{2t}, \qquad t = 1, \ldots, T.$$

(a) Consider the two (logarithmic) price ratios x_{1t} and x_{2t} as exogenous and assume that u_{1t} and u_{2t} are correlated. OLS (Ordinary Least Squares) estimates of Equations (1) and (2) are given in PRINTOUT, PART A, at the end of Exercise 40. Corresponding results when the two equations are estimated as a system of regression equations by FGLS (Feasible Generalized Least Squares) are shown in PRINTOUT, PART B, at the end of Exercise 40. Interpret the two sets of results and explain why the estimates differ.

(b) The standard errors of the coefficient estimates in PRINTOUT, PART B are lower than those in PRINTOUT, PART A. Do you find this an expected result? There is not a similar improvement in the t-values. Do you find the latter finding surprising? Explain briefly.

(c) Are there signs that the assumption of non-autocorrelated disturbances in Equations (1) and (2) is violated? Explain briefly.

(d) The Cobb-Douglas production function is the special case of the CES function where the elasticity of substitution is one ($-\beta_1 = -\beta_2 = 1$). Would you reject the Cobb-Douglas hypothesis from the results in the printouts? Maybe you would need more detailed output to properly answer this question? State briefly the reason for your answer.

An extension of Equations (1)–(2), where both relative input prices are assumed to enter both input equations, is also considered. The model is then specified as

$$(14) \qquad y_{1t} = \alpha_1 + \beta_{11} x_{1t} + \beta_{12} x_{2t} + v_{1t},$$
$$(15) \qquad y_{2t} = \alpha_2 + \beta_{21} x_{1t} + \beta_{22} x_{2t} + v_{2t}, \qquad t = 1, \ldots, T.$$

where v_{1t} and v_{2t} are disturbances.

(e) OLS estimates of Equations (3) and (4) are reported in PRINTOUT, PART C of the printout. Corresponding results when the two equations are estimated jointly as a system of regression equations by FGLS are shown in PRINTOUT, PART D, at the end of Exercise 40. Compare these two sets of results with those obtained in PRINTOUT, PART A and PRINTOUT, PART B and state your conclusion.

(f) A colleague examining PRINTOUT, PART A, ... ,PRINTOUT, PART D claims that since Industry 1 and Industry 2 produce outputs which are strongly different, the restriction $\text{cov}(u_{1t}, u_{2t}) = 0$ should have been imposed on Equations (1)–(2) and $\text{cov}(v_{1t}, v_{2t}) = 0$ should have been imposed on Equations (3)–(4). Would your conclusions above then have been different? Do you agree with you colleague that a difference in the nature of the outputs from the two industries is a valid reason for imposing these zero covariance restrictions? Explain briefly your argument.

EXERCISE 40 (continuation of **EXERCISE 39**). Assume now that an assumed feedback from the rest of the economy, say from the way the labour market functions, gives reason to believe that the input price ratio in Industry 2 affects the input price ratio in Industry 1, but that there is no feedback the other way. To account for this we extend the equation system (1)–(2) by adding a third equation, making x_{1t} endogenous, so that the modified system becomes

$$\begin{aligned} y_{1t} &= \alpha_1 + \beta_1 x_{1t} + u_{1t}, \\ y_{2t} &= \alpha_2 + \beta_2 x_{2t} + u_{2t}, \\ x_{1t} &= \gamma + \delta x_{2t} + \varepsilon_t, \end{aligned}$$

where ε_t is a disturbance.

(a) Give a complete specification of this model. PRINTOUT, PART E at the end gives estimation results for the first [EQ(1)] and third equation [EQ(3)] of the latter model when, because of the assumed endogeneity of x_{1t}, we use x_{2t} as instrument for x_{1t} when estimating the first equation. Explain briefly why x_{2t} is a valid instrument and why we do not need to reestimate the second equation in the model?

(b) Are there situations in which, when the three-equation model is the appropriate specification, you would prefer the methods used in PRINTOUT, PART A and PRINTOUT, PART B to those used in PRINTOUT, PART E? Explain briefly.

(c) Consider the corresponding three-equation system obtained by making x_{1t} in Equations (3)–(4) endogenous:

$$\begin{aligned} y_{1t} &= \alpha_1 + \beta_{11} x_{1t} + \beta_{12} x_{2t} + v_{1t}, \\ y_{2t} &= \alpha_2 + \beta_{21} x_{1t} + \beta_{22} x_{2t} + v_{2t}, \\ x_{1t} &= \gamma + \delta x_{2t} + \varepsilon_t, \end{aligned}$$

where you assume that $\text{cov}(v_{1t}, v_{2t}) \neq 0$. Is it possible to estimate the coefficients in the two first equations consistently? Does you answer depend on whether $\text{cov}(v_{1t}, \varepsilon_t)$ and $\text{cov}(v_{2t}, \varepsilon_t)$ are zero or not?

(d) To account for a possible delayed response of the factor input ratio to changes in the factor price ratio, the following extension of Equation (1) in Problem 1 has been proposed:

$$y_{1t} = \alpha_1 + \beta_1 x_{1t} + \lambda_1 y_{1,t-1} + u_{1t}, \qquad |\lambda_1| < 1,$$

PRINTOUT, PART F of the printout gives OLS estimates for this equation. Can you from this printout (i) estimate the short-run and long-run elasticity of substitution between labour and capital in Industry 1 consistently, and (ii) test whether the long-run elasticity significantly exceeds the short-run elasticity? Which of the two elasticity estimates is closest to the estimate obtained from the corresponding static equation in PRINTOUT, PART A? Could you explain your finding?

(e) At the end of the exercise is shown – for various combinations of the number of observations (T) and the number of coefficients in the equation (K) – the lower (dL) and the upper (dU) bounds of the 5 % critical values of the Durbin-Watson test ($K \leq T-4$). Give, supported by this table and what you know about the relationship between residuals and disturbances, a brief intuitive explanation of why the difference between the upper and lower bound ($dU-dL$) would have been smaller if you had had twice as long time series, i.e., $T = 78$ rather than $T = 39$, with the same value of K. What do you think will happen to ($dU-dL$) when T goes to infinity for a fixed K?

```
PRINTOUT, PART A

EQ(1) Modelling ln(K1/L1)  by OLS. The estimation sample is: 1958 to 1996

                  Coefficient  Std.Error  t-value  t-prob Part.R^2
Constant             -1.64376    0.03979    -41.3   0.000   0.9788
ln(PK1/PL1)         -0.814122    0.05918    -13.8   0.000   0.8365

sigma                0.148029  RSS             0.810768809
R^2                  0.836457  F(1,37) =       189.2 [0.000]
log-likelihood        20.1914  DW                    0.535
no. of observations        39  no. of parameters         2


EQ(2) Modelling ln(K2/L2) by OLS. The estimation sample is: 1958 to 1996

                  Coefficient  Std.Error  t-value  t-prob Part.R^2
Constant            -0.720182    0.03993    -18.0   0.000 0.8980
ln(PK2/PL2)         -0.859562    0.07035    -12.2   0.000 0.8014

sigma                0.155971  RSS             0.900100587
R^2                  0.801366  F(1,37) =       149.3 [0.000]
log-likelihood        18.1532  DW                    0.312
no. of observations        39  no. of parameters         2
************************************************************************

PRINTOUT, PART B

System of regression equations, estimated by FGLS. Version 1.
The estimation sample is: 1958 to 1996

(1) Equation for: ln(K1/L1)
               Coefficient  Std.Error  t-value  t-prob
Constant         -1.654590    0.03975    -41.6   0.000
ln(PK1/PL1)      -0.794065    0.05904    -13.4   0.000
sigma = 0.148259

(2) Equation for: ln(K2/L2)
               Coefficient  Std.Error  t-value  t-prob
Constant         -0.734906    0.03991    -18.4   0.000
ln(PK2/PL2)      -0.826284    0.07029    -11.8   0.000
sigma = 0.156442

Correlation between residuals in equations for ln(K1/L1) and ln(K2/L2): 0.14657
****************************************************************************************

PRINTOUT, PART C

EQ(1) Modelling ln(K1/L1) by OLS. The estimation sample is: 1958 to 1996

                 Coefficient  Std.Error  t-value  t-prob Part.R^2
Constant            -1.61147    0.02380    -67.7   0.000   0.9922
ln(PK1/PL1)        -0.420474    0.05855    -7.18   0.000   0.5889
ln(PK2/PL2)        -0.553437    0.06606    -8.38   0.000   0.6610

sigma              0.0873797   RSS             0.274867547
R^2                 0.944555   F(2,36) =       306.6 [0.000]
log-likelihood       41.2844   DW                    0.662
no. of observations       39   no. of parameters         3


EQ(2) Modelling ln(K2/L2) by OLS. The estimation sample is: 1958 to 1996
```

```
                  Coefficient  Std.Error  t-value  t-prob Part.R^2
Constant            -0.672907    0.03643    -18.5   0.000   0.9046
ln(PK1/PL1)         -0.339129    0.08962    -3.78   0.001   0.2846
ln(PK2/PL2)         -0.552505    0.10110    -5.46   0.000   0.4534

sigma               0.133744   RSS              0.64395202
R^2                 0.857893   F(2,36) =        108.7 [0.000]
log-likelihood       24.6834   DW                    0.304
no. of observations       39   no. of parameters         3
*********************************************************************************
```

PRINTOUT, PART D

```
System of regression equations, estimated by FGLS. Version 2.
The estimation sample is: 1958 to 1996

(1) Equation for: ln(K1/L1)
             Coefficient  Std.Error  t-value  t-prob
Constant       -1.611470    0.02380    -67.7   0.000
ln(PK1/PL1)    -0.420474    0.05855    -7.18   0.000
ln(PK2/PL2)    -0.553437    0.06606    -8.38   0.000
sigma = 0.0873797

(2) Equation for: ln(K2/L2)
             Coefficient  Std.Error  t-value  t-prob
Constant       -0.672907    0.03643    -18.5   0.000
ln(PK1/PL1)    -0.339129    0.08962    -3.78   0.001
ln(PK2/PL2)    -0.552505    0.10110    -5.46   0.000
sigma = 0.133744

Correlation between residuals in equations for ln(K1/L1) and ln(K2/L2):  0.88602
Correlation between actual and fitted values of ln(K1/L1): 0.97188
Correlation between actual and fitted values of ln(K2/L2): 0.92623
********************************************************************************
```

PRINTOUT, PART E

```
EQ(1) Modelling ln(K1/L1) by IVE.  The estimation sample is: 1958 to 1996

                  Coefficient  Std.Error  t-value  t-prob
Constant             -1.52626    0.05415    -28.2   0.000
ln(PK1/PL1)    Y     -1.03172    0.08617    -12.0   0.000

sigma                0.172971  RSS              1.10699539
no. of observations        39  no. of parameters         2
no. endogenous variables    2  no. of instruments        2
Additional instruments: [0] = ln(PK2/PL2)

EQ(3) Modelling ln(PK1/PL1) by OLS.  The estimation sample is: 1958 to 1996

                  Coefficient  Std.Error  t-value  t-prob Part.R^2
Constant             0.139400    0.06278     2.22   0.033   0.1176
ln(PK2/PL2)          0.905429     0.1107     8.18   0.000   0.6440

sigma                0.245347  RSS               2.2272208
R^2                  0.644013  F(1,37) =     66.94   [0.000]
log-likelihood       0.486136  DW                    0.577
no. of observations        39  no. of parameters         2
**************************************************************************
```

PRINTOUT, PART F

```
EQ(1) Modelling ln(K1/L1) by OLS. The estimation sample is: 1959 to 1996

                  Coefficient  Std.Error  t-value  t-prob Part.R^2
Constant            -0.374223    0.09354    -4.00   0.000   0.3138
ln(K1/L1)_1          0.755314    0.05488     13.8   0.000   0.8440
ln(PK1/PL1)         -0.217377    0.04849    -4.48   0.000   0.3647

sigma               0.0578778  RSS             0.117244368
R^2                  0.975197  F(2,35) =        688.1 [0.000]
log-likelihood        55.9209  DW                     1.78
no. of observations        38  no. of parameters         3
*************************************************************************
```

DESCRIPTIVE STATISTICS. The sample is 1958 to 1996 (39 obs.)

```
Means
      ln(K1/L1)    ln(K2/L2)   ln(PK1/PL1)  ln(PK2/PL2)
      -2.0834      -1.1005      0.54001      0.44245
Standard deviations (using T-1)
      ln(K1/L1)    ln(K2/L2)   ln(PK1/PL1)  ln(PK2/PL2)
       0.36119      0.34532      0.40576      0.35964
Correlation matrix:
                  ln(K1/L1)    ln(K2/L2)   ln(PK1/PL1)  ln(PK2/PL2)
ln(K1/L1)           1.0000      0.97829     -0.91458     -0.93012
ln(K2/L2)          0.97829       1.0000     -0.86025     -0.89519
ln(PK1/PL1)       -0.91458     -0.86025       1.0000      0.80250
ln(PK2/PL2)       -0.93012     -0.89519      0.80250       1.0000
***************************************************************************
```

Durbin-Watson 5 % Critical Values (dL=lower,dU=upper):

T = NO. OF OBS. K = NO. OF COEF. (INCL. INTERCEPT)

T	K	dL	dU
6	2	0.61018	1.40015
7	2	0.69955	1.35635
7	3	0.46723	1.89636
8	2	0.76290	1.33238
8	3	0.55907	1.77711
8	4	0.36744	2.28664
9	2	0.82428	1.31988
9	3	0.62910	1.69926
9	4	0.45476	2.12816
9	5	0.29571	2.58810
10	2	0.87913	1.31971
10	3	0.69715	1.64134
10	4	0.52534	2.01632
10	5	0.37602	2.41365
10	6	0.24269	2.82165
11	2	0.92733	1.32409
11	3	0.75798	1.60439
11	4	0.59477	1.92802
11	5	0.44406	2.28327
11	6	0.31549	2.64456
11	7	0.20253	3.00447
12	2	0.97076	1.33137
12	3	0.81221	1.57935
12	4	0.65765	1.86397
12	5	0.51198	2.17662
12	6	0.37956	2.50609
12	7	0.26813	2.83196
12	8	0.17144	3.14940
13	2	1.00973	1.34040
13	3	0.86124	1.56212
13	4	0.71465	1.81593
13	5	0.57446	2.09428
13	6	0.44448	2.38967
13	7	0.32775	2.69204
13	8	0.23049	2.98506
13	9	0.14693	3.26577
14	2	1.04495	1.35027
14	3	0.90544	1.55066
14	4	0.76666	1.77882
14	5	0.63206	2.02955
14	6	0.50516	2.29593
14	7	0.38897	2.57158
14	8	0.28559	2.84769
14	9	0.20013	3.11121
14	10	0.12726	3.36038
15	2	1.07697	1.36054
15	3	0.94554	1.54318
15	4	0.81396	1.75014
15	5	0.68519	1.97735
15	6	0.56197	2.21981
15	7	0.44707	2.47148
15	8	0.34290	2.72698
15	9	0.25090	2.97866
15	10	0.17531	3.21604
15	11	0.11127	3.43819
16	2	1.10617	1.37092
16	3	0.98204	1.53860
16	4	0.85718	1.72773
16	5	0.73400	1.93506
16	6	0.61495	2.15672
16	7	0.50223	2.38813
16	8	0.39805	2.62409
16	9	0.30433	2.86009
16	10	0.22206	3.08954
16	11	0.15479	3.30391
16	12	0.09809	3.50287
17	2	1.13295	1.38122
17	3	1.01543	1.53614
17	4	0.89675	1.71009
17	5	0.77898	1.90047
17	6	0.66414	2.10414
17	7	0.55423	2.31755
17	8	0.45107	2.53660
17	9	0.35639	2.75688
17	10	0.27177	2.97455
17	11	0.19784	3.18400
17	12	0.13763	3.37817
17	13	0.08711	3.55716

T	K	dL	dU
18	2	1.15759	1.39133
18	3	1.04607	1.53525
18	4	0.93310	1.69614
18	5	0.82044	1.87189
18	6	0.70984	2.06000
18	7	0.60301	2.25750
18	8	0.50158	2.46122
18	9	0.40702	2.66753
18	10	0.32076	2.87268
18	11	0.24405	3.07345
18	12	0.17732	3.26497
18	13	0.12315	3.44141
18	14	0.07786	3.60315
19	2	1.18037	1.40118
19	3	1.07430	1.53553
19	4	0.96659	1.68509
19	5	0.85876	1.84815
19	6	0.75231	2.02262
19	7	0.64870	2.20614
19	8	0.54938	2.39602
19	9	0.45571	2.58939
19	10	0.36889	2.78312
19	11	0.29008	2.97399
19	12	0.22029	3.15930
19	13	0.15979	3.33481
19	14	0.11082	3.49566
19	15	0.07001	3.64241
20	2	1.20149	1.41073
20	3	1.10040	1.53668
20	4	0.99755	1.67634
20	5	0.89425	1.82828
20	6	0.79179	1.99079
20	7	0.69146	2.16189
20	8	0.59454	2.33937
20	9	0.50220	2.52082
20	10	0.41559	2.70374
20	11	0.33571	2.88535
20	12	0.26349	3.06292
20	13	0.19978	3.23417
20	14	0.14472	3.39540
20	15	0.10024	3.54250
20	16	0.06327	3.67619
21	2	1.22115	1.41997
21	3	1.12461	1.53849
21	4	1.02624	1.66942
21	5	0.92719	1.81157
21	6	0.82856	1.96350
21	7	0.73149	2.12355
21	8	0.63710	2.28988
21	9	0.54645	2.46051
21	10	0.46055	2.63324
21	11	0.38035	2.80588
21	12	0.30669	2.97600
21	13	0.24033	3.14129
21	14	0.18198	3.29979
21	15	0.13166	3.44827
21	16	0.09111	3.58322
21	17	0.05747	3.70544
22	2	1.23949	1.42888
22	3	1.14713	1.54079
22	4	1.05292	1.66398
22	5	0.95783	1.79744
22	6	0.86285	1.93996
22	7	0.76898	2.09015
22	8	0.67719	2.24646
22	9	0.58843	2.40718
22	10	0.50363	2.57051
22	11	0.42363	2.73452
22	12	0.34926	2.89726
22	13	0.28119	3.05662
22	14	0.22003	3.21061
22	15	0.16642	3.35756
22	16	0.12028	3.49463
22	17	0.08315	3.61880
22	18	0.05242	3.73092

T	K	dL	dU
23	2	1.25665	1.43747
23	3	1.16815	1.54346
23	4	1.07778	1.65974
23	5	0.98639	1.78546
23	6	0.89488	1.91958
23	7	0.80410	2.06093
23	8	0.71493	2.20816
23	9	0.62821	2.35988
23	10	0.54478	2.51449
23	11	0.46541	2.67038
23	12	0.39083	2.82585
23	13	0.32172	2.97919
23	14	0.25866	3.12852
23	15	0.20216	3.27216
23	16	0.15274	3.40865
23	17	0.11029	3.53549
23	18	0.07619	3.65007
23	19	0.04801	3.75327
24	2	1.27276	1.44575
24	3	1.18781	1.54639
24	4	1.10100	1.65649
24	5	1.01309	1.77526
24	6	0.92486	1.90184
24	7	0.83706	2.03522
24	8	0.75048	2.17427
24	9	0.66589	2.31774
24	10	0.58400	2.46431
24	11	0.50554	2.61260
24	12	0.43119	2.76111
24	13	0.36156	2.90835
24	14	0.29723	3.05282
24	15	0.23869	3.19285
24	16	0.18635	3.32700
24	17	0.14066	3.45402
24	18	0.10150	3.57167
24	19	0.07006	3.67769
24	20	0.04413	3.77297
25	2	1.28791	1.45371
25	3	1.20625	1.54954
25	4	1.12276	1.65403
25	5	1.03811	1.76655
25	6	0.95297	1.88634
25	7	0.86803	2.01252
25	8	0.78400	2.14412
25	9	0.70154	2.28007
25	10	0.62133	2.41924
25	11	0.54401	2.56041
25	12	0.47019	2.70229
25	13	0.40046	2.84360
25	14	0.33536	2.98300
25	15	0.27536	3.11913
25	16	0.22090	3.25058
25	17	0.17231	3.37604
25	18	0.12995	3.49447
25	19	0.09371	3.60384
25	20	0.06465	3.70220
25	21	0.04070	3.79041
26	2	1.30219	1.46139
26	3	1.22358	1.55281
26	4	1.14319	1.65225
26	5	1.06158	1.75911
26	6	0.97937	1.87274
26	7	0.89717	1.99240
26	8	0.81561	2.11722
26	9	0.73529	2.24629
26	10	0.65683	2.37862
26	11	0.58079	2.51315
26	12	0.50775	2.64877
26	13	0.43825	2.78436
26	14	0.37279	2.91872
26	15	0.31182	3.05067
26	16	0.25578	3.17904
26	17	0.20499	3.30253
26	18	0.15977	3.42006
26	19	0.12041	3.53067
26	20	0.08677	3.63257
26	21	0.05983	3.72404

T	K	dL	dU
27	2	1.31568	1.46878
27	3	1.23991	1.55620
27	4	1.16239	1.65101
27	5	1.08364	1.75274
27	6	1.00421	1.86079
27	7	0.92463	1.97449
27	8	0.84546	2.09313
27	9	0.76726	2.21588
27	10	0.69057	2.34190
27	11	0.61593	2.47026
27	12	0.54385	2.59997
27	13	0.47482	2.73007
27	14	0.40933	2.85950
27	15	0.34780	2.98721
27	16	0.29062	3.11215
27	17	0.23816	3.23327
27	18	0.19072	3.34944
27	19	0.14853	3.45967
27	20	0.11188	3.56318
27	21	0.08057	3.65833
28	2	1.32844	1.47589
28	3	1.25534	1.55964
28	4	1.18051	1.65025
28	5	1.10444	1.74728
28	6	1.02762	1.85022
28	7	0.95052	1.95851
28	8	0.87366	2.07148
28	9	0.79754	2.18844
28	10	0.72265	2.30862
28	11	0.64947	2.43122
28	12	0.57848	2.55540
28	13	0.51013	2.68025
28	14	0.44486	2.80489
28	15	0.38308	2.92838
28	16	0.32517	3.04976
28	17	0.27146	3.16812
28	18	0.22228	3.28249
28	19	0.17787	3.39189
28	20	0.13843	3.49546
28	21	0.10421	3.59248
29	2	1.34054	1.48275
29	3	1.26992	1.56312
29	4	1.19762	1.64987
29	5	1.12407	1.74260
29	6	1.04971	1.84088
29	7	0.97499	1.94420
29	8	0.90036	2.05196
29	9	0.82626	2.16358
29	10	0.75316	2.27837
29	11	0.68148	2.39562
29	12	0.61166	2.51459
29	13	0.54413	2.63447
29	14	0.47929	2.75449
29	15	0.41753	2.87381
29	16	0.35918	2.99160
29	17	0.30461	3.10700
29	18	0.25409	3.21917
29	19	0.20790	3.32728
29	20	0.16625	3.43042
29	21	0.12931	3.52786
30	2	1.35204	1.48936
30	3	1.28373	1.56661
30	4	1.21380	1.64981
30	5	1.14262	1.73860
30	6	1.07060	1.83259
30	7	0.99815	1.93133
30	8	0.92564	2.03432
30	9	0.85351	2.14102
30	10	0.78217	2.25080
30	11	0.71202	2.36307
30	12	0.64345	2.47714
30	13	0.57685	2.59233
30	14	0.51259	2.70793
30	15	0.45105	2.82319
30	16	0.39255	2.93738
30	17	0.33740	3.04971
30	18	0.28590	3.15946
30	19	0.23830	3.26584
30	20	0.19485	3.36811
30	21	0.15572	3.46549

T	K	dL	dU	T	K	dL	dU	T	K	dL	dU	T	K	dL	dU
31	2	1.36298	1.49574	35	2	1.40194	1.51914	39	2	1.43473	1.53963	43	2	1.46278	1.55773
31	3	1.29685	1.57011	35	3	1.34332	1.58382	39	3	1.38210	1.59686	43	3	1.41507	1.60905
31	4	1.22915	1.65002	35	4	1.28330	1.65282	39	4	1.32827	1.65754	43	4	1.36629	1.66319
31	5	1.16021	1.73518	35	5	1.22214	1.72593	39	5	1.27338	1.72152	43	5	1.31655	1.72002
31	6	1.09040	1.82522	35	6	1.16007	1.80292	39	6	1.21761	1.78863	43	6	1.26600	1.77944
31	7	1.02008	1.91976	35	7	1.09735	1.88351	39	7	1.16116	1.85870	43	7	1.21476	1.84132
31	8	0.94962	2.01834	35	8	1.03424	1.96743	39	8	1.10419	1.93153	43	8	1.16298	1.90552
31	9	0.87940	2.12046	35	9	0.97099	2.05436	39	9	1.04692	2.00692	43	9	1.11080	1.97189
31	10	0.80979	2.22562	35	10	0.90788	2.14395	39	10	0.98953	2.08460	43	10	1.05837	2.04027
31	11	0.74115	2.33323	35	11	0.84516	2.23585	39	11	0.93220	2.16437	43	11	1.00581	2.11047
31	12	0.67387	2.44273	35	12	0.78311	2.32966	39	12	0.87514	2.24594	43	12	0.95328	2.18231
31	13	0.60828	2.55347	35	13	0.72197	2.42501	39	13	0.81853	2.32904	43	13	0.90093	2.25562
31	14	0.54474	2.66484	35	14	0.66200	2.52146	39	14	0.76257	2.41340	43	14	0.84891	2.33017
31	15	0.48358	2.77618	35	15	0.60346	2.61858	39	15	0.70743	2.49872	43	15	0.79734	2.40577
31	16	0.42513	2.88680	35	16	0.54659	2.71593	39	16	0.65333	2.58469	43	16	0.74639	2.48220
31	17	0.36966	2.99604	35	17	0.49162	2.81306	39	17	0.60044	2.67100	43	17	0.69619	2.55922
31	18	0.31748	3.10322	35	18	0.43878	2.90951	39	18	0.54891	2.75733	43	18	0.64688	2.63664
31	19	0.26882	3.20762	35	19	0.38829	3.00481	39	19	0.49896	2.84336	43	19	0.59860	2.71419
31	20	0.22392	3.30859	35	20	0.34034	3.09851	39	20	0.45072	2.92876	43	20	0.55149	2.79164
31	21	0.18298	3.40545	35	21	0.29513	3.19013	39	21	0.40437	3.01320	43	21	0.50568	2.86878
32	2	1.37340	1.50190	36	2	1.41065	1.52451	40	2	1.44214	1.54436	44	2	1.46920	1.56193
32	3	1.30932	1.57358	36	3	1.35365	1.58716	40	3	1.39083	1.59999	44	3	1.42257	1.61196
32	4	1.24371	1.65046	36	4	1.29530	1.65387	40	4	1.33835	1.65889	44	4	1.37490	1.66467
32	5	1.17688	1.73226	36	5	1.23583	1.72447	40	5	1.28484	1.72092	44	5	1.32631	1.71996
32	6	1.10916	1.81867	36	6	1.17545	1.79873	40	6	1.23047	1.78594	44	6	1.27692	1.77772
32	7	1.04088	1.90931	36	7	1.11441	1.87643	40	7	1.17541	1.85378	44	7	1.22685	1.83784
32	8	0.97239	2.00381	36	8	1.05294	1.95730	40	8	1.11983	1.92426	44	8	1.17624	1.90017
32	9	0.90401	2.10171	36	9	0.99128	2.04104	40	9	1.06391	1.99717	44	9	1.12522	1.96460
32	10	0.83609	2.20255	36	10	0.92967	2.12737	40	10	1.00782	2.07233	44	10	1.07390	2.03095
32	11	0.76897	2.30583	36	11	0.86836	2.21594	40	11	0.95174	2.14950	44	11	1.02245	2.09907
32	12	0.70299	2.41102	36	12	0.80759	2.30642	40	12	0.89585	2.22843	44	12	0.97099	2.16881
32	13	0.63847	2.51758	36	13	0.74759	2.39844	40	13	0.84035	2.30888	44	13	0.91964	2.23997
32	14	0.57573	2.62493	36	14	0.68861	2.49162	40	14	0.78539	2.39060	44	14	0.86856	2.31237
32	15	0.51510	2.73248	36	15	0.63089	2.58557	40	15	0.73115	2.47330	44	15	0.81787	2.38581
32	16	0.45685	2.83963	36	16	0.57463	2.67990	40	16	0.67782	2.55672	44	16	0.76771	2.46011
32	17	0.40129	2.94576	36	17	0.52008	2.77418	40	17	0.62556	2.64056	44	17	0.71822	2.53505
32	18	0.34866	3.05028	36	18	0.46745	2.86800	40	18	0.57454	2.72455	44	18	0.66953	2.61043
32	19	0.29923	3.15253	36	19	0.41692	2.96095	40	19	0.52492	2.80836	44	19	0.62177	2.68601
32	20	0.25319	3.25193	36	20	0.36871	3.05259	40	20	0.47687	2.89172	44	20	0.57507	2.76161
32	21	0.21078	3.34784	36	21	0.32299	3.14249	40	21	0.43054	2.97431	44	21	0.52954	2.83698
33	2	1.38335	1.50784	37	2	1.41900	1.52971	41	2	1.44927	1.54895	45	2	1.47538	1.56602
33	3	1.32119	1.57703	37	3	1.36354	1.59044	41	3	1.39922	1.60307	45	3	1.42980	1.61482
33	4	1.25756	1.65110	37	4	1.30678	1.65501	41	4	1.34803	1.66028	45	4	1.38320	1.66618
33	5	1.19272	1.72978	37	5	1.24891	1.72327	41	5	1.29584	1.72048	45	5	1.33571	1.71999
33	6	1.12698	1.81282	37	6	1.19014	1.79499	41	6	1.24280	1.78353	45	6	1.28744	1.77618
33	7	1.06065	1.89986	37	7	1.13071	1.86998	41	7	1.18907	1.84926	45	7	1.23849	1.83462
33	8	0.99402	1.99057	37	8	1.07081	1.94799	41	8	1.13481	1.91753	45	8	1.18899	1.89520
33	9	0.92743	2.08455	37	9	1.01066	2.02876	41	9	1.08019	1.98813	45	9	1.13907	1.95778
33	10	0.86115	2.18137	37	10	0.95051	2.11203	41	10	1.02536	2.06089	45	10	1.08886	2.02222
33	11	0.79554	2.28061	37	11	0.89057	2.19749	41	11	0.97050	2.13561	45	11	1.03846	2.08839
33	12	0.73086	2.38177	37	12	0.83105	2.28481	41	12	0.91576	2.21204	45	12	0.98802	2.15611
33	13	0.66745	2.48437	37	13	0.77219	2.37369	41	13	0.86132	2.28998	45	13	0.93765	2.22524
33	14	0.60559	2.58789	37	14	0.71421	2.46378	41	14	0.80736	2.36919	45	14	0.88750	2.29558
33	15	0.54558	2.69181	37	15	0.65734	2.55471	41	15	0.75402	2.44941	45	15	0.83769	2.36698
33	16	0.48769	2.79558	37	16	0.60177	2.64613	41	16	0.70146	2.53039	45	16	0.78833	2.43924
33	17	0.43219	2.89865	37	17	0.54771	2.73765	41	17	0.64987	2.61187	45	17	0.73955	2.51218
33	18	0.37933	3.00046	37	18	0.49537	2.82891	41	18	0.59940	2.69358	45	18	0.69149	2.58559
33	19	0.32935	3.10046	37	19	0.44494	2.91951	41	19	0.55018	2.77525	45	19	0.64427	2.65929
33	20	0.28246	3.19808	37	20	0.39661	3.00907	41	20	0.50238	2.85660	45	20	0.59801	2.73306
33	21	0.23887	3.29275	37	21	0.35054	3.09719	41	21	0.45615	2.93734	45	21	0.55282	2.80672
34	2	1.39285	1.51358	38	2	1.42702	1.53475	42	2	1.45615	1.55340	46	2	1.48136	1.56999
34	3	1.33251	1.58045	38	3	1.37301	1.59368	42	3	1.40730	1.60608	46	3	1.43677	1.61763
34	4	1.27074	1.65189	38	4	1.31774	1.65625	42	4	1.35733	1.66172	46	4	1.39121	1.66769
34	5	1.20779	1.72770	38	5	1.26140	1.72229	42	5	1.30640	1.72019	46	5	1.34477	1.72012
34	6	1.14393	1.80758	38	6	1.20418	1.79164	42	6	1.25463	1.78137	46	6	1.29756	1.77482
34	7	1.07944	1.89129	38	7	1.14627	1.86409	42	7	1.20218	1.84512	46	7	1.24969	1.83167
34	8	1.01462	1.97849	38	8	1.08787	1.93942	42	8	1.14918	1.91130	46	8	1.20127	1.89058
34	9	0.94973	2.06882	38	9	1.02919	2.01742	42	9	1.09581	1.97972	46	9	1.15242	1.95141
34	10	0.88506	2.16190	38	10	0.97045	2.09782	42	10	1.04219	2.05023	46	10	1.10325	2.01404
34	11	0.82091	2.25735	38	11	0.91183	2.18033	42	11	0.98851	2.12262	46	11	1.05388	2.07834
34	12	0.75755	2.35473	38	12	0.85356	2.26470	42	12	0.93489	2.19670	46	12	1.00443	2.14416
34	13	0.69527	2.45359	38	13	0.79583	2.35061	42	13	0.88151	2.27227	46	13	0.95503	2.21134
34	14	0.63433	2.55348	38	14	0.73886	2.43775	42	14	0.82852	2.34909	46	14	0.90578	2.27974
34	15	0.57503	2.65392	38	15	0.68284	2.52581	42	15	0.77607	2.42694	46	15	0.85681	2.34918
34	16	0.51760	2.75442	38	16	0.62799	2.61444	42	16	0.72431	2.50558	46	16	0.80825	2.41950
34	17	0.46231	2.85449	38	17	0.57448	2.70332	42	17	0.67341	2.58480	46	17	0.76020	2.49051
34	18	0.40939	2.95361	38	18	0.52253	2.79207	42	18	0.62350	2.66432	46	18	0.71278	2.56205
34	19	0.35907	3.05127	38	19	0.47229	2.88036	42	19	0.57474	2.74389	46	19	0.66611	2.63391
34	20	0.31155	3.14697	38	20	0.42396	2.96784	42	20	0.52726	2.82328	46	20	0.62032	2.70593
34	21	0.26704	3.24020	38	21	0.37769	3.05412	42	21	0.48121	2.90220	46	21	0.57550	2.77790

T	K	dL	dU	T	K	dL	dU	T	K	dL	dU	T	K	dL	dU
47	2	1.48715	1.57386	51	2	1.50856	1.58835	55	2	1.52755	1.60144	59	2	1.54455	1.61336
47	3	1.44352	1.62038	51	3	1.46838	1.63088	55	3	1.49031	1.64062	59	3	1.50985	1.64967
47	4	1.39894	1.66923	51	4	1.42734	1.67538	55	4	1.45232	1.68149	59	4	1.47448	1.68745
47	5	1.35350	1.72033	51	5	1.38554	1.72179	55	5	1.41362	1.72399	59	5	1.43848	1.72663
47	6	1.30731	1.77361	51	6	1.34305	1.77005	55	6	1.37431	1.76807	59	6	1.40191	1.76720
47	7	1.26047	1.82895	51	7	1.29995	1.82007	55	7	1.33442	1.81368	59	7	1.36481	1.80908
47	8	1.21309	1.88627	51	8	1.25632	1.87178	55	8	1.29403	1.86074	59	8	1.32723	1.85226
47	9	1.16526	1.94545	51	9	1.21224	1.92510	55	9	1.25319	1.90921	59	9	1.28923	1.89665
47	10	1.11710	2.00636	51	10	1.16780	1.97994	55	10	1.21199	1.95902	59	10	1.25086	1.94223
47	11	1.06873	2.06889	51	11	1.12308	2.03620	55	11	1.17049	2.01008	59	11	1.21218	1.98893
47	12	1.02026	2.13290	51	12	1.07818	2.09378	55	12	1.12875	2.06233	59	12	1.17325	2.03668
47	13	0.97178	2.19824	51	13	1.03319	2.15258	55	13	1.08685	2.11568	59	13	1.13410	2.08543
47	14	0.92342	2.26478	51	14	0.98817	2.21249	55	14	1.04485	2.17003	59	14	1.09482	2.13510
47	15	0.87529	2.33235	51	15	0.94324	2.27338	55	15	1.00284	2.22532	59	15	1.05545	2.18564
47	16	0.82751	2.40080	51	16	0.89847	2.33515	55	16	0.96087	2.28146	59	16	1.01605	2.23698
47	17	0.78018	2.46998	51	17	0.85396	2.39767	55	17	0.91902	2.33833	59	17	0.97668	2.28902
47	18	0.73341	2.53970	51	18	0.80978	2.46083	55	18	0.87736	2.39585	59	18	0.93739	2.34171
47	19	0.68732	2.60980	51	19	0.76604	2.52448	55	19	0.83597	2.45392	59	19	0.89826	2.39495
47	20	0.64200	2.68011	51	20	0.72282	2.58848	55	20	0.79492	2.51244	59	20	0.85932	2.44869
47	21	0.59759	2.75044	51	21	0.68021	2.65272	55	21	0.75427	2.57131	59	21	0.82065	2.50283
48	2	1.49275	1.57762	52	2	1.51352	1.59174	56	2	1.53197	1.60452	60	2	1.54853	1.61617
48	3	1.45004	1.62308	52	3	1.47410	1.63339	56	3	1.49541	1.64295	60	3	1.51442	1.65184
48	4	1.40640	1.67076	52	4	1.43388	1.67692	56	4	1.45810	1.68300	60	4	1.47965	1.68891
48	5	1.36192	1.72061	52	5	1.39290	1.72228	56	5	1.42012	1.72461	60	5	1.44427	1.72735
48	6	1.31672	1.77253	52	6	1.35124	1.76942	56	6	1.38152	1.76776	60	6	1.40832	1.76711
48	7	1.27087	1.82645	52	7	1.30899	1.81827	56	7	1.34237	1.81238	60	7	1.37186	1.80817
48	8	1.22447	1.88226	52	8	1.26622	1.86874	56	8	1.30271	1.85841	60	8	1.33493	1.85045
48	9	1.17764	1.93987	52	9	1.22299	1.92076	56	9	1.26263	1.90579	60	9	1.29758	1.89393
48	10	1.13046	1.99915	52	10	1.17941	1.97426	56	10	1.22217	1.95448	60	10	1.25987	1.93856
48	11	1.08306	2.05999	52	11	1.13553	2.02913	56	11	1.18141	2.00438	60	11	1.22183	1.98427
48	12	1.03552	2.12227	52	12	1.09146	2.08528	56	12	1.14040	2.05542	60	12	1.18354	2.03101
48	13	0.98794	2.18586	52	13	1.04727	2.14263	56	13	1.09922	2.10755	60	13	1.14505	2.07873
48	14	0.94045	2.25062	52	14	1.00304	2.20106	56	14	1.05793	2.16067	60	14	1.10640	2.12734
48	15	0.89314	2.31641	52	15	0.95887	2.26046	56	15	1.01659	2.21470	60	15	1.06764	2.17681
48	16	0.84614	2.38309	52	16	0.91481	2.32074	56	16	0.97530	2.26956	60	16	1.02885	2.22705
48	17	0.79951	2.45049	52	17	0.87099	2.38176	56	17	0.93408	2.32515	60	17	0.99007	2.27800
48	18	0.75340	2.51847	52	18	0.82745	2.44341	56	18	0.89304	2.38140	60	18	0.95135	2.32958
48	19	0.70789	2.58687	52	19	0.78431	2.50559	56	19	0.85222	2.43820	60	19	0.91276	2.38173
48	20	0.66309	2.65552	52	20	0.74163	2.56816	56	20	0.81170	2.49546	60	20	0.87435	2.43437
48	21	0.61909	2.72427	52	21	0.69949	2.63099	56	21	0.77155	2.55309	60	21	0.83616	2.48742
49	2	1.49819	1.58129	53	2	1.51833	1.59505	57	2	1.53628	1.60754	61	2	1.55240	1.61892
49	3	1.45635	1.62573	53	3	1.47967	1.63585	57	3	1.50036	1.64524	61	3	1.51886	1.65396
49	4	1.41362	1.67230	53	4	1.44022	1.67845	57	4	1.46372	1.68449	61	4	1.48468	1.69035
49	5	1.37007	1.72095	53	5	1.40002	1.72282	57	5	1.42642	1.72526	61	5	1.44989	1.72808
49	6	1.32580	1.77159	53	6	1.35918	1.76890	57	6	1.38852	1.76751	61	6	1.41455	1.76708
49	7	1.28090	1.82415	53	7	1.31774	1.81661	57	7	1.35008	1.81119	61	7	1.37871	1.80732
49	8	1.23546	1.87852	53	8	1.27579	1.86590	57	8	1.31114	1.85622	61	8	1.34240	1.84876
49	9	1.18958	1.93463	53	9	1.23340	1.91668	57	9	1.27177	1.90257	61	9	1.30568	1.89137
49	10	1.14336	1.99236	53	10	1.19063	1.96889	57	10	1.23203	1.95018	61	10	1.26860	1.93507
49	11	1.09687	2.05160	53	11	1.14757	2.02244	57	11	1.19198	1.99896	61	11	1.23120	1.97984
49	12	1.05024	2.11224	53	12	1.10430	2.07723	57	12	1.15168	2.04887	61	12	1.19355	2.02560
49	13	1.00354	2.17415	53	13	1.06090	2.13318	57	13	1.11121	2.09982	61	13	1.15567	2.07232
49	14	0.95690	2.23723	53	14	1.01743	2.19019	57	14	1.07060	2.15175	61	14	1.11763	2.11992
49	15	0.91040	2.30131	53	15	0.97399	2.24817	57	15	1.02994	2.20456	61	15	1.07950	2.16835
49	16	0.86415	2.36628	53	16	0.93065	2.30700	57	16	0.98929	2.25820	61	16	1.04129	2.21755
49	17	0.81824	2.43199	53	17	0.88749	2.36659	57	17	0.94871	2.31257	61	17	1.00309	2.26744
49	18	0.77278	2.49829	53	18	0.84459	2.42682	57	18	0.90825	2.36758	61	18	0.96492	2.31796
49	19	0.72786	2.56505	53	19	0.80204	2.48757	57	19	0.86800	2.42316	61	19	0.92686	2.36904
49	20	0.68358	2.63211	53	20	0.75990	2.54874	57	20	0.82802	2.47920	61	20	0.88896	2.42062
49	21	0.64003	2.69930	53	21	0.71826	2.61021	57	21	0.78836	2.53563	61	21	0.85126	2.47262
50	2	1.50345	1.58486	54	2	1.52300	1.59829	58	2	1.54047	1.61048	62	2	1.55619	1.62161
50	3	1.46246	1.62833	54	3	1.48506	1.63825	58	3	1.50517	1.64747	62	3	1.52318	1.65605
50	4	1.42059	1.67385	54	4	1.44636	1.67998	58	4	1.46918	1.68598	62	4	1.48957	1.69180
50	5	1.37793	1.72135	54	5	1.40693	1.72339	58	5	1.43254	1.72594	62	5	1.45536	1.72881
50	6	1.33457	1.77077	54	6	1.36687	1.76844	58	6	1.39532	1.76733	62	6	1.42061	1.76708
50	7	1.29059	1.82203	54	7	1.32622	1.81508	58	7	1.35755	1.81009	62	7	1.38536	1.80655
50	8	1.24607	1.87504	54	8	1.28506	1.86324	58	8	1.31931	1.85418	62	8	1.34967	1.84718
50	9	1.20110	1.92972	54	9	1.24345	1.91283	58	9	1.28063	1.89954	62	9	1.31356	1.88893
50	10	1.15579	1.98597	54	10	1.20149	1.96381	58	10	1.24159	1.94610	62	10	1.27709	1.93176
50	11	1.11021	2.04368	54	11	1.15921	2.01609	58	11	1.20224	1.99382	62	11	1.24031	1.97561
50	12	1.06445	2.10276	54	12	1.11672	2.06959	58	12	1.16263	2.04262	62	12	1.20326	2.02044
50	13	1.01862	2.16307	54	13	1.07408	2.12420	58	13	1.12283	2.09245	62	13	1.16599	2.06620
50	14	0.97280	2.22452	54	14	1.03136	2.17987	58	14	1.08289	2.14323	62	14	1.12856	2.11282
50	15	0.92709	2.28698	54	15	0.98864	2.23647	58	15	1.04288	2.19489	62	15	1.09100	2.16026
50	16	0.88159	2.35032	54	16	0.94600	2.29392	58	16	1.00287	2.24735	62	16	1.05338	2.20844
50	17	0.83638	2.41440	54	17	0.90349	2.35213	58	17	0.96289	2.30054	62	17	1.01573	2.25732
50	18	0.79156	2.47910	54	18	0.86122	2.41097	58	18	0.92304	2.35436	62	18	0.97812	2.30681
50	19	0.74723	2.54428	54	19	0.81925	2.47036	58	19	0.88335	2.40875	62	19	0.94058	2.35687
50	20	0.70348	2.60978	54	20	0.77766	2.53019	58	20	0.84389	2.46362	62	20	0.90319	2.40742
50	21	0.66040	2.67548	54	21	0.73651	2.59033	58	21	0.80473	2.51889	62	21	0.86597	2.45840

T	K	dL	dU	T	K	dL	dU	T	K	dL	dU	T	K	dL	dU
63	2	1.55987	1.62425	67	2	1.57378	1.63427	71	2	1.58648	1.64352	75	2	1.59813	1.65209
63	3	1.52741	1.65810	67	3	1.54328	1.66596	71	3	1.55771	1.67331	75	3	1.57091	1.68020
63	4	1.49433	1.69321	67	4	1.51221	1.69877	71	4	1.52844	1.70409	75	4	1.54323	1.70920
63	5	1.46068	1.72957	67	5	1.48063	1.73267	71	5	1.49868	1.73584	75	5	1.51511	1.73904
63	6	1.42650	1.76712	67	6	1.44856	1.76762	71	6	1.46849	1.76854	75	6	1.48659	1.76975
63	7	1.39183	1.80584	67	7	1.41604	1.80360	71	7	1.43787	1.80214	75	7	1.45767	1.80127
63	8	1.35672	1.84569	67	8	1.38311	1.84060	71	8	1.40686	1.83664	75	8	1.42840	1.83360
63	9	1.32121	1.88663	67	9	1.34979	1.87856	71	9	1.37551	1.87202	75	9	1.39877	1.86670
63	10	1.28534	1.92860	67	10	1.31613	1.91744	71	10	1.34381	1.90823	75	10	1.36884	1.90057
63	11	1.24915	1.97159	67	11	1.28216	1.95723	71	11	1.31182	1.94524	75	11	1.33863	1.93516
63	12	1.21269	2.01552	67	12	1.24792	1.99787	71	12	1.27957	1.98304	75	12	1.30815	1.97046
63	13	1.17602	2.06035	67	13	1.21345	2.03934	71	13	1.24707	2.02157	75	13	1.27744	2.00643
63	14	1.13917	2.10603	67	14	1.17878	2.08158	71	14	1.21437	2.06081	75	14	1.24652	2.04304
63	15	1.10219	2.15250	67	15	1.14396	2.12453	71	15	1.18150	2.10073	75	15	1.21542	2.08028
63	16	1.06512	2.19971	67	16	1.10903	2.16819	71	16	1.14851	2.14128	75	16	1.18418	2.11811
63	17	1.02803	2.24761	67	17	1.07401	2.21248	71	17	1.11539	2.18242	75	17	1.15281	2.15649
63	18	0.99096	2.29612	67	18	1.03897	2.25735	71	18	1.08222	2.22412	75	18	1.12135	2.19540
63	19	0.95394	2.34518	67	19	1.00394	2.30277	71	19	1.04900	2.26634	75	19	1.08982	2.23480
63	20	0.91703	2.39474	67	20	0.96894	2.34868	71	20	1.01579	2.30903	75	20	1.05825	2.27465
63	21	0.88029	2.44473	67	21	0.93402	2.39503	71	21	0.98261	2.35215	75	21	1.02668	2.31492
64	2	1.56348	1.62683	68	2	1.57706	1.63665	72	2	1.58949	1.64571	76	2	1.60090	1.65413
64	3	1.53152	1.66011	68	3	1.54701	1.66784	72	3	1.56112	1.67507	76	3	1.57404	1.68185
64	4	1.49897	1.69463	68	4	1.51642	1.70011	72	4	1.53226	1.70539	76	4	1.54673	1.71043
64	5	1.46587	1.73033	68	5	1.48531	1.73345	72	5	1.50293	1.73664	76	5	1.51900	1.73985
64	6	1.43223	1.76720	68	6	1.45373	1.76781	72	6	1.47317	1.76881	76	6	1.49086	1.77009
64	7	1.39813	1.80520	68	7	1.42171	1.80318	72	7	1.44300	1.80187	76	7	1.46233	1.80113
64	8	1.36359	1.84429	68	8	1.38928	1.83952	72	8	1.41245	1.83581	76	8	1.43346	1.83295
64	9	1.32865	1.88444	68	9	1.35647	1.87679	72	9	1.38154	1.87059	76	9	1.40425	1.86553
64	10	1.29336	1.92561	68	10	1.32332	1.91497	72	10	1.35030	1.90618	76	10	1.37473	1.89886
64	11	1.25775	1.96775	68	11	1.28987	1.95403	72	11	1.31877	1.94256	76	11	1.34493	1.93288
64	12	1.22188	2.01081	68	12	1.25614	1.99393	72	12	1.28698	1.97970	76	12	1.31488	1.96761
64	13	1.18576	2.05475	68	13	1.22218	2.03462	72	13	1.25495	2.01756	76	13	1.28458	2.00299
64	14	1.14949	2.09952	68	14	1.18803	2.07606	72	14	1.22272	2.05611	76	14	1.25408	2.03900
64	15	1.11306	2.14507	68	15	1.15372	2.11823	72	15	1.19031	2.09532	76	15	1.22340	2.07563
64	16	1.07655	2.19134	68	16	1.11929	2.16106	72	16	1.15776	2.13516	76	16	1.19257	2.11283
64	17	1.04000	2.23829	68	17	1.08477	2.20453	72	17	1.12510	2.17558	76	17	1.16161	2.15057
64	18	1.00345	2.28584	68	18	1.05021	2.24857	72	18	1.09237	2.21655	76	18	1.13056	2.18883
64	19	0.96694	2.33395	68	19	1.01563	2.29315	72	19	1.05959	2.25803	76	19	1.09942	2.22757
64	20	0.93053	2.38255	68	20	0.98109	2.33822	72	20	1.02680	2.29997	76	20	1.06825	2.26676
64	21	0.89425	2.43159	68	21	0.94663	2.38371	72	21	0.99403	2.34236	76	21	1.03706	2.30638
65	2	1.56699	1.62936	69	2	1.58027	1.63898	73	2	1.59243	1.64788	77	2	1.60361	1.65614
65	3	1.53553	1.66210	69	3	1.55066	1.66970	73	3	1.56446	1.67681	77	3	1.57710	1.68348
65	4	1.50349	1.69602	69	4	1.52052	1.70146	73	4	1.53599	1.70667	77	4	1.55015	1.71166
65	5	1.47092	1.73110	69	5	1.48988	1.73425	73	5	1.50709	1.73745	77	5	1.52279	1.74065
65	6	1.43782	1.76731	69	6	1.45877	1.76803	73	6	1.47775	1.76911	77	6	1.49503	1.77044
65	7	1.40426	1.80462	69	7	1.42723	1.80279	73	7	1.44801	1.80164	77	7	1.46690	1.80102
65	8	1.37027	1.84298	69	8	1.39529	1.83849	73	8	1.41789	1.83502	77	8	1.43842	1.83235
65	9	1.33589	1.88238	69	9	1.36298	1.87512	73	9	1.38743	1.86923	77	9	1.40961	1.86443
65	10	1.30115	1.92276	69	10	1.33032	1.91262	73	10	1.35663	1.90422	77	10	1.38048	1.89722
65	11	1.26611	1.96408	69	11	1.29737	1.95098	73	11	1.32556	1.93999	77	11	1.35108	1.93071
65	12	1.23080	2.00631	69	12	1.26415	1.99014	73	12	1.29421	1.97649	77	12	1.32143	1.96487
65	13	1.19525	2.04939	69	13	1.23069	2.03009	73	13	1.26262	2.01370	77	13	1.29155	1.99969
65	14	1.15952	2.09329	69	14	1.19704	2.07078	73	14	1.23084	2.05159	77	14	1.26146	2.03511
65	15	1.12364	2.13795	69	15	1.16322	2.11216	73	15	1.19889	2.09013	77	15	1.23119	2.07113
65	16	1.08767	2.18331	69	16	1.12928	2.15421	73	16	1.16678	2.12927	77	16	1.20076	2.10772
65	17	1.05165	2.22934	69	17	1.09524	2.19688	73	17	1.13456	2.16899	77	17	1.17020	2.14485
65	18	1.01560	2.27597	69	18	1.06115	2.24012	73	18	1.10226	2.20925	77	18	1.13954	2.18248
65	19	0.97960	2.32315	69	19	1.02704	2.28388	73	19	1.06991	2.25001	77	19	1.10881	2.22059
65	20	0.94367	2.37083	69	20	0.99295	2.32813	73	20	1.03753	2.29124	77	20	1.07801	2.25914
65	21	0.90785	2.41894	69	21	0.95892	2.37281	73	21	1.00517	2.33290	77	21	1.04721	2.29811
66	2	1.57043	1.63184	70	2	1.58341	1.64127	74	2	1.59530	1.65001	78	2	1.60626	1.65812
66	3	1.53945	1.66404	70	3	1.55422	1.67152	74	3	1.56772	1.67852	78	3	1.58010	1.68509
66	4	1.50790	1.69740	70	4	1.52452	1.70278	74	4	1.53966	1.70793	78	4	1.55351	1.71287
66	5	1.47583	1.73188	70	5	1.49434	1.73505	74	5	1.51115	1.73825	78	5	1.52651	1.74145
66	6	1.44326	1.76745	70	6	1.46369	1.76827	74	6	1.48222	1.76943	78	6	1.49912	1.77081
66	7	1.41023	1.80409	70	7	1.43262	1.80245	74	7	1.45289	1.80144	78	7	1.47136	1.80093
66	8	1.37677	1.84175	70	8	1.40115	1.83754	74	8	1.42321	1.83429	78	8	1.44325	1.83178
66	9	1.34293	1.88041	70	9	1.36932	1.87353	74	9	1.39316	1.86793	78	9	1.41483	1.86337
66	10	1.30874	1.92004	70	10	1.33716	1.91037	74	10	1.36281	1.90235	78	10	1.38610	1.89565
66	11	1.27424	1.96058	70	11	1.30469	1.94805	74	11	1.33217	1.93752	78	11	1.35711	1.92862
66	12	1.23947	2.00200	70	12	1.27196	1.98652	74	12	1.30127	1.97341	78	12	1.32785	1.96224
66	13	1.20447	2.04426	70	13	1.23899	2.02574	74	13	1.27013	2.01000	78	13	1.29836	1.99650
66	14	1.16928	2.08731	70	14	1.20582	2.06569	74	14	1.23878	2.04724	78	14	1.26867	2.03136
66	15	1.13394	2.13110	70	15	1.17249	2.10634	74	15	1.20725	2.08511	78	15	1.23879	2.06680
66	16	1.09850	2.17559	70	16	1.13902	2.14762	74	16	1.17559	2.12359	78	16	1.20876	2.10279
66	17	1.06298	2.22074	70	17	1.10544	2.18951	74	17	1.14379	2.16263	78	17	1.17860	2.13932
66	18	1.02744	2.26648	70	18	1.07182	2.23197	74	18	1.11192	2.20220	78	18	1.14832	2.17634
66	19	0.99192	2.31277	70	19	1.03816	2.27495	74	19	1.07998	2.24227	78	19	1.11797	2.21384
66	20	0.95646	2.35954	70	20	1.00451	2.31840	74	20	1.04801	2.28280	78	20	1.08756	2.25177
66	21	0.92111	2.40676	70	21	0.97091	2.36230	74	21	1.01605	2.32375	78	21	1.05712	2.29011

T	K	dL	dU	T	K	dL	dU	T	K	dL	dU	T	K	dL	dU
79	2	1.60887	1.66006	83	2	1.61880	1.66751	87	2	1.62804	1.67448	91	2	1.63664	1.68102
79	3	1.58304	1.68667	83	3	1.59423	1.69276	87	3	1.60461	1.69851	91	3	1.61425	1.70395
79	4	1.55679	1.71407	83	4	1.56928	1.71874	87	4	1.58083	1.72320	91	4	1.59154	1.72747
79	5	1.53015	1.74225	83	5	1.54395	1.74541	87	5	1.55670	1.74852	91	5	1.56850	1.75157
79	6	1.50312	1.77118	83	6	1.51828	1.77278	87	6	1.53224	1.77448	91	6	1.54516	1.77625
79	7	1.47572	1.80086	83	7	1.49226	1.80080	87	7	1.50748	1.80103	91	7	1.52154	1.80147
79	8	1.44800	1.83126	83	8	1.46593	1.82950	87	8	1.48242	1.82819	91	8	1.49763	1.82725
79	9	1.41994	1.86237	83	9	1.43930	1.85882	87	9	1.45707	1.85592	91	9	1.47345	1.85356
79	10	1.39160	1.89416	83	10	1.41239	1.88877	87	10	1.43146	1.88423	91	10	1.44903	1.88040
79	11	1.36299	1.92661	83	11	1.38522	1.91933	87	11	1.40561	1.91310	91	11	1.42437	1.90774
79	12	1.33411	1.95970	83	12	1.35780	1.95048	87	12	1.37951	1.94250	91	12	1.39948	1.93557
79	13	1.30501	1.99342	83	13	1.33017	1.98219	87	13	1.35320	1.97243	91	13	1.37440	1.96389
79	14	1.27571	2.02773	83	14	1.30233	2.01444	87	14	1.32671	2.00285	91	14	1.34911	1.99268
79	15	1.24622	2.06261	83	15	1.27430	2.04723	87	15	1.30002	2.03377	91	15	1.32365	2.02192
79	16	1.21658	2.09804	83	16	1.24612	2.08052	87	16	1.27317	2.06515	91	16	1.29803	2.05159
79	17	1.18679	2.13398	83	17	1.21779	2.11429	87	17	1.24617	2.09699	91	17	1.27226	2.08168
79	18	1.15690	2.17041	83	18	1.18934	2.14853	87	18	1.21906	2.12925	91	18	1.24637	2.11217
79	19	1.12693	2.20730	83	19	1.16080	2.18320	87	19	1.19183	2.16192	91	19	1.22035	2.14305
79	20	1.09689	2.24464	83	20	1.13217	2.21827	87	20	1.16450	2.19498	91	20	1.19424	2.17430
79	21	1.06680	2.28237	83	21	1.10349	2.25373	87	21	1.13710	2.22841	91	21	1.16803	2.20590
80	2	1.61143	1.66197	84	2	1.62118	1.66929	88	2	1.63024	1.67615	92	2	1.63870	1.68259
80	3	1.58592	1.68823	84	3	1.59691	1.69424	88	3	1.60709	1.69990	92	3	1.61656	1.70526
80	4	1.56001	1.71526	84	4	1.57225	1.71987	88	4	1.58358	1.72429	92	4	1.59410	1.72851
80	5	1.53370	1.74304	84	5	1.54723	1.74619	88	5	1.55974	1.74929	92	5	1.57132	1.75232
80	6	1.50703	1.77156	84	6	1.52188	1.77318	88	6	1.53557	1.77491	92	6	1.54824	1.77670
80	7	1.47999	1.80081	84	7	1.49618	1.80084	88	7	1.51109	1.80112	92	7	1.52488	1.80161
80	8	1.45262	1.83077	84	8	1.47018	1.82912	88	8	1.48633	1.82792	92	8	1.50125	1.82707
80	9	1.42495	1.86142	84	9	1.44388	1.85804	88	9	1.46129	1.85529	92	9	1.47736	1.85304
80	10	1.39698	1.89272	84	10	1.41731	1.88756	88	10	1.43599	1.88321	92	10	1.45321	1.87953
80	11	1.36873	1.92469	84	11	1.39048	1.91768	88	11	1.41044	1.91168	92	11	1.42883	1.90652
80	12	1.34024	1.95727	84	12	1.36340	1.94837	88	12	1.38466	1.94068	92	12	1.40423	1.93399
80	13	1.31151	1.99046	84	13	1.33611	1.97962	88	13	1.35867	1.97019	92	13	1.37943	1.96194
80	14	1.28259	2.02423	84	14	1.30862	2.01140	88	14	1.33248	2.00018	92	14	1.35444	1.99033
80	15	1.25348	2.05857	84	15	1.28094	2.04370	88	15	1.30611	2.03067	92	15	1.32927	2.01918
80	16	1.22422	2.09343	84	16	1.25310	2.07649	88	16	1.27958	2.06160	92	16	1.30393	2.04845
80	17	1.19481	2.12881	84	17	1.22512	2.10976	88	17	1.25290	2.09298	92	17	1.27846	2.07813
80	18	1.16529	2.16467	84	18	1.19701	2.14348	88	18	1.22609	2.12478	92	18	1.25285	2.10821
80	19	1.13568	2.20099	84	19	1.16880	2.17762	88	19	1.19918	2.15699	92	19	1.22713	2.13867
80	20	1.10600	2.23772	84	20	1.14051	2.21218	88	20	1.17217	2.18959	92	20	1.20129	2.16949
80	21	1.07628	2.27487	84	21	1.11215	2.24712	88	21	1.14507	2.22254	92	21	1.17538	2.20066
81	2	1.61393	1.66385	85	2	1.62350	1.67105	89	2	1.63242	1.67780	93	2	1.64073	1.68414
81	3	1.58875	1.68976	85	3	1.59952	1.69568	89	3	1.60951	1.70127	93	3	1.61883	1.70656
81	4	1.56316	1.71643	85	4	1.57516	1.72100	89	4	1.58628	1.72536	93	4	1.59661	1.72954
81	5	1.53719	1.74384	85	5	1.55045	1.74697	89	5	1.56271	1.75006	93	5	1.57409	1.75308
81	6	1.51085	1.77196	85	6	1.52540	1.77361	89	6	1.53883	1.77535	93	6	1.55127	1.77716
81	7	1.48417	1.80079	85	7	1.50003	1.80089	89	7	1.51465	1.80123	93	7	1.52818	1.80176
81	8	1.45715	1.83031	85	8	1.47434	1.82879	89	8	1.49017	1.82768	93	8	1.50480	1.82690
81	9	1.42984	1.86051	85	9	1.44837	1.85730	89	9	1.46542	1.85469	93	9	1.48117	1.85255
81	10	1.40223	1.89135	85	10	1.42212	1.88641	89	10	1.44042	1.88223	93	10	1.45730	1.87870
81	11	1.37434	1.92282	85	11	1.39562	1.91610	89	11	1.41518	1.91032	93	11	1.43321	1.90534
81	12	1.34622	1.95492	85	12	1.36889	1.94635	89	12	1.38970	1.93892	93	12	1.40889	1.93246
81	13	1.31787	1.98760	85	13	1.34194	1.97714	89	13	1.36402	1.96802	93	13	1.38437	1.96004
81	14	1.28931	2.02085	85	14	1.31477	2.00845	89	14	1.33814	1.99760	93	14	1.35966	1.98806
81	15	1.26058	2.05466	85	15	1.28744	2.04028	89	15	1.31208	2.02766	93	15	1.33477	2.01652
81	16	1.23168	2.08898	85	16	1.25993	2.07259	89	16	1.28585	2.05816	93	16	1.30972	2.04540
81	17	1.20264	2.12381	85	17	1.23229	2.10536	89	17	1.25949	2.08910	93	17	1.28453	2.07469
81	18	1.17348	2.15911	85	18	1.20451	2.13858	89	18	1.23299	2.12046	93	18	1.25920	2.10436
81	19	1.14424	2.19486	85	19	1.17664	2.17223	89	19	1.20638	2.15221	93	19	1.23376	2.13441
81	20	1.11491	2.23103	85	20	1.14868	2.20627	89	20	1.17967	2.18434	93	20	1.20821	2.16482
81	21	1.08555	2.26760	85	21	1.12064	2.24070	89	21	1.15289	2.21683	93	21	1.18259	2.19556
82	2	1.61639	1.66569	86	2	1.62579	1.67277	90	2	1.63454	1.67942	94	2	1.64272	1.68567
82	3	1.59152	1.69128	86	3	1.60209	1.69711	90	3	1.61190	1.70262	94	3	1.62106	1.70784
82	4	1.56625	1.71759	86	4	1.57802	1.72210	90	4	1.58893	1.72642	94	4	1.59908	1.73055
82	5	1.54060	1.74462	86	5	1.55360	1.74775	90	5	1.56564	1.75082	94	5	1.57681	1.75382
82	6	1.51461	1.77237	86	6	1.52885	1.77404	90	6	1.54202	1.77580	94	6	1.55424	1.77761
82	7	1.48826	1.80079	86	7	1.50378	1.80095	90	7	1.51812	1.80135	94	7	1.53140	1.80192
82	8	1.46159	1.82989	86	8	1.47842	1.82848	90	8	1.49393	1.82745	94	8	1.50829	1.82675
82	9	1.43462	1.85964	86	9	1.45277	1.85659	90	9	1.46947	1.85411	94	9	1.48493	1.85209
82	10	1.40736	1.89003	86	10	1.42684	1.88530	90	10	1.44476	1.88129	94	10	1.46133	1.87791
82	11	1.37984	1.92105	86	11	1.40066	1.91457	90	11	1.41982	1.90900	94	11	1.43750	1.90421
82	12	1.35207	1.95265	86	12	1.37426	1.94439	90	12	1.39464	1.93721	94	12	1.41345	1.93097
82	13	1.32408	1.98485	86	13	1.34762	1.97474	90	13	1.36926	1.96592	94	13	1.38921	1.95820
82	14	1.29590	2.01760	86	14	1.32081	2.00561	90	14	1.34368	1.99510	94	14	1.36478	1.98586
82	15	1.26752	2.05088	86	15	1.29379	2.03697	90	15	1.31792	2.02474	94	15	1.34016	2.01394
82	16	1.23898	2.08469	86	16	1.26662	2.06881	90	16	1.29200	2.05483	94	16	1.31540	2.04244
82	17	1.21030	2.11897	86	17	1.23931	2.10111	90	17	1.26594	2.08533	94	17	1.29049	2.07134
82	18	1.18150	2.15373	86	18	1.21187	2.13384	90	18	1.23974	2.11626	94	18	1.26544	2.10062
82	19	1.15260	2.18894	86	19	1.18432	2.16700	90	19	1.21344	2.14756	94	19	1.24027	2.13027
82	20	1.12364	2.22455	86	20	1.15667	2.20054	90	20	1.18703	2.17925	94	20	1.21500	2.16027
82	21	1.09461	2.26056	86	21	1.12896	2.23446	90	21	1.16053	2.21129	94	21	1.18965	2.19061