A note to question 3 in exercise set to seminar 5

The question was:

Explain how you would estimate the parameters in each equation of the following model.

$$
\begin{aligned}
y_{1t} + \beta_{12} y_{2t} + \gamma_{11} + \gamma_{12} x_{2t} &= \varepsilon_{1t} \\
y_{2t} + \gamma_{21} + \gamma_{23} x_{3t} &= \varepsilon_{2t} \\
\beta_{32} y_{2t} + y_{3t} + \gamma_{31} + \gamma_{33} x_{3t} &= \varepsilon_{3t}
\end{aligned}
$$

where y_{1t}, y_{2t} and y_{3t} are endogenous variables.

HINT: Take care to base your answer on a complete econometric specification of the model.

Remarks:

It is important to base the answer on a full econometric specification of the system of equations. The two possibilities we have discussed are recursive model and "not-recursive model" (dependency via correlated disturbances).

i) Recursive model: In addition to econometric exogeneity of x_{2t} and x_{3t}, and (for simplicity) classical assumptions for each disturbance, ε_{it} $(i = 1, 2, 3)$, we assume that each equation's error term is uncorrelated with all the other error terms in all time periods. In this case we have a recursive econometric model, because the covariance matrix of the disturbances is diagonal and the equations can be re-ordered

$$
\begin{aligned}
y_{1t} + 0 y_{3t} + \beta_{12} y_{2t} &= -\gamma_{11} - \gamma_{12} x_{2t} + \varepsilon_{1t} \\
y_{3t} + \beta_{32} y_{2t} &= -\gamma_{31} - \gamma_{33} x_{3t} + \varepsilon_{3t} \\
y_{2t} &= -\gamma_{21} - \gamma_{23} x_{3t} + \varepsilon_{2t}
\end{aligned}
$$

showing the characteristic triangular form of the matrix that holds the coefficients of the endogenous variables. All the equations are identified and the OLS estimators (equation by equation) are BLUE (given the classical assumptions made for each disturbance).

ii) If we do not make the assumption about uncorrelated disturbances, we have a different econometric model. When we use the order condition we get the following provisional conclusions (referring to the original ordering of the equations): The first equation is just identified. 2SLS, IV and ILS are all equivalent (give the same estimates) and provide consistent estimators: x_{3t} is used as instrument for y_{2t} in this equation.

The second equation contains no endogenous explanatory variables, hence OLS of y_{2t} on x_{3t} and a constant provides unbiased and consistent estimators (in fact, they are BLUE given the classical assumptions that we have made above). Evaluated by the order condition, the second equation is over-identified, but since the reduced form equation for y_{2t} is the same as the structural equation, we do in fact get one and only one consistent estimator for the parameter vector $(\gamma_{21}, \gamma_{23})$ from the reduced form.

The third equation is exactly identified on the order condition. But this is a provisional result (as noted!). To see whether it really is identified, try to answer the question: which variable should we use as an instrument for y_{2t} in the case where y_{2t} is correlated with ε_{3t}? The obvious candidate is x_{2t}, but given the specification of the system of equations, there is no way that x_{2t} can influence y_{2t}! Hence there is after all no available valid instrument for y_{2t} in the third equation, and therefore there is no consistent estimator, meaning that the parameters of the third equation are not identified generally (the rank condition would confirm this).