Econ 4160: Seminar 4

October 25, 2011

Problems from postponed exam spring 2006

1a

Specify the reduced form of this macro model and determine which of its equations are exactly identified and which are overidentified.

Solving for Y_t

$$Y_t = \frac{a_0+u_t+b_0+b_2Y_{t-1}+v_t+X_t}{1-a_1-b_1}$$

$$= \prod_{10} + \prod_{11} Y_{t-1} + \prod_{12} X_t + \varepsilon_{1t}$$

$$\prod_{10} = \frac{a_0+b_0}{1-a_1-b_1}$$

$$\prod_{11} = \frac{b_2}{1-a_1-b_1}$$

$$\prod_{12} = \frac{1}{1-a_1-b_1}$$

$$\varepsilon_{1t} = \frac{u_t+v_t}{1-a_1-b_1}$$

Solving for C_t

$$C_t = a_0 + a_1(\frac{a_0+u_t+b_0+b_2Y_{t-1}+v_t+X_t}{1-a_1-b_1}) + u_t$$

$$= \prod_{20} + \prod_{21} Y_{t-1} + \prod_{22} X_t + \varepsilon_{2t}$$

$$\prod_{20} = \frac{a_1b_0-b_1}{(1-a_1-b_1)}$$

$\prod_{21} = \frac{a_1 b_2}{(1-a_1-b_1)}$

$\prod_{22} = \frac{a_1}{(1-a_1-b_1)}$

$\varepsilon_{2t} = \frac{u_t + a_1 v_t - u_t b_1}{(1-a_1-b_1)}$

Solving for I_t

$I_t = b_0 + b_1(\frac{a_0+u_t+b_0+b_2Y_{t-1}+v_t+X_t}{1-a_1-b_1}) + b_2Y_{t-1} + v_t$

$= \prod_{30} + \prod_{31} Y_{t-1} + \prod_{32} X_t + \varepsilon_{3t}$

$\prod_{30} = \frac{b_0 - b_0 a_1 - b_1 a_0}{1-a_1-b_1}$

$\prod_{31} = \frac{b_1 b_2 - b_2 a_1}{1-a_1-b_1}$

$\prod_{32} = \frac{b_1}{1-a_1-b_1}$

$\varepsilon_{3t} = \frac{v_t - v_t a_1 + b_1 u_t}{1-a_1-b_1}$

Problem 3

3a

The model contains four equations: which are its four endogenous variables, and which of them are observable and which are latent? The consumption function (1) is specified without a disturbance. Do you have any comment to this simplifying assumption?

a)

We have four endogenous variables : η_i, ξ_i, y_i, x_i

Where the first two are latent variables and the two latter are observed consumption and income.

When equation 1 and two are combined we get:

$y_i = \alpha + \beta\xi_i + \varepsilon$

Adding a disturbance term to (1) would not give any extra information.

3b

$var(y_i) = var(\eta_i + \varepsilon_i) = var(\eta_i) + var(\varepsilon_i) + 2cov(\eta, \varepsilon_i)$

$2cov(\eta, \varepsilon_i)$ is 0 by assumption

$=var(\alpha+\beta\xi)+var(\varepsilon_i) = var(\alpha)+\beta^2 var(\xi)+var(\varepsilon_i) = \beta^2\gamma_x^2\sigma_q^2+\beta^2\sigma_u^2+\sigma_\varepsilon^2$

$var(x_i) = \gamma_x^2\sigma_q^2 + \sigma_u^2 + \sigma_\delta^2$

$cov(y_i,q_i) = \beta\gamma_x\sigma_q^2$

$cov(y_i,x_i) = \beta\gamma_x^2\sigma_q^2 + \beta\sigma_u^2$

$cov(x_i,q_i) = \gamma_x\sigma_q^2$

3c

$y_i = (\alpha+\beta(x_i-\delta_i)+\varepsilon_i) = (\alpha+\beta x_i-\beta\delta_i+\varepsilon_i)$ where we can put $\omega_i = -\beta\delta_i+\varepsilon_i$

- Correlated with thte variable it is supposed to represent (X)
- Not correlated with residual (w_i)

3d

$$\hat{\beta}(z) = \frac{M[y,z]}{M[x,z]}$$

$$plim\hat{\beta}(z) = \frac{plimM[y,z]}{plimM[x,z]} = \frac{cov(y,z)}{var(z)}$$

i) $z_i = x_i$

$\frac{cov(y,x)}{var(x)} = \frac{\beta\gamma_x^2\sigma_q^2+\beta\sigma_u^2}{\gamma_x^2\sigma_q^2+\sigma_u^2+\sigma_\delta^2} = \beta\frac{\gamma_x^2\sigma_q^2+\sigma_u^2}{\gamma_x^2\sigma_q^2+\sigma_u^2+\sigma_\delta^2} < \beta$

Inconsistent estimator. Underestimation
ii) $z_i = y_i$

$\frac{var(y)}{cov(x,y)} = \frac{\beta^2\gamma_x^2\sigma_q^2+\beta^2\sigma_u^2+\sigma_\varepsilon^2}{\beta\gamma_x^2\sigma_q^2+\beta\sigma_u^2} = \beta + \frac{\sigma_\varepsilon^2}{\gamma_x^2\sigma_q^2+\sigma_u^2} > \beta$

Inconsistent estimator. Overestimation

iii) $z_i = \hat{x}_i = \hat{\lambda}_x + \hat{\gamma_x}qi$

$\frac{cov(y,\lambda_x+\gamma_x qi)}{cov(x,\lambda_x+\gamma_x qi)} = \frac{cov(\alpha+\beta x_i+\varepsilon_i-\beta\delta_i,\lambda_x+\gamma_x q_i)}{cov(-\frac{\alpha}{\beta}+\frac{1}{\beta}y_i+\delta_i-\frac{1}{\beta}\varepsilon_i,\lambda_x+\gamma_x q_i)} = \frac{\beta\gamma_x cov(x,q)}{\frac{1}{\beta}\gamma_x cov(y,q)} = \frac{\beta^2\gamma_x\sigma_q^2}{\beta\gamma_x\sigma_q^2} = \beta$

Consistent estimator for β

iv) $z_i = \hat{y}_i = \hat{\lambda}_y + \hat{\gamma}_y qi$

$$\frac{cov(y, \lambda_y + \gamma_y qi)}{cov(x, \lambda_y + \gamma_y qi)} = \frac{cov(\alpha + \beta x_i + \varepsilon_i - \beta\delta_i, \lambda_y + \gamma_y q_i)}{cov(-\frac{\alpha}{\beta} + \frac{1}{\beta} y_i + \delta_i - \frac{1}{\beta}\varepsilon_i, \lambda_y + \gamma_y q_i)} = \frac{\beta\gamma_y cov(x,q)}{\frac{1}{\beta}\gamma_y cov(y,q)} = \frac{\beta^2 \gamma_y \sigma_q^2}{\beta\gamma_y \sigma_q^2} = \beta$$

Consistent estimator for β

x and y endogenous -> cannot be used as IV shown by inconsistency in i) and ii)

We know that q is uncorrelated with the error terms in both equations -> $(\hat{\lambda}_x, \hat{\gamma_x})$consistent estimators of (λ_x, γ_x)

If you find any misprints/mistakes, don't hesitate to let us know! (sofie.kjernli-wijnen@econ.uio.no)