Exercise 1: The ADL model and Equilibrium Correction form

a)

Known properties of VAR:

We insert equation (4) into equation (3) to get:

$$y_t = (\varphi_1 + \beta_1 a_{21})y_{t-1} + (\beta_2 + \beta_1 a_{22})x_t + \beta_1 \varepsilon_{xt} + \varepsilon_t,$$

Where $(\varphi_1 + \beta_1 a_{21}) = a_{11}$, $(\beta_2 + \beta_1 a_{22}) = a_{12}$ in (1), and $\beta_1 \varepsilon_{xt} + \varepsilon_t = \varepsilon_{yt}$

From the slides to CC 2 we have that

$$\varphi_1 = a_{11} + \frac{\omega_{xy}}{\sigma_x^2} a_{21} = 0{,}4 \approx 0{,}447$$

$$\beta_1 = \frac{\omega_{xy}}{\sigma_x^2} = 0{,}5 \approx 0{,}42$$

$$\beta_2 = a_{12} - \frac{\omega_{xy}}{\sigma_x^2} a_{22} = 0{,}05 \approx 0{,}088$$

And we see that the estimated parameter values come close to what we expect from the known properties of this artificial VAR.

AR 1-2 test:
Regress the OLS residuals on all independent variables, including the lagged variables (and the intercept). What we really do is to test whether the errors in different time periods are correlated with each other. We the obtain the F test for joint significance of u_t and u_{xt} to test the null hypothesis that that the errors are serially uncorrelated;

H_0: ρ=0 vs H_1≠0.

In Pc Give we obtain F(2, 195) = 0,59 (0,56) (Significant 5% lever of the F-test with 2 degrees of freedom and 120 observations is 3,07, so it is far from significant).

Conclusion: No serial correlation in the residuals.

ARCH 1-1 Test:
"Auto Regressive Conditional Heteroskedasticity" test.

First, the errors should not be serially correlated; any serial correlation will generally invalidate a heteroskedasticity test.

In the ARCH test we look at the conditional variance of u_t given past errors.

$$Var(y_t|x_t, x_{t-1}, y_{t-1}) = Var(u_t|x_t, x_{t-1}, y_{t-1}) = \alpha_0 + \alpha_1 u_{t-1}^2,$$

$where\ u_t = y_t - E(y_t|x_t, x_{t-1}, y_{t-1})$

In Pc Give we obtain F(1, 198) = 0,015 (0,90) which again is not statistically significant different from zero. (Significant 5% lever of the F-test with 1 degrees of freedom and 120 observations is 3,92)

Conclusion: No heteroskedasticity in the residuals.

Normality Test:
Test the goodness of fit for the system to follow a normal distribution.

The Chi Square Test in Pc Give Chi^2(2)=0,35 (0,84), which means that we cannot reject the null hypothesis that the residual follows a normal distribution.

RESET Test:
Used to test for whether unknown variables have been omitted from a regression specification.

b)

The static long run solution of (3) is defined as

$$y^* = \frac{\beta_1 + \beta_2}{1 - \phi_1} x^*, where\ we\ can\ define\ B \equiv \frac{\beta_1 + \beta_2}{1 - \phi_1}\ as\ the\ long\ run\ derivative\ coefficient.$$

From the static long run solution in PcGive we get the estimate of the long run derivative coefficient $\hat{B}$ = 0.919469 (0.04159).

We received the following value from the regression in PcGive:

$$\widehat{\beta_1} = 0.420176\ \widehat{\beta_2} = 0.0883763\ and\ \widehat{\phi_1} = 0.446907$$

Another way to find B is to plot the estimates of the coefficient from the regression into the definition of B:

$$\hat{B} = \frac{0.420176 + 0.0883763}{1 - 0.446907} = 0.919469$$

We are now going to test the hypothesis H_0: B=1 versus H_1: B≠1

Using the t-test:

$t = \frac{(\hat{B}-1)}{se(\widehat{B})} = \frac{0.919469-1}{0.04159} = -1{,}93630$ so we can reject the null hypothysis that B=1 at the 5% significance level, but not at the 1% level.

c)

We'll use eq (4):

$$y_t = \varphi_1 y_{t-1} + \beta_1 x_t + \beta_2 x_{t-1} + \varepsilon_t$$

and $\Delta y_t = Y_t - y_{t-1}\ ,\ \ \Delta x_t = x_t - x_{t-1}$

First subtract $y_{t-1}\ and\ \beta_1 x_{t-1}$ on both side of eq(4):

$$y_t - y_{t-1} - \beta_1 x_{t-1} = (\varphi_1 - 1) y_{t-1} + \beta_1 x_t + \beta_2 x_{t-1} - \beta_1 x_{t-1} + \varepsilon_t \longrightarrow$$

$\Delta y_t = (\varphi_1 - 1)y_{t-1} + \beta_1 \Delta x_t + (\beta_2 + \beta_1)x_{t-1} + \varepsilon_t.$

We define $b_1 = (\varphi_1 - 1), b_2 = \beta_1 \; and \; b_3 = \beta_1 + \beta_2$ as the new coefficients.

We then create two new variables, Δx_t and Δy_t in PcGive and regress (5):

Comments:

- When we estimated eq (5) in PcGive we got the following estimates of the new coefficients: $\widehat{b_1} = -0.553093, \widehat{b_2} = 0.420176 \; and \; \widehat{b_3} = 0.508552$ which is exactly the same values we get if we insert the estimated coefficients from the previous regression into the definitions of the b's.
- All the tests we went through in 1a) has exactly the same values except from the Hetero-test and the Reset23-test. The result from the Reset23-test has increased from 0.15202 [0.8591] to 0.40360 [0.6685]. This means that the probability that we have omitted a variable has increased, but the test result is still not significant enough to reject the null hypothesis.

d)

If we want to look at the long term static solution we have to transform the model:

In the long run we assume that there is no change between the short term periods:

$Y_t - y_{t-1} = \Delta y = 0$ and $x_t - x_{t-1} = \Delta x = 0$

If we use these assumptions, we can rewrite equation (5) and find the long run coefficient B:

$$\Delta y_t = (\phi_1 - 1)y_{t-1} + \beta_1 \Delta x_t + (\beta_1 + \beta_2)x_t \longrightarrow 0 = (\phi_1 - 1)y + (\beta_1 + \beta_2)x \longrightarrow$$

$$y = \left(\frac{\beta_1 + \beta_2}{1 - \phi_1}\right) x$$

So alternative 3 for estimating B is then simply to see that $B = -\frac{b_3}{b_1}$. To see this;

$-\frac{\widehat{b_3}}{\widehat{b_1}} = -\frac{0.508552}{-0.553093}$=0,919469.

Exercise 3:

We interpret Y(Ya in PcGive) to be private consumption and X(Yb in PcGive) to be disposable income. We are going to look at a permanent change in the endogenous variable x (Yb), and we model this as a sunspot shock in ε_{xt}.

Comments:

We have simulated at 0,1 permanet change in X(Yb) and as we can see on the graph, the long term change on Y(Ya) is approximately 0.95. This means that we will use 95% of the increase in income on consumption. This coincides with exercise 1b where we found that we could not reject the H_0: B=1.

Excercise 4

We are given the two equations

(6) $y_t = \beta_0 + \beta_1 E(x_t|x_{t-1}) + \varepsilon_t$

(7) $x_t = \lambda x_{t-1} + \varepsilon_{xt}$,

And we assume that

- $Cov(\varepsilon_{xt}, \varepsilon_t) = 0$ (the disturbances are uncorrelated)
- $E(\varepsilon_t|x_{t-1}) = E(\varepsilon_{xt}|x_{t-1}) = 0$ (zero conditional expectations)
- $Cov(\varepsilon_{xt}, \varepsilon_{xs}|x_{t-1}, x_{s-1}) = \begin{cases} \sigma^2 \ for\ t = s \\ 0 \ for\ t \neq s \end{cases}$, $Cov(\varepsilon_t, \varepsilon_s|x_{t-1}, x_{s-1}) = \begin{cases} s^2 \ for\ t = s \\ 0 \ for\ t \neq s \end{cases}$ (The errors are homoscedastic and that there is no autocorrelation).

Two alternative ways of showing that the OLS estimator in general will be an inconsistent estimator for the parameter β_1:

Alternative 1:

Regressing y_t on x_{t-1} instead of x_t:

Taking conditional expectations on both sides of (7) yields:

$E(x_t|x_{t-1}) = E(\lambda x_{t-1} + \varepsilon_{xt}|x_{t-1}) = \lambda E(x_{t-1}|x_{t-1}) + E(\varepsilon_{xt}|x_{t-1}) = \lambda x_{t-1}$

Inserting the expression $E(x_t|x_{t-1}) = \lambda x_{t-1}$ into (6) yields:

$y_t = \beta_0 + \beta_1 \lambda x_{t-1} + \varepsilon_t$, where $\beta_1 \lambda$ is the new OLS estimator.

If we define $\gamma \equiv \beta_1 \lambda$, we see that when we regress y_t on x_{t-1} we will get

$\widehat{y_t} = \widehat{\beta_0} + \hat{\gamma} x_{t-1}$,

and $\hat{\gamma}$ will in general not be a consistent estimator for β_1 since $E(\hat{\gamma}) = E(\beta_1 \lambda) = E(\beta_1)E(\lambda)$. So as long as $E(\lambda) \neq 1$, then the OLS estimator will be an inconsistent estimator.

Alternative 2:

When $y_t = \beta_0 + \beta_1 E(x_t|x_{t-1}) + \varepsilon_t$, then we see from (7) that

$x_t = E(x_t|x_{t-1}) + \varepsilon_{xt}$

$E(x_t|x_{t-1}) = x_t - \varepsilon_{xt}$

Inserting this expression into (6) yields:

$y_t = \beta_0 + \beta_1 E(x_t|x_{t-1}) + \varepsilon_t = y_t = \beta_0 + \beta_1(x_t - \varepsilon_{xt}) + \varepsilon_t = \beta_0 + \beta_1 x_t - \beta_1\varepsilon_{xt} + \varepsilon_t.$

Since y_t now is a function of both x_t and ε_{xt}, the regression of the above equation will therefore in general produce inconsistent estimates of β_1. That is because the explanatory variables should not be correlated with the disturbances, but that will now be the case, since x_t is in general correlated with ε_{xt}.

Exercise 5: Exercise 16 in Forty Exercises

"Explain precisely the terms structural form and reduced form of a simultaneous, linear equation system. Explain precisely the difference between a simultaneous linear equation system and a system of linear regressions equations."

With structural form we mean a set of structural relations put together to one determined system, with specified endogenous and exogenous variables, and with specified assumptions about the distributions of the error terms. The structural form has as many equations as endogenous variables.

Structural equations also satisfy the normalization restrictions: Each equation has at least one known coefficient.

The reduced form is found by solving the structural form with respect to the endogenous variables.

In systems of regressions equations there are no endogenous variables on the RHS, while in a simultaneous linear equation system that can be the case. It follows that a system of linear regressions equations often consist of reduced form equations.

Exercise 6:

An explanatory variable that is determined simultaneously with the dependent variable is generally correlated with the error term, which leads to bias and inconsistency in OLS.

EX:

Look at the simultaneous linear equation system for demand and supply as functions of the same price p_t:

(8) $x_t^D = \beta_0 + \beta_1 p_t + \varepsilon_{xD}$

(9) $x_t^S = \alpha_0 + \alpha_1 p_t + \varepsilon_{xS}$

Defining a new equation;

(10) $x_t^D = x_t^S = x_t$

And solving out for p_t we get

$$\beta_0 + \beta_1 p_t + \varepsilon_{xD} = \alpha_0 + \alpha_1 p_t + \varepsilon_{xS}$$

$$p_t(\alpha_1 - \beta_1) = \alpha_0 - \beta_0 + \varepsilon_{xS} - \varepsilon_{xD}$$

$$p_t = \frac{\alpha_0 - \beta_0}{\alpha_1 - \beta_1} + \frac{\varepsilon_{xS} - \varepsilon_{xD}}{\alpha_1 - \beta_1}$$

And we see that p_t is a function of both the disturbance in (8) and the disturbance in (9). This means that the explanatory variable in both equations will be correlated with the disturbance, and this will create simultaneity bias.

Exercise 2: Exercise in practical estimation and use of a conditional model of the VAR (system).

a)

LRC – Log of private disposable income.

LRCa – Disposable income corrected for dividends.

LPC – Log of private consumption.

LF – Log of Wealth where including houses.

Fourth order VAR for the three variables. This means that each explanatory variable is lagged with four periods.

1. First regression: The unrestricted model:
   - The regression on LCP (consumption): Her all variables are significant except from the lags of LRCa variables.
   - Regression on LRCa(income): All the variables are not significant except from the lagged LRCa variables; disposable income mainly depends on historical income.
   - Regression on LF(wealth): What's interesting here is that only the previous period of wealth is significant and the second to fourth lag is insignificant.
   - The F-test result shows that all the variables except LF_3, LRCa_2 and LF_4 are jointly significant, most of them at 1% significance level. This could suggest that a model without these three variables would perform equally well.
   - The log-likelihood value is 1101.91716

2. Next regression: In this regression we have included the LRCa in the LCP regression. Otherwise the model is equal to the unrestricted model.
   - The new variable LRCa is significant with 1% significance level.
   - The log-likelihood value is has decreased 1101.52409; almost nothing.

3. Next regression: In this model we have removed all lagged LCP variable in the LCP regression, and removed the dummy variable in the other equations.
    - The log-likelihood variable I almost the same; decreased to 1101.22203. So the dummy variable in LRCa and LF has no prediction power.
    - The p-value from the LR-test is 0.8459 witch means that the new model is as good as the unrestricted model. So it doesn't matter for the models prediction power that we removed these variables.

b)

Based on the batch file and the significance level of the variables, we estimate the model without LF_3 and LF_4.

The Log-likelihood value is 1098.66019 witch is still very high and the P-value from the LR-test(0.9634) so the model I still good.