ECON 4160: Econometrics–Modelling and Systems Estimation:

Computer Class # 6

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Outline

Data sets

The Durbin-Wu-Hausman test and (a bit more) on tests for overidentifying restrictions

The SUR model

A model of the VAR: The ECM

Single equation typology: special cases of ARDL

Data sets for today:

- *KeynesID1.in7*
- *ECON4160_JORG121314.in7*
- *KonsData2Nor_old.in7*

Batch files for today:

- *Keynes_for_CC6.fl*
- *klem1213_model.fl*
- *Conditional consumption equation.fl*

Durbin-Wu-Hausman test of exogeneity

- Suppose we are interested in the following theoretical relationship

$$C_t = \theta Y_t$$

where θ is the true derivative coefficient (MPC)

- The motivation of the Hausman-test is that we test

$$H_0\colon \ \text{plim}(\hat{\theta}_{OLS}) = \text{plim}(\hat{\theta}_{IV})$$

Why is this the test of interest?

1. Under the null, both $\hat{\theta}_{OLS}$ and $\hat{\theta}_{IV}$ are consistent, but $\hat{\theta}_{OLS}$ is efficient
2. Under the alternative, $\hat{\theta}_{OLS}$ is inconsistent, while $\hat{\theta}_{IV}$ is consistent

Hence:

- If we do not reject the null, we stick to $\hat{\theta}_{OLS}$, since it is efficient
- If we reject the null, we opt for $\hat{\theta}_{IV}$, since it is consistent, while $\hat{\theta}_{OLS}$ is not!

The test uses the information implied by the statistical model of C and Y under the alternative hypothesis, to test the equivalent H_0 above:

$$H_0: \ \delta = 0$$

in the regression model:

$$C_t = \beta Y_t + \delta \hat{v}_t + \varepsilon_t$$

where $\hat{v}_t$ is the residual from the marginal (reduced form) model of Y_t, for example

$$Y_t = \gamma Z_t + e_t$$

which can only be formulated by specifying the statistical system for C, Y and Z

- The Hausman test can be interpreted as a test of *Weak Exogeneity*
- The definition of WE is that it allow us to do efficient inference about the *parameter of interest* (θ) by only considering the conditional relationship
- If the test rejects, the parameter of interest is not "in" the regression model—we need a different statistical model, where the marginal model for Y is allowed to play a role in the estimation of θ

At CC # 4, we looked at a version of the (old) Keynesian model:

$$Y_t = C_t + I_t + X_t \tag{1}$$
$$C_t = 0.75Y_t \tag{2}$$
$$I_t = 1.5Y_t - 0.5Y_{t-1} \tag{3}$$

and showed that (both based on OC and RC):

1. (2) is overidentified
2. (3) is exactly identified

Remember that (1) is an identity

Use *Keynes_for_CC6.fl* together with *KeynesID1.in7* to:

1. Test by use of the **Durbin-Wu-Hausman test** for exogeneity whether Y_t is weakly exogenous in (2)
2. Estimate (2) by IVE and test the validity of the overidentifying restriction using the **Sargan-Hansen specification test**
3. Estimate the full system (1–2) by FIML (treating (1) as an identity) and test the validity of the overidentifying restriction using a **Likelihood ratio test**

Confer slides from CC # 5 for details on the latter two tests

The SUR model

- As, we have discussed, there are several times where we are not interested in the parameters of the conditional model, but in the parameters of the system

We could for instance be interested in $\beta_{10}, \beta_{20}, \gamma_{11}, \gamma_{22}$ from the following system:

$$y_t = \beta_{10} + \gamma_{11} z_{1,t} + \epsilon_{y,t} \quad (4)$$
$$x_t = \beta_{20} + \gamma_{22} z_{2,t} + \epsilon_{x,t} \quad (5)$$

with $Cov(z_{1,t}, \epsilon_{y,t}) = Cov(z_{2,t}, \epsilon_{x,t}) = 0$

- Since the correlation between the disturbances in the system in general is different from zero, i.e. $Cov(\epsilon_{y,t}, \epsilon_{x,t}) = \omega_{yx} \neq 0$, the two equations are Seemingly Unrelated Regression Equations, SURE
- From Lecture # 9, you know that the efficient estimator in this case is the Feasible Generalized Least Squares estimator, FGLS (the SUR estimator)

There are, however two special cases where the SUR estimator collapses to the OLS estimator:

1. If $z_{1,t} = z_{2,t}$
2. If $\omega_{yx} = 0$

Example

Use the data set in the *klem1213.zip* to estimate a SUR model in PcGive. HINT: Use constrained FIML (CFIML) to estimate the following system

$$Lkl12_i = \beta_{10} + \gamma_{11} Lpkpl12_i + \epsilon_{Lkl12,i} \quad (6)$$

$$Lkl13_i = \beta_{20} + \gamma_{22} Lpkpl13_i + \epsilon_{Lkl13,i} \quad (7)$$

At CC #2, we considered a bi-variate VAR model of first order:

$$\begin{pmatrix} y_{1,t} \\ y_{2,t} \end{pmatrix} = \begin{pmatrix} \mu_1 \\ \mu_2 \end{pmatrix} + \begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{pmatrix} \begin{pmatrix} y_{1,t-1} \\ y_{2,t-1} \end{pmatrix} + \begin{pmatrix} \varepsilon_{1,t} \\ \varepsilon_{2,t} \end{pmatrix} \tag{8}$$

A more compact notation gives:

$$\mathbf{y}_t = \boldsymbol{\mu} + \mathbf{A}\mathbf{y}_{t-1} + \boldsymbol{\varepsilon}_t \tag{9}$$

Assume that $\boldsymbol{\varepsilon}_t \sim MVN(\mathbf{0}_{2\times1}, \boldsymbol{\Sigma})$, i.e.:

$$\begin{pmatrix} \varepsilon_{1,t} \\ \varepsilon_{2,t} \end{pmatrix} \sim MVN\left(\mathbf{0}_{2\times1}, \boldsymbol{\Sigma} = \begin{pmatrix} \sigma_1^2 & \sigma_{12} \\ \sigma_{21} & \sigma_2^2 \end{pmatrix}\right) \tag{10}$$

We have also shown that the bi-variate VAR(1) represented by (8) can expressed in **model form**:

$$\begin{aligned} y_{1,t} &= \phi_0 + \beta_0 y_{2,t} + \beta_1 y_{2,t-1} + \phi_1 y_{1,t-1} + \epsilon_t & (11) \\ y_{2,t} &= \mu_2 + a_{21} y_{1,t-1} + a_{22} y_{2,t-1} + \varepsilon_{2,t} & (12) \end{aligned}$$

$$\begin{aligned} E(\epsilon_t) &= 0 \;\forall\; t \\ Var(\epsilon_t) &= \sigma_1^2 (1 - \rho_{12}) \;\forall\; t \\ E(y_{2,t}\epsilon_t) &= 0 \;\forall\; t \\ E(\varepsilon_{2,t}\epsilon_t) &= 0 \;\forall\; t \end{aligned}$$

The same applies to a bi-variate VAR model of p^{th} order. Without loss of generality, we shall consider a VAR(2) model:

$$\begin{pmatrix} y_{1,t} \\ y_{2,t} \end{pmatrix} = \begin{pmatrix} \mu_1 \\ \mu_2 \end{pmatrix} + \begin{pmatrix} a_{11,1} & a_{12,1} \\ a_{21,1} & a_{22,1} \end{pmatrix} \begin{pmatrix} y_{1,t-1} \\ y_{2,t-1} \end{pmatrix} + \begin{pmatrix} a_{11,2} & a_{12,2} \\ a_{21,2} & a_{22,2} \end{pmatrix} \begin{pmatrix} y_{1,t-2} \\ y_{2,t-2} \end{pmatrix} + \begin{pmatrix} \varepsilon_{1,t} \\ \varepsilon_{2,t} \end{pmatrix} \quad (13)$$

A more compact notation gives:

$$\mathbf{y}_t = \boldsymbol{\mu} + \mathbf{A}_1\mathbf{y}_{t-1} + \mathbf{A}_2\mathbf{y}_{t-2} + \boldsymbol{\varepsilon}_t \quad (14)$$

Assume that $\boldsymbol{\varepsilon}_t \sim MVN(\mathbf{0}_{2\times 1}, \boldsymbol{\Sigma})$

In a similar way as for the VAR(1) model, we can represent the VAR(2) in (13) on **model form**:

$$\begin{aligned} y_{1,t} &= \phi_0 + \beta_0 y_{2,t} + \beta_1 y_{2,t-1} + \phi_1 y_{1,t-1} \\ &\quad + \beta_2 y_{2,t-2} + \phi_2 y_{1,t-2} + \epsilon_t \quad (15) \\ y_{2,t} &= \mu_2 + a_{21,1} y_{1,t-1} + a_{22,1} y_{2,t-1} \\ &\quad + a_{21,2} y_{1,t-2} + a_{22,2} y_{2,t-2} + \varepsilon_{2,t} \quad (16) \end{aligned}$$

$$\begin{aligned} E(\epsilon_t) &= 0 \; \forall \; t \\ Var(\epsilon_t) &= \sigma_1^2 (1 - \rho_{12}) \; \forall \; t \\ E(y_{2,t}\epsilon_t) &= 0 \; \forall \; t \\ E(\varepsilon_{2,t}\epsilon_t) &= 0 \; \forall \; t \end{aligned}$$

The equilibrium correction model (ECM) as a reparameterization of the ARDL model

The conditional model for $y_{1,t}$ in (15) is called an ARDL(2,2) model:

$$y_{1,t} = \phi_0 + \phi_1 y_{1,t-1} + \phi_2 y_{1,t-2} + \beta_0 y_{2,t} + \beta_1 y_{2,t-1} + \beta_2 y_{2,t-2} + \epsilon_t$$

Subtract and add $\phi_2 y_{1,t-1}$ and $\beta_2 y_{2,t-1}$ on the RHS. We then get:

$$\begin{aligned} y_{1,t} = {} & \phi_0 + \phi_1 y_{1,t-1} + \phi_2 y_{1,t-2} + \beta_0 y_{2,t} + \beta_1 y_{2,t-1} + \beta_2 y_{2,t-2} \\ & + \phi_2 y_{1,t-1} - \phi_2 y_{1,t-1} + \beta_2 y_{2,t-1} - \beta_2 y_{2,t-1} + \epsilon_t \end{aligned}$$

Collecting some terms, we have:

$$\begin{aligned} y_{1,t} = {} & \phi_0 + (\phi_1 + \phi_2) y_{1,t-1} - \phi_2 (y_{1,t-1} - y_{1,t-2}) + \beta_0 y_{2,t} \\ & + (\beta_1 + \beta_2) y_{2,t-1} - \beta_2 (y_{2,t-1} - y_{2,t-2}) + \epsilon_t \end{aligned}$$

Outline Data sets Hausman test + recap tests for OIR The SUR model ECM Typology

ECM cont'd

Let $\Delta z_{t-i} == L^i z_t - L^{i+1} z_t = L^i(1-L) z_t = z_{t-i} - z_{t-i-1}$ for any variable z (in our case y_1 and y_2). We can then write the above expression as:

$$\begin{aligned} y_{1,t} = {} & \phi_0 + (\phi_1 + \phi_2) y_{1,t-1} - \phi_2 \Delta y_{1,t-1} + \beta_0 y_{2,t} + (\beta_1 + \beta_2) y_{2,t-1} \\ & - \beta_2 \Delta y_{2,t-1} + \epsilon_t \end{aligned}$$

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ECM cont'd

Now, subtract and add $\beta_0 y_{2,t-1}$ on the RHS and also subtract $y_{1,t-1}$ on both the RHS and the LHS side. We then get:

$$y_{1,t} - y_{1,t-1} = \phi_0 + (\phi_1 + \phi_2)y_{1,t-1} - \phi_2 \Delta y_{1,t-1} + \beta_0 y_{2,t} + (\beta_1 + \beta_2)y_{2,t-1} - \beta_2 \Delta y_{2,t-1} - y_{1,t-1} + \beta_0 y_{2,t-1} - \beta_0 y_{2,t-1} + \epsilon_t$$

Again, just collecting terms and using $\Delta z_{t-i} = z_{t-i} - z_{t-i-1}$, we have:

$$\Delta y_{1,t} = \phi_0 + (\phi_1 + \phi_2 - 1)y_{1,t-1} - \phi_2 \Delta y_{1,t-1} + \beta_0 \Delta y_{2,t} - \beta_2 \Delta y_{2,t-1} + (\beta_0 + \beta_1 + \beta_2)y_{2,t-1} + \epsilon_t$$

But then, we can also write this as:

$$\Delta y_{1,t} = \phi_0 + (\phi_1 + \phi_2 - 1)\left(y_{1,t-1} - \frac{(\beta_0 + \beta_1 + \beta_2)}{(1 - \phi_1 + \phi_2)} y_{2,t-1}\right)$$

$$\phi_2 \Delta y_{1,t-1} + \beta_0 \Delta y_{2,t} - \beta_2 \Delta y_{2,t-1} + \epsilon_t$$

ECM cont'd

Now, let us define:

$$\alpha = \phi_1 + \phi_2 - 1$$

$$\gamma = \frac{\beta_0 + \beta_1 + \beta_2}{1 - \phi_1 + \phi_2} = -\frac{\beta_0 + \beta_1 + \beta_2}{\alpha}$$

But then, we have:

$$\Delta y_{1,t} = \phi_0 + \alpha(y_{1,t-1} - \gamma y_{2,t-1}) - \phi_2 \Delta y_{1,t-1} + \beta_0 \Delta y_{2,t} - \beta_2 \Delta y_{2,t-1} + \epsilon_t \quad (17)$$

Which is nothing but the equilibrium/error correction model!

ECM cont'd

For estimation purposes (OLS), we would consider:

$$\Delta y_{1,t} = \phi_0 + \alpha y_{1,t-1} + \eta y_{2,t-1} - \phi_2 \Delta y_{1,t-1} + \beta_0 \Delta y_{2,t} - \beta_2 \Delta y_{2,t-1} + \epsilon_t$$

where I just defined $\eta = -\alpha\gamma$. How would you estimate γ? How would you get the standard error of gamma?

Why do we call this model an ECM?

Remember that the conditional model for $y_{1,t}$ is:

$$y_{1,t} = \phi_0 + \phi_1 y_{1,t-1} + \phi_2 y_{1,t-2} + \beta_0 y_{2,t} + \beta_1 y_{2,t-1} + \beta_2 y_{2,t-2} + \epsilon_t$$

In a static long-run equilibrium, we have:

$$\begin{aligned} y_{1,t} &= y_{1,t-1} = y_{1,t-2} = y_1^* \\ y_{2,t} &= y_{2,t-1} = y_{2,t-2} = y_2^* \\ \epsilon_t &= \epsilon^* = 0 \end{aligned}$$

Thus, we find that:

$$y_1^* = \phi_0 + \phi_1 y_1^* + \phi_2 y_1^* + \beta_0 y_2^* + \beta_1 y_2^* + \beta_2 y_2^*$$

Solving for y_1^* , we get:

$$y_1^* = \frac{\phi_0}{1-\phi_1-\phi_2} + \frac{\beta_0+\beta_1+\beta_2}{1-\phi_1-\phi_1} y_2^* = -\frac{\phi_0}{\alpha} - \frac{\beta_0+\beta_1+\beta_2}{\alpha} y_2^* = -\frac{\phi_0}{\alpha} + \gamma y_2^*$$

But this means that $\gamma y_2^* = y_1^* + \frac{\phi_0}{\alpha}$! Substituting this back into (17), we find:

$$\Delta y_{1,t} = \phi_0 + \alpha(y_{1,t-1} - y_{1,t-1}^* - \frac{\phi_0}{\alpha}) - \phi_2 \Delta y_{1,t-1} + \beta_0 \Delta y_{2,t} - \beta_2 \Delta y_{2,t-1} + \epsilon_t$$

which also can be written as:

$$\Delta y_{1,t} = \phi_0 - \alpha\frac{\phi_0}{\alpha} + \alpha(y_{1,t-1} - y_{1,t-1}^*) - \phi_2 \Delta y_{1,t-1} + \beta_0 \Delta y_{2,t} - \beta_2 \Delta y_{2,t-1} + \epsilon_t$$

Thus, we see that, as long as $\alpha = \phi_1 + \phi_2 - 1 < 0 \Rightarrow \phi_1 + \phi_2 < |1|$, Δy_1 will fall (equilibrium correct) when $y_1 > y_1^*$ in the previous period!!

Back to the conditional consumption model!

At Seminar # 3, we looked at the following ARDL(4,4) model for consumption in Norway:

$$LCP_t = \mu + \sum_{i=1}^{4} a_{CC,i} LCP_{t-i} + \sum_{i=0}^{4} a_{CR,i} LRCa_{t-i} + \sum_{i=0}^{4} a_{CF,i} LF_{t-i}$$
$$Seasonals + \alpha_{CD} D_t + \varepsilon_t$$

Ignoring the dummies for simplicity, we can rewrite the conditional consumption model in a similar way as above:

$$\Delta LCP_t = \mu + \sum_{i=1}^{3} \phi_i \Delta LCP_{t-i} + \sum_{i=0}^{3} \gamma_i \Delta LRCa_{t-i} + \sum_{i=0}^{3} \eta_i \Delta LF_{t-i}$$
$$+ \alpha(LCP_{t-1} - \beta_1 LRCa_{t-1} - \beta_2 LF_{t-1}) + \varepsilon_t$$

where the $\phi's$, $\gamma's$ and $\eta's$ are functions of the parameters in the

ECM cont'd

α, β_1 and β_2 are defined in the following way:

$$\alpha = a_{CC1} + a_{CC2} + a_{CC3} + a_{CC4} - 1$$

$$\beta_1 = \frac{a_{CR1} + a_{CR2} + a_{CR3} + a_{CR4}}{1 - a_{CC1} + a_{CC2} + a_{CC3} + a_{CC4}} = -\frac{a_{CR1} + a_{CR2} + a_{CR3} + a_{CR4}}{\alpha}$$

$$\beta_2 = \frac{a_{CF1} + a_{CF2} + a_{CF3} + a_{CF4}}{1 - a_{CC1} + a_{CC2} + a_{CC3} + a_{CC4}} = -\frac{a_{CF1} + a_{CF2} + a_{CF3} + a_{CF4}}{\alpha}$$

Set all Δ −terms to zero. Then solve for the stationary value of LCP

$$LCP = \mu + \beta_1 LFRCa + \beta_2 LF$$

We can get the estimates for the two long-run coefficients β_1 and β_2 from the estimates of the parameters in the conditional consumption model:

$$\beta_1 = \frac{\hat{a}_{CR1} + \hat{a}_{CR2} + \hat{a}_{CR3} + \hat{a}_{CR4}}{1 - \hat{a}_{CC1} + \hat{a}_{CC2} + \hat{a}_{CC3} + \hat{a}_{CC4}}$$

$$\beta_2 = \frac{\hat{a}_{CF1} + \hat{a}_{CF2} + \hat{a}_{CF3} + \hat{a}_{CF4}}{1 - \hat{a}_{CC1} + \hat{a}_{CC2} + \hat{a}_{CC3} + \hat{a}_{CC4}}$$

Note that the ECM:

$$\Delta LCP_t = \mu + \sum_{i=1}^{3} \phi_i \Delta LCP_{t-i} + \sum_{i=0}^{3} \gamma_i \Delta LRCa_{t-i} + \sum_{i=0}^{3} \eta_i \Delta LF_{t-i} + \alpha(LCP_{t-1} - \beta_1 LRCa_{t-1} - \beta_2 LF_{t-1}) + \varepsilon_t$$

Can also be written as:

$$\Delta LCP_t = \mu + \sum_{i=1}^{3} \phi_i \Delta LCP_{t-i} + \sum_{i=0}^{3} \gamma_i \Delta LRCa_{t-i} + \sum_{i=0}^{3} \eta_i \Delta LF_{t-i} + \alpha LCP_{t-1} + \psi_1 LRCa_{t-1} + \psi_2 LF_{t-1} + \varepsilon_t$$

where $\psi_1 = -\alpha\beta_1$ and $\psi_2 = -\alpha\beta_2$. How would you calculate the two long run coefficients now? (Nice exercise: How would you find the standard errors of $\hat{\beta}_1$ and $\hat{\beta}_2$?)

Calculate the long run multipliers for the conditional consumption model using the **KonsDataNor2.in7** and run the batch **Conditional consumption equation.fl**. Also, estimate the equilibrium correction model.

Remember the ARDL model in its general form:

$$y_t \quad = \quad \beta_0 + \phi_1 y_{t-1} + \beta_1 x_t + \beta_2 x_{t-1} + \varepsilon_t \qquad (18)$$

	Distributed lag (DL
$\phi_1 = 0$	$y_t = \beta_0 + \beta_1 x_t + \beta_2 x_{t-1} + \varepsilon_t.$
	Growth rate (GR)
$\phi_1 = 1$ $\beta_1 + \beta_2 = 0$	$\Delta y_t = \beta_0 + \beta_1 \Delta x_t + \varepsilon_t.$
	Random Walk (RW)
$\phi_1 = 1$ $\beta_1 = \beta_2 = 0$	$y_t = \beta_0 + y_{t-1} + \varepsilon_t.$
	Static model
$\phi_1 = 0$ $\beta_2 = 0$	$y_t = \beta_0 + \beta_1 x_t + \varepsilon_t.$
	ECM
$\lvert\phi_1\rvert < 1$	$\Delta y_t = \beta_0 + \beta_1 \Delta x_t + (\phi_1 - 1) y_{t-1}$ $+ (\beta_1 + \beta_2) x_{t-1} \varepsilon_t.$

And many more, including polynomial lag distributions and geometric lag distribution, see EB note DL which can be used to represent longer lags by a small number of parameters

- Note in particular that the ECM representation does not impose any restrictions on the ARDL model, except $|\phi_1| < 1$ (dynamic stability).
- The partial derivatives
$$\frac{\partial y_t}{\partial x_{t-j}}$$
also called *dynamic multipliers*, or l*ag-weights*, are easy to obtain in PcGive after estimation, as are the *long-run multipliers*, as we have seen
- As all parameters of an econometric model, also dynamic and long-run multipliers can become badly biased if a mis-specified model is estimated

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