Simultaneous equations bias in a macro model

Extra slides to Lecture 7

Ragnar Nymoen

University of Oslo

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Medium term macro model

- Medium term macro model of the Keynesian type.
  - C_t : private consumption in year t (in constant prices, eg.2010)
  - GDP_t, TAX_t and I_t are *gross domestic product*, *net taxes* and *investments and gov.exp*.
  - a - e are parameters of the macroeconomic model
  - ϵ_{Ct} and ϵ_{TAXt} are independent disturbances with *classical properties* conditional on I_t and C_{t-1}.

$$C_t = a + b(GDP_t - TAX_t) + cC_{t-1} + \epsilon_{Ct} \tag{1}$$

$$TAX_t = d + eGDP_t + \epsilon_{TAXt} \tag{2}$$

$$GDP_t = C_t + I_t \tag{3}$$

- C_t, GDP_t and TAX_t are endogenous, C_{t-1} is predetermined.
- Assume that I_t is strictly exogenous with $E(I_t) = \mu_I$ and $Var(I_t) = \sigma_I^2$. For simplicity, we will use

$$I_t = \mu_I + \epsilon_{It} \tag{4}$$

where is independent of ϵ_{Ct} and ϵ_{TAXt}, and has classical properties conditional on I_t and C_{t-1}.

Reduced form

- The reduced form equation for C_t and GDP_t from (1)-(3)

$$C_t = \frac{a+bd}{(1-b(1-e))} + \frac{b(1-e)}{(1-b(1-e))} I_t + \frac{c}{(1-b(1-e))} C_{t-1} + \frac{\epsilon_{Ct} - (be)\epsilon_{TAXt}}{(1-b(1-e))}$$

$$GDP_t = \frac{a+bd}{(1-b(1-e))} + \frac{(1-be)}{(1-b(1-e))} I_t + \frac{c}{(1-b(1-e))} C_{t-1} + \frac{\epsilon_{Ct} - (be)\epsilon_{TAXt}}{(1-b(1-e))}$$

- Both C_t and GDP_t depend on ϵ_{Ct} (and ϵ_{TAXt}).

Structural parameter

- Assume now that our parameter of interest is the estimation of the structural parameter b, the "marginal propensity" to consume b in the above model. For exposition, we can simplified and static structural model

$$C_t = a + b(GDP_t) + \epsilon_{Ct} \tag{5}$$

$$GDP_t = C_t + I_t \tag{6}$$

$$I_t = \mu_I + \epsilon_{It} \tag{7}$$

- Assumptions about structural disturbances:

$$E(\epsilon_{Ct} \mid I_t) = 0,\ Var(\epsilon_{Ct} \mid I_t) = \sigma_C^2 \tag{8}$$

$$E(\epsilon_{It}) = 0,\ Var(\epsilon_{Ct}) = \sigma_I^2 \tag{9}$$

$$Kov(\epsilon_{Ct}, \epsilon_{It}) = 0 \tag{10}$$

- In this model, the OLS estimator of b is inconsistent.

OLS estimator

We know that the OLS estimator is

$$\hat{b} = \frac{\sum_{t=1}^{T}(GDP_t - \overline{GDP})C_t}{\sum_{t=1}^{T}\left(GDP_t - \overline{GDP}\right)^2} = b + \frac{\sum_{t=1}^{T} GDP_t(\epsilon_{Ct} - \bar{\epsilon}_C)}{\sum_{t=1}^{T}\left(GDP_t - \overline{GDP}\right)^2}$$

To asses the probability limit of the bias term we need the reduced form:

$$GDP_t = \frac{a + \mu_I}{1-b} + \frac{\epsilon_{Ct}}{1-b} + \frac{\epsilon_{It}}{1-b}, \text{ for } 0 < b < 1$$

$$C_t = \frac{(a + b\mu_I)}{1-b} + \frac{\epsilon_{Ct}}{1-b} + \frac{b}{1-b}\epsilon_{It}$$

which together with the assumptions give the properties of the two random variables GDP_t and C_t

Simultaneous equations bias I

$$plim(\hat{b}-b) = plim\frac{\sum_{t=1}^{T} GDP_t(\epsilon_{Ct}-\bar{\epsilon}_C)}{\sum_{t=1}^{T}\left(GDP_t-\overline{GDP}\right)^2}$$

$$plim(\hat{b}-b) = plim\frac{\sum_{t=1}^{T}\left(\frac{a+\mu_I}{1-b}+\frac{\epsilon_{Ct}}{1-b}+\frac{\epsilon_{It}}{1-b}\right)(\epsilon_{Ct}-\bar{\epsilon}_C)}{\sum_{t=1}^{T}\left(\frac{\epsilon_{Ct}-\bar{\epsilon}_C}{1-b}+\frac{\epsilon_{It}-\bar{\epsilon}_I}{1-b}\right)^2}$$

From the assumptions of the model:

$$plim(\hat{b}-b) = \frac{\sigma_C^2}{\sigma_C^2+\sigma_I^2} > 0 \qquad (11)$$

Simultaneous equations bias II

- OLS is overestimating the structural parameter b in the model given by (5)-(10)
- Trygve Haavelmo: *The statistical implications of system of simultaneous equations*, Econometrica (1943)
- Inconsistency of OLS is true for *all* simultaneous equations models: Market supply and demand equations for example, as illustrated by Monte Carlo simulation in Computer Class 4.