E 4160
Lecture 11: Lucas critique and stationary non-causal VARs

Ragnar Nymoen

Department of Economics, University of Oslo

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References

- HN 20.
- Lecture Note 5 to this course
- (BN2011: § 5.12, § 8.5.3)

- So far, we have covered *stationary* and *causal* dynamic systems.
- Dynamic systems with one or more roots "larger than one" are stationary but *non-causal.*
- In economics they can be motivated by for example forward-looking models.
- Eventually, we are interested in solving, estimating and testing such models
- This lecture gives an introduction to this large research area.

Rational expectations (RE) model I

- In Lecture Note 5 (19 Sept 2014) we have the model

$$Y_t = \beta_1 E(X_{t+1} \mid \mathcal{I}_{t-1}) + \varepsilon_t \quad (1)$$

$$X_t = \lambda X_{t-1} + \epsilon_{xt}, \quad -1 < \lambda < 1 \quad (2)$$

$$\varepsilon_t \sim IID(0, \sigma^2) \quad (3)$$

$$\epsilon_{xt} \sim IID(0, \sigma_x^2) \quad (4)$$

$$Cov(\varepsilon_t, \epsilon_{xs}) = 0 \text{ for all } t \text{ and } s \quad (5)$$

The asymptotic bias of the OLS estimator of β_1, is

$$plim(\hat{\beta}_1) - \beta_1 = \beta_1(\lambda^4 - 1) < 0 \quad (6)$$

For the case where X_{t+1}replaces X_{t+1} in (1) it is:

$$plim(\hat{\beta}_1) - \beta_1 = \beta_1(\lambda^2 - 1) < 0$$

Rational expectations (RE) model II

- For both versions of this RE model, we therefore have that:
- The parameter of interest β_1 cannot be estimated consistently by only considering the conditional model for Y given X: *Lack of Weak Exogeneity*
- If the expectations parameter λ changes, there will also be a structural break in the conditional parameter, $plim(\hat{\beta}_1)$. *Lack of invariance*.
- The Lucas-critique:
  - Expectations typically change when there are changes in economic policies (in particular in policy rules).
  - Conditional econometric models cannot be used for analysis of the effect of policy changes, because the parameter of these models will not be invariant to the policy change.

Rational expectations (RE) model III

- The **relevance** of the Lucas-Critique (LC) is however **testable** by testing whether the parameters of the marginal model change when the policy changes, and whether the parameters of the conditional model change as a consequence.
- Testing the Lucas-critique is a special case of testing super-exogeneity.
- See §20.4 and § 20.5 in HN for examples of empirical tests of the Lucas-critique
- Those sub-chapters also offer an interesting (but of course contested) reconciliation of the two broad observations that expectations about the future is important for agents, yet the LC is not very important empirically.

Causal and non-causal processes I

- A time series (stochastic process) Y_t is *causal* if the solution can be expressed by $\varepsilon_t + \psi_1 \varepsilon_{t-1} + \psi_2 \varepsilon_{t-2} \dots$ with decaying weights, ψ_i: a *linear filtering*.
- This is assured if the associated characteristic equation of Y_t has *all* its roots inside the unit circle
- This definition generalizes, as we have seen, to the case of the n dimensional vector process $\boldsymbol{y}_t$
- Y_t is a *non-causal,* or *future dependent*, process if the stable solution is a well defined linear filtering of $\varepsilon_{t+1}, \varepsilon_{t+2}, \dots$.
- For a non-causal process, the associated characteristic equation has one or more roots that are larger than one "in absolute value".

Causal and non-causal processes II

- The simplest example of a covariance-stationary and non-causal model is

$$Y_t = \phi_1 Y_{t-1} + \varepsilon_t, \quad \phi_1 > 1, \tag{7}$$

where ε_t is white-noise.
(7) has one root which is larger than unity. The non-causal *solution* is:

$$Y_t = (\phi_1^{-1})^N y_{t+N} + \textstyle\sum_{i=1}^{N-1} (-\phi_1^{-1})^i \varepsilon_{t+i} \tag{8}$$

where Y_{t+N} is a terminal condition.

Causal and non-causal processes III

- The solution is stable since, if we look at the homogenous part,

$$Y_t^h \underset{N \longrightarrow \infty}{\longrightarrow} 0 \text{ if } \phi_1 > 1$$

  as we have assumed.
- Y_t is also stationary since it can be express as a well defined linear filter of stationary variables (namely ε_{t+i}):

$$Y_t = \sum_{i=1}^{\infty} (-\phi_1^{-1})^i \varepsilon_{t+i} \tag{9}$$

- The solution (9),for a single time-series is not so practical.
- However in linear multivariate RE models, where some, but not all, characteristic roots are larger than unity, the solution (if it exists) takes the forms of a casual and stationary VAR!
- This important result means that we can test forward-looking economic theories within the framework of the VAR.

VAR for consumption and income I

- Define c_t as the log of private consumption and y_t as the log of disposable income
- Assume the following dynamic model for Δc_t, Δy_t and a third stationary variable x_t

$$\Delta c_t = \kappa - \alpha_c x_{t-1} + e_{c,t}, \qquad 0 \leq \alpha_c < 1, \tag{10}$$
$$\Delta y_t = \varphi + \alpha_y x_{t-1} + e_{y,t}, \qquad 0 \leq \alpha_y < 1, \tag{11}$$
$$x_t = \Delta c_t - \Delta y_t + x_{t-1} \tag{12}$$

- x_t is approximately the minus of the savings rate:

$$-x_t = -c_t + y_t \approx \text{savings rate}$$

VAR for consumption and income II

- It is often relevant to have the equilibrium of x_t as a parameter. Define
$$\mu_x = \mathsf{E}(x_t)$$
and decompose κ and φ as:
$$\kappa = \eta_c + \alpha_c \mu_x$$
$$\varphi = \eta_y - \alpha_y \mu_x$$
Thus we can rewrite this system into
$$\Delta c_t = \eta_c - \alpha_c [x_{t-1} - \mu_x] + e_{c,t}, \tag{13}$$
$$\Delta y_t = \eta_y + \alpha_y [x_{t-1} - \mu_x] + e_{y,t}, \tag{14}$$
$$x_t = \Delta c_t - \Delta y_t + x_{t-1} \tag{15}$$

VAR for consumption and income III

where the two intercepts must have the interpretation:
$\eta_c = \mathsf{E}(\Delta c_t)$ and $\eta_y = \mathsf{E}(\Delta y_t)$.

RE Permanent income hypothesis (RE-PIH) I

- A well known implication of the PIH is that the savings rate ($-x_t$) is a function of expected future income.
- The logic is very similar to the market fundamentals solution for P_t above.
- Intuitively, x_t is Granger causing income growth because the savings rate is a carrier or households expectations about future income growth—
  - the *saving for a rainy day feature of* the PIH model, see Campbell (1987).

RE Permanent income hypothesis (RE-PIH) II

(13)-(14) can be written in model form:

$$\Delta c_t = \eta_c + e_{c,t}, \tag{16}$$
$$\Delta y_t = \eta_y + \gamma_y + \pi_y \Delta c_t + \alpha_y [x_{t-1} - \mu_x] + \varepsilon_{y,t}, \tag{17}$$

where (16) is the marginal model for consumption and (17) is a conditional model for real income

If we assume normal distrbution

$$\begin{gathered} \pi_y = \rho_{cy} \frac{\sigma_y}{\sigma_c}, \\ \gamma_y = -\eta_y \pi_y, \\ \varepsilon_{y,t} = e_{y,t} - \pi_y e_{c,t}. \end{gathered} \tag{18}$$

Since $\alpha_c = 0$, cointegration implies that $0 < \alpha_y < 1$, with *rainy-day* as the economic interpretation.

Consumption function (CF) I

$$\Delta c_t = \eta_c + \gamma_c + \pi_c \Delta y_t - (\alpha_c + \pi_c \alpha_y)[x_{t-1} - \mu_x] + \varepsilon_{ct} \tag{19}$$

$$\Delta y_t = \eta_y + \alpha_y [x_{t-1} - \mu_x] + e_{y,t} \tag{20}$$

(19) is the consumption function,(20) is the marginal income equation.

$$\begin{aligned} \alpha_c &= (1 - \phi_{cc}) \\ \pi_c &= \rho_{c,y} \frac{\sigma_c}{\sigma_y}, \\ \gamma_c &= -\eta_y \pi_c, \\ \varepsilon_{c,t} &= e_{c,t} - \pi_c e_{y,t}. \end{aligned} \tag{21}$$

- Underlying the consumption function approach is $0 < \alpha_c < 1$.

Consumption function (CF) II

- Note that the hypothesis H_0: $\alpha_c = 0$ must be tested separately, since finding $[x_{t-1} - \mu_x]$ significant in (19) could be due to $0 < \alpha_y$.
- For the coefficient α_y there are two possibilities.
- $0 < \alpha_y < 1$ is consistent with hours worked etc. being "demand determined" and with y_t adjusting to past disequilibria. In econometric terms there is mutual (Granger) causation between income and consumption.
- The second possibility is that $\alpha_y = 0$, reflecting that income is "supply-side" determined

Consumption function (CF) III

- Of course $\alpha_y = 0$ gives the clearest contrast to the PIH: (19)-(20) simplify to

$$\Delta c_t = \eta_c + \gamma_c + \pi_c \Delta y_t - \alpha_c [x_{t-1} - \mu_x] + \varepsilon_{c,t} \tag{22}$$

$$\Delta y_t = \eta_y + e_{y,t}, \tag{23}$$

  with $\eta_y = \varphi$, since there is no equilibrium correction in income.

- The analysis of the different implications of CF and PIH carry over to the realistic case of
  - VAR(p) and
  - deterministic terms
  - Unit-root non-stationarity

Analysis of Norwegian consumption-income data I

- Quarterly data 1968(2)-1984(4)
  Refer to this as the *Before break* sample.
- Full results in Eitrheim et al (2002)

Analysis of Norwegian consumption-income data II

Table: Diagnostics for the I(0) VAR

The sample is 1968(2) to 1984(4), 67 observations.		
$\hat{\sigma}_{\Delta c}$	$=$	1.63%
$\hat{\sigma}_{\Delta y}$	$=$	1.60%
$AR\ 1-5\ F(20,82)$	$=$	$0.8921[0.5973]$
$Normality\ \chi^2(4)$	$=$	$8.0609[0.0894]$
$Heteroscedasticity\ F(75,72)$	$=$	$0.5222[0.9971]$

Analysis of Norwegian consumption-income data III

Table: FIML consumption function estimates. 1968(2)-84(2)—Before Break

The consumption function

$$\widehat{\Delta c}_t = -\underset{(0.070)}{0.302}\,\Delta c_{t-1} + \underset{(0.073)}{0.227}\,\Delta c_{t-4} + \underset{(0.181)}{0.471}\,\Delta y_t - \underset{(0.048)}{0.128}\,(c-y)_{t-1}$$

$$\hat{\sigma} = 1.53\%$$

The income equation

$$\widehat{\Delta y}_t = \underset{(0.002)}{0.009} - \underset{(0.109)}{0.477}\,\Delta y_{t-1} + \underset{(0.109)}{0.311}\,\Delta y_{t-4}$$

$$\hat{\sigma} = 1.60\%$$

Analysis of Norwegian consumption-income data IV

Table: *Before break* FIML estimates for consumption function

Diagnostics		
Overidentification $\chi^2(14)$	$=$	$15.7597[0.3283]$
AR $1-5$ $F(20,96)$	$=$	$0.7830[0.7274]$
Normality $\chi^2(4)$	$=$	$5.057[0.2815]$
Heteroscedasticity $F(75,93)$	$=$	$0.6530[0.9717]$
FIML estimation. The sample is 1968(2) to 1984(4), 67 observations.		

Analysis of Norwegian consumption-income data V

Table: *Before break* FIML estimates for PIH .

The Euler equation

$$\widehat{\Delta c}_t = \underset{(0.003)}{0.0059} + \underset{(0.089)}{0.236}\, \Delta c_{t-4}$$

$$\hat{\sigma} = 2.10\%$$

The savings equation

$$\widehat{\Delta y}_t = \underset{(0.003)}{0.0144} - \underset{(0.098)}{0.308}\, \Delta y_{t-1} + \underset{(0.01)}{0.231}\, \Delta y_{t-4}$$

$$+ \underset{(0.068)}{0.158}\, (c-y)_{t-1} + \underset{(0.013)}{0.041}\, VAT$$

$$\hat{\sigma} = 1.62\%$$

The sample is 1968(1) to 1984(4), 68 observations.

Analysis of Norwegian consumption-income data VI

Table: *Before break* FIML estimates for PIH

Diagnostics		
Overidentification $\chi^2(15)$	=	45.3776[0.0001]
AR $1-5$ $F(20, 98)$	=	1.5548[0.0804]
Normality $\chi^2(4)$	=	7.4287[0.1149]
Heteroscedasticity $F(75, 96)$	=	1.0269[0.4481]
The sample is 1968(1) to 1984(4), 68 observations.		

- After 1984(4), the conditional+marginal model started to forecast badly and the parameters of the consumption function changed.
- Since the *rainy-day* hypothesis has already rejected on the Before-break sample, the explanation was not likely to be that the LC applied.

Analysis of Norwegian consumption-income data VII

- Found instead that the break-down of the consumption was caused by the financial deregulation, and the effect of housing prices on wealth and household financial balances.
- Since then, Norwegian macro models have taken account of the relationship between housing prices and consumption and saving.

The New Keynesian Phillips curve I

In the slides to Lecture 8, the NPC was introduced:

- $$\pi_t = \underset{\geq 0}{a^f} E_t[\pi_{t+1}] + \underset{\geq 0}{a^b} \pi_{t-1} + \underset{>0}{b} s_t + \varepsilon_{\pi t} \tag{24}$$
- π_t denotes the rate of inflation
- In this model s_t denotes the logarithm of the wage-share which is the most used operational definition of real marginal costs. $\varepsilon_{\pi t}$ is a white noise disturbance.
- In many applications, notably Gali and Gertler (1999), the disturbance term is omitted, which is often referred to as the NPC holding in "exact form"

The New Keynesian Phillips curve II

- s_t is referred to as a forcing variable, in the same way as D_t in the stock price model. We assume the following model for s_t

$$s_t = c_{s1} s_{t-1} + \cdots + c_{sk} s_{t-k} + \varepsilon_{s,t} \ . \tag{25}$$

- It can be show that $k \geq 2$ is necessary for identification of the parameters in the NPC.

NPC solution I

- Following Bårdsen et al (2005), Ch 7, we first find a partial solution for π_t as

$$\pi_t = r_1 \pi_{t-1} + \frac{b}{a^f r_2} \sum_{i=o}^{\infty} (\frac{1}{r_2})^i E_t s_{t+i} + \frac{1}{a^f r_2} \varepsilon_{\pi,t} \qquad (26)$$

where $r_{1,2}$ are the two roots of

$$r^2 - (1/a^f)r + (a^b/a^f) = 0$$

A stable solution of the pure NPC, with $a^b = 0$, requires $a^f > 1$.

NPC solution II

With $a^f > 0$, $a^b > 0$ and $a^f + a^b \leq 1$ it is implied that r_1 and r_2 are real and positive.
It is usual to define r_1 as

$$r_1 = \frac{1 - \sqrt{1 - 4a^f a^b}}{2a^f} \tag{27}$$

which is $0 \leq r_1 < 1$ under the assumption of $a^f + a^b < 1$.
Under these assumption the solution is (cf Nymoen, Swensen and Tveter (2012)):

$$\pi_t = r_1 \pi_{t-1} + \frac{b}{a^f r_2} K_{s1} s_t + \frac{b}{a^f r_2} K_{s2} s_{t-1} + \frac{1}{a^f r_2} \varepsilon_{\pi,t}, \tag{28}$$

written here for the case of $k = 2$ in the forcing process (25).

NPC solution III

- K_{s1} and K_{s2} are constants that are defined when

$$\left|\frac{r_{si}}{r_2}\right| < 1 \text{ for } i = 1, 2 \tag{29}$$

  where $r_{s1}$and r_{s2} are the roots of the characteristic equation associated with the forcing variable.
- (28) can be solved from known initial conditions, "as if" the model was causal.

Testing the NPC

Different "types" of tests:

- Robustness and strength of instruments (as we did in seminar)
- Testing the implications of $E_t[\pi_{t+1}]$ on the VAR, for example Boug et al (2010)
- Testing the encompassing implications of the NPC against existing models of the wage-price spiral—encompassing approach
- Testing invariance of NPC with respect to regime shifts.

Encompassing type test I

Start from:

$$s_t = ulc_t - q_t, \tag{30}$$

where *ulc* denotes unit labour costs (in logs) and q is the log of the price level on domestic goods and services. Let $(1-\gamma)$ denote a constant import share, and *pi* the import price index.
The aggregate price level is defined as

$$p_t = \gamma q_t + (1-\gamma)\, pi_t \tag{31}$$

Encompassing type test II

Using (30) and (33), we can re-write the NPC (24), after dropping the disturbance for simplicity:

$$\pi_t = \frac{a^f}{\left(1+\frac{b}{\gamma}\right)} E_t[\pi_{t+1}] + \frac{a^b}{\left(1+\frac{b}{\gamma}\right)} \pi_{t-1}$$
$$- \frac{b}{(\gamma+b)} \left[p_{t-1} - \gamma ulc_{t-1} - (1-\gamma)\, pi_{t-1}\right]$$
$$+ \frac{\gamma b}{(\gamma+b)} \Delta ulc_t + \frac{b\,(1-\gamma)}{(\gamma+b)} \Delta pi_t$$

Encompassing type test III

or

$$
\begin{aligned}
\pi_t = {} & \alpha^f \Delta E_t[\pi_{t+1}] + \alpha^b \pi_{t-1} \\
& + \beta(ulc_{t-1} - p_{t-1}) - \beta\,(1-\gamma)\,(ulc_{t-1} - pi_{t-1}) \\
& + \beta\;\gamma\;\Delta ulc_t + \beta\,(1-\gamma)\,\Delta pi_t
\end{aligned} \tag{32}
$$

with α^f, α^b, β and ψ as new coefficients.

- This shows hat the NPC has an interpretation as an EqCM for the price level.
- An alternative model for price and wage formation is the *imperfect competition model*, ICM.

Encompassing type test IV

- The ICM the price equation with a lead-term added is

$$\begin{aligned}\pi_t = {} & \alpha^f \Delta E_t[\pi_{t+1}] + \alpha^b \Delta\pi_{t-1} \\ & + \beta_1(ulc_{t-1} - p_{t-1}) + \beta_2(ulc_{t-1} - pi_{t-1}) \\ & + \beta_3 \Delta ulc_t + \beta_4 \Delta pi_t.\end{aligned} \tag{33}$$

- The NPC implies restrictions on (33)
  - H_0^a: $\beta_3 = \beta_1 + \beta_2$ and
  - H_0^b: $\beta_4 = -\beta_2$.
- For the ICM, the only requirement is $\beta_1 > 0$ and $\beta_1 > -\beta_2$.
- Non rejection of H_0^a and/or H_0^b would mean that the NPC encompasses the ICM :

$$\text{NPC } \mathcal{E} \text{ ICM} \quad \text{if} \quad \beta_3 = \beta_1 + \beta_2 \text{ and } \beta_4 = -\beta_2$$

Encompassing type test V

- Conversely, if $\beta_3 = \beta_1 + \beta_2$ and/or $\beta_4 = -\beta_2$ is rejected, the ICM implies that the NPC has omitted variables bias.
  - In particular $ulc_{t-1} - p_{t-1}$ and $ulc_{t-1} - pi_{t-1}$ are predictors of π_{t+1} so omission (misrepresentation) of theses variables in the NPC will affect the *IV / GMM* estimate of the parameter a^f in the NPC.
- This test is formulated and applied to panel data in Bjørnstad and Nymoen (2008)

Testing invariance of NPC I

- If the data generating process is characterized by intermittent structural break, the significance of the forward term in the NPC may be overestimated.
- Structural breaks represented by impulse indicators
- The impulse indicators are selected in the implied reduced form forecasting equation
- Then tested for significance in the NPC.
  - Under the null of correct specification, few such impulse indicators will be selected,
  - and those that are should not be significant when added to NPC;
  - moreover, parameter estimates should not alter much.

Testing invariance of NPC II

- Under the alternative that there are unmodelled outliers or breaks, there will be significant impulse indicators in the 'forecasting' equation, and these will remain significant when added to the NPC.
- Castle et al (2014)

References (ie. additional reading) I

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