Supplement to Lecture 3: Confidence interval for the "natural rate of unemployment". NLS and delta-Method

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PCM natural rate of unemployment I

- The **natural rate of unemployment** is a central concept in macro economics
- In order to estimate the natural rate, we must specify a model where it is a parameter (explicitly or implicitly)
- There are many such models, but we consider a **linear Phillips curve model** (PCM)

$$\pi_t = \beta_1 + \beta_2 U_t + \varepsilon_t \tag{1}$$

where π_t is Norwegian inflation in year t and U_t is the unemployment percentage U_t.

PCM natural rate of unemployment II

- The natural rate can defined in such a way that it becomes a parameter in (1). Re-write the PCM as

$$\begin{aligned}\pi_t &= \beta_2(U_t - \frac{-\beta_1}{\beta_2}) + \varepsilon_t \\ &= \beta_2(U_t - U^{nat}) + \varepsilon_t \qquad (2)\end{aligned}$$

  and define

$$U^{nat} := \frac{-\beta_1}{\beta_2}$$

  as the natural rate of unemployment.

- U^{nat} is at parameter in both (1) and (2), though implicit in (1).

PCM natural rate of unemployment III

- (2) is however NOT linear in parameters. To estimate U^{nat} from (2) requires Non-linear Least Squares, (NLS).
- However, with the use of the **delta method** we can make inference about U^{nat} by estimating the linear-in parameter model (1)

With annual data from 1981 to 2010 **(**$T = 30$) **we estimate:**

$$\hat{\pi}_t = \underset{(1.551)}{8.37527} - \underset{(0.4573)}{1.36632} U_t$$

Nat-rate ($\hat{U}^{nat}$)	$\frac{(8.37527)}{1.36632} = 6.1298$
IT-rate ($\hat{U}^{it}$)	$\frac{(8.37527 - 2.5)}{1.36632} = 4.3001$

Note:

- U^{it} is the" inflation target rate of unemployment": the "natural rate for $\pi_t = 2.5$, instead of 0

Use the **delta-method formula** :

$$var(\hat{U}^{nat}) = var(\frac{-\hat{\beta}_1}{\hat{\beta}_2}) \approx \left(\frac{1}{\hat{\beta}_2}\right)^2 \left[var(-\hat{\beta}_1) + \left(\hat{U}^{nat}\right)^2 var(\hat{\beta}_2) - 2\left(\hat{U}^{nat}\right) cov(-\hat{\beta}_1, \hat{\beta}_2)\right]$$

From the estimation: $cov(\hat{\beta}_1, \hat{\beta}_2) = -0.66876$, $Var(\hat{\beta}_1) = 2.4043$, $Var(\hat{\beta}_2) = 0.20915$

$$\begin{aligned} var(\hat{U}^{nat}) &= \\ var(\frac{-\hat{\beta}_0}{\hat{\beta}_1}) &\approx \left(\frac{1}{-1.36632}\right)^2 \cdot \left[2.4043 + (6.1298)^2 \cdot 0.20915 - 2 \cdot (5.73) \cdot 0.66876\right] \\ &= 0.535\,67 * 2.599 = 1.392\,2 \end{aligned}$$

Approximate 95 % confidence interval for U^{nat} is therefore

$$6.\,1298 \pm 2 \cdot \sqrt{1.392\,2} = 5.73 - 2 \cdot 1.\,1799$$

or

$$[3.372\% \; ; \; 8.\,089\,8\%]$$

Memo: Direct estimation using the Non Linear Least Squares (NLS) gives:
$var(\hat{U}^{nat}) = 1.052^2 = 1.\,1067$