ECON 4160: Seminars autumn semester 2015—SECOND SEMINAR

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Exercise set to seminar 2 (14 & 16 Sep)

Question A

Consider the bivariate normal model where the two random variables are denoted X and Y. Let the marginal expectations of X and Y be $E(X) = \mu_X$ and $E(Y) = \mu_Y$. The variances are σ_X^2 , σ_Y^2, and σ_{XY} is the covariance. The correlation coefficient is $\rho_{XY} = \frac{\sigma_{XY}}{\sigma_Y \sigma_X}$.

Using the notation $z_Y = (y - \mu_Y)/\sigma_Y$ and $z_X = (x - \mu_X)/\sigma_X$, the bivariate normal probability density function (pdf) can be written as:

$$f_{XY}(y,x) = \frac{1}{\sigma_Y \sigma_X 2\pi \sqrt{(1-\rho_{XY}^2)}} \times \exp\left[-\frac{1}{2}\frac{(z_Y^2 - 2\rho_{XY} z_Y z_X + z_X^2)}{(1-\rho_{XY}^2)}\right] \quad (1)$$

1. From the joint density $f_{XY}(y,x)$, the marginal pdf $f_X(x)$ can be shown to be (non-trivial in the case of $\rho_{XY} \neq 0$).

$$f_X(x) = \frac{1}{\sigma_X \sqrt{2\pi}} \exp\left[-\frac{1}{2}\left(\frac{x-\mu_X}{\sigma_X}\right)^2\right] \quad (2)$$

Take (1) and (2) as given and show that the conditional pdf for Y is the normal pdf:

$$f_{Y|X}(y \mid x) = \frac{1}{\sqrt{2\pi\sigma_{Y|X}^2}} \times \exp\left\{-\frac{1}{2}\frac{\left[y - \mu_{Y|X}\right]^2}{\sigma_{Y|X}^2}\right\} \quad (3)$$

with parameters

$$\mu_{Y|X} = E(Y \mid x) = \beta_0 + \beta_1 x \quad (4)$$

$$\beta_0 = \mu_Y - \frac{\sigma_{YX}}{\sigma_X^2}\mu_X \quad (5)$$

$$\beta_1 = \frac{\sigma_{YX}}{\sigma_X^2} \quad (6)$$

$$\sigma_{Y|X}^2 = Var(Y \mid x) = \sigma_Y^2(1 - \frac{\sigma_{YX}^2}{\sigma_Y^2 \sigma_X^2}) \quad (7)$$

2. Assume that we have data that can be modelled as n independent repetitions of a bivariate vector:

$$\mathbf{y}_i = \begin{pmatrix} Y_i \\ X_i \end{pmatrix} \overset{D}{=} N(\mu, \mathbf{\Sigma})$$

i.e., $\mathbf{y}_1$, $\mathbf{y}_2$, ...,$\mathbf{y}_n$ are mutually independent. The elements in the parameter vector μ and matrix $\mathbf{\Sigma}$ are:

$$\mu = \begin{pmatrix} \mu_Y \\ \mu_X \end{pmatrix}, \qquad \mathbf{\Sigma} = \begin{pmatrix} \sigma_Y^2 & \sigma_{XY} \\ \sigma_{XY} & \sigma_X^2 \end{pmatrix}$$

(a) Formulate the "equation form" of the conditional model of Y given X, i.e., the regression equation model and the equation that represent the marginal model for X.

(b) Argue that the independence assumptions above translate into independence of the equation disturbances of the model you formulate in (a).

(Note: H&N Ch 10 is a concise reference to this exercise. In Bårdsen and Nymoen (2011), see Kap 4.5.4 and 5.7)

Question B

Download the *SimdataLECT3.fl* posted on the web page (under **Data set and code**: *Batch file for AR(2) example in Lecture 3 slide set*).

1. Read the code down to the *break;* command and note that there are three variables that are created here: *eps1*, *YM1* and *YM1H*. *eps1* is the white-noise input series in the AR(2) model. It is generated as $N(0, 1)$. Line 18 and 19 give the solutions of the AR(2) with autoregressive coefficients 1.6 and −0.9. Line 18 is for the full solution, and line 19 is for the homogenous solutions. The two solutions are dubbed *YM1* (full) and *YM1H* (homogenous). (The initial condition is 4 for both.)

   (a) Run SimdataLECT3.fl and plot the three variables *eps1*, *YM1* and *YM1H* in a single figure. Use the full sample (default). Explain briefly the differences and similarities between the three graphs.

   (b) Change the autoregressive coefficient to 0.6 (first lag) and 0.20 (second lag). Run the modified file and make a plot with the three series. Compare with the figure from (a) and explain the differences.

   (c) Change the autoregressive coefficients back to 1.6 and −0.9. Multiply *rann()* by 0.5. What is the effect of this? Run the modified file, make the graphs and compare to the graph from (a). What has happened?

   (d) Keep *rann()*0.5*, but change the file in such a way that the expectation *YM1* is 1.0 and not zero. Run the file and graph *YM1H* as a check that you got it right.

2. Assume that the we know that *YM1* is generated by an AR(2) process, but that we need to estimate the parameters. Use the data set from (1d) and use PcGive (*Models for time series data—Single Equation Dynamic Modeling*) to obtain the OLS estimates of the two parameters. Use a sample that begins in 1962 and ends in 2012. Comment on the results.

3. Use the Menu *Test-Dynamic Analysis* and check the box *Roots of the lag polynomial*. What do you find? Are the estimated roots logically consistent with the assumed stationarity of AR(2) in this case? Explain briefly.

4. What is the (approximate) ML estimate of the expectation of *YM1*? Give also an approximate 95 % interval for this parameter.

5. Assume that we do not know the order of the AR process. How could you proceed to try to identify empirically the correct lag-order? Apply your approach in PcGive, and report the result.

6. Since there are no observable explanatory variables in this models, we do not get any lag-weights (aka dynamic multipliers). To obtain them, go to *Models for time series data—Multiple Equation Dynamic Modeling*. Formulate the AR(2) model with Constant included. Use *Unrestricted estimation* in the next menu (*Choose a model type*). Observe that the OLS estimates are exactly the same as you obtained in the *Single Equation Dynamic Modeling*. Since we have now estimated a (single equation) system, a VAR(2) with only one row, we can get hold of the impulse response functions! After estimation, go to *Test-Dynamic Simulation and Impulse Responses,* choose *Impulse responses* and check *Impulse responses* and *Cumulated Impulse Responses. (*Note the similarity between the graph for the impulse responses (the responses to a change in the disturbance) and the graph of the homogenous solution that you found in (1d) above!