ECON 4160: Seminars autumn semester 2015—SIXTH SEMINAR

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Exam 2013

Question A (1/3)

1. Consider the regression model

   (1) $$Y_t = \phi_0 + \beta X_t + \varepsilon_t$$

   where Y_t and X_t are random variables, and where ε_t has the classical properties of a regression model disturbance. Explain briefly, what is meant by the following two exogeneity concepts:

   (a) Weak exogeneity of X_t with respect to the parameters of interest, ϕ_0 and β.

   (b) Super exogeneity of X_t.

2. Assume that the regression model is not (1), but

   (2) $$Y_t = \phi_0 + \phi_1 Y_{t-1} + \beta_1 X_t + \varepsilon_t,$$

   and that the parameters of interest are the characteristic root(s) that determine whether Y_t is a weakly stationary (covariance stationary) time series variable or not. Is, in general, X_t weakly exogenous in this case? Motivate your answer.

3. Consider an econometric equation for Y_t $\{Y_t; t = 1, 2, \ldots, T\}$ with k explanatory variables. The equation can be written in matrix notation as

$$\mathbf{y} = \mathbf{X}\boldsymbol{\beta} + \boldsymbol{\varepsilon} \tag{3}$$

The vector $\mathbf{y}$ is $T \times 1$ and the matrix $\mathbf{X}$ is $T \times k$. $\boldsymbol{\varepsilon}$ is the $T \times 1$ disturbance vector. $\boldsymbol{\beta}$ is the $k \times 1$ coefficient vector.

In each of the three cases below, give, without proofs, the expression for the consistent and asymptotically efficient *method of moments* estimator of $\boldsymbol{\beta}$. Assume $E(\boldsymbol{\varepsilon}) = \mathbf{0}$ in all cases:

(a) $E(\boldsymbol{\varepsilon}\boldsymbol{\varepsilon}') = \sigma^2\mathbf{I}$, where $\mathbf{I}$ is the identity matrix of dimension $T \times T$, $\sigma^2 > 0$, and $\text{plim}(\frac{1}{T}\mathbf{X}'\boldsymbol{\varepsilon}) = \mathbf{0}$.

(b) $E(\boldsymbol{\varepsilon}\boldsymbol{\varepsilon}') = \sigma^2\mathbf{I}$, and $\text{plim}(\frac{1}{T}\mathbf{X}'\boldsymbol{\varepsilon}) \neq \mathbf{0}$, $\text{plim}(\frac{1}{T}\mathbf{W}'\boldsymbol{\varepsilon}) = \mathbf{0}$, $\text{plim}(\frac{1}{T}\mathbf{W}'\mathbf{X}) = \boldsymbol{\Sigma}_{WX}$ (invertible) and $\text{plim}(\frac{1}{T}\mathbf{W}'\mathbf{W}) = \boldsymbol{\Sigma}_{WW}$ (positive definite) where the matrix $\mathbf{W}$ is $T \times k$.

(c) As in (a), but $E(\boldsymbol{\varepsilon}\boldsymbol{\varepsilon}') = \sigma^2\mathbf{I}$ is replaced by $E(\boldsymbol{\varepsilon}\boldsymbol{\varepsilon}') = \sigma^2\boldsymbol{\Omega}$, where $\boldsymbol{\Omega}$ is a symmetric and positive definite $T \times T$ matrix.

Question B (2/3)

In this exercise, we shall first consider the following open VAR, often called a VAR-X, with two exogenous variables, Z_1 and Z_2:

$$\begin{aligned}\begin{pmatrix} Y_t \\ X_t \end{pmatrix} &= \begin{pmatrix} a_{10} \\ a_{20} \end{pmatrix} + \begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{pmatrix}\begin{pmatrix} Y_{t-1} \\ X_{t-1} \end{pmatrix} \\ &+ \begin{pmatrix} \gamma_{11} & \gamma_{12} \\ \gamma_{21} & \gamma_{22} \end{pmatrix}\begin{pmatrix} Z_{1,t} \\ Z_{2,t} \end{pmatrix} + \begin{pmatrix} \varepsilon_{1,t} \\ \varepsilon_{2,t} \end{pmatrix}\end{aligned} \tag{4}$$

$\boldsymbol{\varepsilon}_t = (\varepsilon_{1t}, \varepsilon_{2t})'$ is bivariate normal with $E(\boldsymbol{\varepsilon}_t) = \mathbf{0}$ and $Var(\boldsymbol{\varepsilon}_t) = \boldsymbol{\Omega}$.

1. It can be shown that (you are not supposed to show this) if we multiply on both sides of the equality sign by the matrix

$$\boldsymbol{B} = \begin{pmatrix} 1 & -b_{12} \\ -b_{21} & 1 \end{pmatrix}$$

(4) can be written as a simultaneous equation system in the two variables Y_t and X_t:

$$Y_t - b_{12}X_t = \pi_{10} + \pi_{11}Y_{t-1} + \pi_{12}X_{t-1} + \eta_{11}Z_{1,t} + \eta_{12}Z_{2,t} + \epsilon_{1t} \tag{5}$$

$$-b_{21}Y_t + X_t = \pi_{20} + \pi_{21}Y_{t-1} + \pi_{22}X_{t-1} + \eta_{21}Z_{1,t} + \eta_{22}Z_{2,t} + \epsilon_{2t} \tag{6}$$

where all the parameters of the SEM in (5) and (6) are functions of the parameters of the reduced form VAR system in (4). Discuss the identification of the two equations in the following two cases:

(a) $\eta_{11} = 0$ and $\eta_{22} = 0$

(b) $\pi_{12} = 0$, $\pi_{21} = 0$ and $\eta_{22} = 0$

2. Which estimation method would you use to estimate the parameters of (5) and(6) in the cases where you have concluded that one or both of the equations are identified? Does it matter for your choice of estimation method whether the two structural disturbances ϵ_{1t} and ϵ_{2t} are contemporaneously uncorrelated or not?

3. For the remaining questions, we consider the case of a closed VAR (corresponding to setting $\gamma_{11} = \gamma_{12} = \gamma_{21} = \gamma_{22} = 0$ in (4)). At the end of the question set, you find OLS estimation results for such a VAR.

   Use the relevant results to test, first, the null hypothesis that Y is not Granger-causing X, and, second, that X is not Granger-causing Y.

4. Based on your answer in 3., explain why X_t can be regarded as strongly exogenous in the ARDL model:

$$Y_t = \beta_0 + \rho Y_{t-1} + \beta_1 X_t + \beta_2 X_{t-1} + \varepsilon_t \tag{7}$$

5. Under *Estimation results for question B* you also find results for the estimation of the ECM version of (7), together with a marginal model for ΔX_t.

   (a) The endogenous variables are different in the two models. The numbers of parameters are also different. Nevertheless, the "log-likelihood" is the same in the VAR as in the second model. Why is that?

   (b) The VAR residuals are correlated contemporaneously, as the results show. Will the disturbances of the ECM equation and the marginal model for ΔX_t, in the second model, also be contemporaneously correlated?

6. Assume that we get to know that Y_t and X_t are $I(1)$ variables. Under this assumption, and using the estimation results at the end of the question set, show that the null-hypothesis of no long-run relationship between Y_t and X_t can be rejected.

   For reference: the 5 % critical value of the relevant test statistic is -3.21, and the 1 % critical value is -3.79.

7. What is the estimated long-run effect on Y_t of a change in X_t? What additional results would you need to be able to construct a confidence interval for that parameter, and how would you proceed?

Estimation results for question B.

Results for question B3 and B4.

Estimation method is OLS

```
        The estimation sample is: 3 - 201

URF equation for: Y
                 Coefficient  Std.Error  t-value  t-prob
Y_1                 0.507680    0.07350     6.91  0.0000
X_1                 0.394077    0.05876     6.71  0.0000
Constant       U    0.962592     0.1687     5.71  0.0000

sigma = 0.994659   RSS = 193.9118976

URF equation for: X
                 Coefficient  Std.Error  t-value  t-prob
Y_1                0.0126066    0.07717    0.163  0.8704
X_1                 0.990516    0.06169     16.1  0.0000
Constant       U     1.03115     0.1771     5.82  0.0000

sigma = 1.04433   RSS = 213.7638469

log-likelihood    -526.222426  -T/2log|Omega|     38.5151101
|Omega|           0.679032334  log|Y'Y/T|         8.24714056
no. of observations       199  no. of parameters           6

correlation of URF residuals (standard deviations on diagonal)
                 Y            X
Y          0.99466      0.59269
X          0.59269       1.0443
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Memo: Residual diagnostics (have been omitted to save space) do not give any indication of misspecification of the two equation's disturbances. They can be assumed to be white-noise.

Results for question B5-B7.

Estimation method is OLS

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        The estimation sample is: 3 - 201

Equation for: DY
                 Coefficient  Std.Error  t-value  t-prob
Y_1                -0.499436    0.05935    -8.41  0.0000
DX                  0.564496    0.05493     10.3  0.0000
X_1                 0.399431    0.04745     8.42  0.0000
Constant       U    0.380511     0.1475     2.58  0.0106

sigma = 0.80113

Equation for: DX
                 Coefficient  Std.Error  t-value  t-prob
Y_1                0.0126066    0.07717    0.163  0.8704
X_1              -0.00948438    0.06169   -0.154  0.8780
Constant       U     1.03115     0.1771     5.82  0.0000

sigma = 1.04433

log-likelihood    -526.222426  -T/2log|Omega|     38.5151101
no. of observations       199  no. of parameters           7
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