4.

The model

$$Y_t = \beta_0 + \rho Y_{t-1} + \beta_1 X_t + \beta_2 X_{t-1} + \varepsilon_t$$

is a conditional regression model derived from the bivariate VAR(1) that we have estimation results for here. Since Granger causation is found to be one-way from X to Y in Q3, X is strongly-exogenous in the ARDL, and ΔX_t will also be in the ECM version of this equation.

5.

a) Based on the VAR we can formulate the condition-marginal equation system:

$$\begin{aligned} Y_t &= \beta_0 + \rho Y_{t-1} + \beta_1 X_t + \beta_2 X_{t-1} + \varepsilon_t \\ X_t &= a_{20} + a_{21} Y_{t-1} + a_{22} X_{t-1} + \varepsilon_{2,t} \end{aligned}$$

which is a re-parameterization of the VAR. For example $Cov(\varepsilon_t, \varepsilon_{2,t}) = 0$. The maximised "log-likelihood" of this model is the same as for the unrestricted VAR. The same is true for the model we have estimation result for, namely

$$\begin{aligned} \Delta Y_t &= \beta_0 + (\rho - 1) Y_{t-1} + \beta_1 \Delta X_t + (\beta_1 + \beta_2) X_{t-1} + \varepsilon_t \\ \Delta X_t &= a_{20} + a_{21} Y_{t-1} + a_{22} X_{t-1} + \varepsilon_{2,t} \end{aligned}$$

since the two disturbances are unaffected by this (second) re-paramerization that changes the left-hand side variables form levels to differences. The increase in parameters from 6 to 7 is because the correlation of the VAR disturbances has been "moved to" the regression parameter β_1. The endogenous variables are different in the two models.

b) If not already noted: No: The disturbances of the (valid) conditional model and the marginal model are uncorrelated by construction. This is not affected by writing the model in ECM form.