ECON3215/4215, extra exercise, 19 Nov. 2010, D.L.

This is an extra exercise following naturally after Cowell's 8.13. In that exercise we showed that a risk averse owner of a firm would prefer knowing a price with certainty over a situation of uncertainty in which the choice of output must be made before the output price is known. We also showed that if output can be chosen after the price is known, a risk neutral owner would prefer uncertainty over certainty. This is due to the convexity of the profit function. A natural question is, what happens in the latter case if the owner is slightly risk averse? That is the topic of this exercise. The answer is available in Fronter.

Conceptually, this topic is within the syllabus of the course, and the students should be able to find the solution method. However, the calculations are more evolved than typical exam questions in this course.

1. Let g be a constant (a real number) and consider the function

$$U(C) = \frac{1}{1-g} C^{1-g}.$$

What condition(s) must g satisfy for $E[U(C)]$ to be a meaningful expected-utility function for a risk averse individual? (You can disregard requirements that the U function should be restricted.) What are the indices of absolute and relative risk aversion?

2. Consider an individual who is planning for two periods, $t = 0$ and $t = 1$, with the utility function

$$V = U(C_0) + \theta E[U(C_1)],$$

where U is specified above, the condition(s) from part 1 is/are satisfied, and where $\theta \in (0, 1)$ is a constant. The individual has wealth, H, which at $t = 0$ can be divided between consumption, C_0, and a real investment, K. The prices of the consumption good and the capital good are both equal to unity. The consumption at $t = 1$, C_1, is equal to operating profits from a firm owned by the individual. The firm has a production function

$$X = L^{\alpha} K^{1-\alpha},$$

where $\alpha \in (0, 1)$ is a constant. L is labor, hired at $t = 1$ at a wage w. X is the produced quantity, which is sold at $t = 1$ at a price p.

For this part of the exercise (but not for part 3), assume that when K is chosen at $t = 0$, w is known, while for p there is known a probability distribution. Explain how L and K will be chosen. Show that if L and K are optimally chosen, the utility level can be written as

$$V^* = \frac{1}{1-g} H^{1-g} \left[\frac{1+Z}{Z}\right]^g,$$

where

$$Z \equiv (RY)^{-1/g}, \quad R \equiv E\left[\left(\frac{p}{w^\alpha}\right)^{\frac{1-g}{1-\alpha}}\right],$$

and

$$Y \equiv \theta \alpha^{\alpha(1-g)/(1-\alpha)} (1-\alpha)^{1-g}.$$

(This formulation also covers part 3 below.) Discuss whether the individual will get a higher expected utility if p is announced before K is chosen, with p equal to its expected value from the uncertain case. Show that the answer depends on $g > \alpha$. Interpret the result. (Hint: A function $f(s) = s^b$ is concave if and only if $b \in [0, 1]$. Find first the effect on R.)

3. Repeat part 2, but with w uncertain and p certain. This means, you should end up with a comparison of an uncertain w and a known w, equal to the expected w from the uncertain case. The limit is now not $g > \alpha$. What is it?