Summing up demand and supply, Cowell 2.1–2.3

- In sections 2.1–2.4: Single-product firm
- Firm takes prices of input factors, $w_1, \ldots, w_m$, as given
- For any output q that the firm wishes to produce, the firm wants to minimize the costs of production, which gives
  - Conditional factor demands $z_i = H^i(w_1, \ldots, w_m, q)$ for $i = 1, \ldots, m$
  - Cost function $C(w_1, \ldots, w_m, q) = \Sigma_{i=1}^m w_i H^i(w_1, \ldots, w_m, q)$
- In section 2.2.3, firm also takes output price, p, as given
- Chooses q to maximize profits, $\max_q[pq - C(w_1, \ldots, w_m, q)]$, which gives
  - Unconditional factor demands $z_i = D^i(w_1, \ldots, w_m, p)$ for $i = 1, \ldots, m$
  - Supply $q = S(w_1, \ldots, w_m, p)$
  - Profit function $\Pi(w_1, \ldots, w_m, p) = pS(w_1, \ldots, w_m, p) - C(w_1, \ldots, w_m, S(w_1, \ldots, w_m, p))$

Supply curve of competitive, single-product firm

Assume now that functions are differentiable

- First-order condition for profit maximization: $p = C'_q(w_1, \ldots, w_m, q)$
- Second-order cond. for profit max.: $C''_{qq}(w_1, \ldots, w_m, q) > 0$
- Condition for positive solution: $p > C/q$, average cost (AC)
- Derivative of AC with respect to q is
$$\frac{d[C(q)/q]}{dq} = \frac{qC'(q) - C}{q^2} = \frac{1}{q}\left(C' - \frac{C}{q}\right)$$
- Shows AC is increasing in q if and only if $C' >$AC
- Assume minimum AC occurs for some $\underline{q} \geq 0$, with first-order condition $C'(\underline{q}) = C(\underline{q})/\underline{q}$, and that AC is increasing for all $q > \underline{q}$
- When $p >$ minimum AC, supply function is inverse of marginal cost function, Cowell fig. 2.12

Short run, one input factor fixed, Cowell 2.4

- Analysis so far is interpreted as long run
- In contrast, short run means one input factor fixed
- Typically we think of this as capital equipment
- Costly and time-consuming to change amount of capital
- Analytically: Keep m'th input fixed at $\bar{z}_m$
- This could have been just any value of z_m
- But a more specific definition is given in Cowell:
- First: Consider long-run cost minimization for $q = \bar{q}$
- $\bar{z}_i = H^i(w_1, \ldots, w_m, \bar{q})$ for $i = 1, \ldots, m$
- Then keep z_m fixed at this specific level $\bar{z}_m$
- Firm is now allowed to change its output level
- Short-run cost minimization: Optimize $z_1, \ldots, z_{m-1}$:

$$\tilde{C} = \min_{z_i \geq 0} \sum_{i=1}^{m} w_i z_i \text{ s.t. } \phi(z_1, \ldots, z_m) \geq q \text{ and } z_m = \bar{z}_m$$

- Solutions are *short-run conditional factor demands*

$$\tilde{H}(w_1, \ldots, w_m, q, \bar{z}_m) \text{ for } i = 1, \ldots, m-1$$

- Gives short-run cost function $\tilde{C}(w_1, \ldots, w_m, q, \bar{z}_m)$
- Still including cost $w_m \bar{z}_m$ (see mini problem 34)

Some results about the short-run cost function

- Obviously, $\tilde{C}(w_1, \ldots, w_m, q, \bar{z}_m) \geq C(w_1, \ldots, w_m, q)$
- And, $\tilde{C}(w_1, \ldots, w_m, \bar{q}, \bar{z}_m) = C(w_1, \ldots, w_m, \bar{q})$
- Short-run marginal cost is defined as $\partial\tilde{C}/\partial q$
- Will show: At $q = \bar{q}$, this equals long-run marginal cost
- Use the fact that $\partial\tilde{C}(w_1, \ldots, w_m, \bar{q}, \bar{z}_m)/\partial\bar{z}_m = 0$
  - If this derivative had been different from zero, it would have been possible at $q = \bar{q}$ to reduce long-run and short-run cost (which are equal) by varying z_m away from $\bar{z}_m$
  - But that cannot be true, since $\bar{z}_m$ is part of long-run cost-minimizing solution at $q = \bar{q}$
- Start now from equation in second bullet point above
- Plug in $\bar{z}_m = H^m(w_1, \ldots, w_m, \bar{q})$, and rewrite equation,

$$\tilde{C}(w_1, \ldots, w_m, \bar{q}, H^m(w_1, \ldots, w_m, \bar{q})) = C(w_1, \ldots, w_m, \bar{q})$$

- Derivatives of left-hand and right-hand side w.r.t. $\bar{q}$:

$$\frac{\partial\tilde{C}(w_1, \ldots, w_m, \bar{q}, H^m(w_1, \ldots, w_m, \bar{q}))}{\partial\bar{q}} + 0 = \frac{\partial C(w_1, \ldots, w_m, \bar{q})}{\partial\bar{q}}$$

  i.e., short-run and long-run marginal costs are equal

- "+0" signifies the term which we just proved to be zero

$$\frac{\partial\tilde{C}(w_1, \ldots, w_m, \bar{q}, H^m(w_1, \ldots, w_m, \bar{q}))}{\partial\bar{z}_m} \cdot \frac{\partial\bar{z}_m}{\partial\bar{q}}$$

Illustrating short-run and long-run cost

- Assume again: Long-run average cost is U-shaped
- Know long-run marginal cost "goes through" minimum point
- Pick some $\bar{q}$ for which long-run AC is increasing
- Short-run costs satisfy
  - Short-run AC is equal to long-run AC at $\bar{q}$
  - Short-run MC is equal to long-run MC at $\bar{q}$
  - Short-run MC "goes through" minimum of short-run AC

Multi-product firm, Cowell 2.5

- Consider now firm with more than one output
- Cowell defines the *net output vector*

$$\mathbf{q} = (q_1, \ldots, q_m, q_{m+1}, \ldots, q_r, q_{r+1}, \ldots, q_n)$$

  in which quantities of inputs, intermediate goods, and outputs all have the same notation, q_i
- When good i is used as net input, q_i is negative
- Could imagine production process allowing some goods to be inputs in some situations, outputs in others, but will not consider this possibility here
  - Thus, for each i we assume always $q_i < 0$, $q_i = 0$, or $q_i > 0$
  - Can then choose to arrange inputs first, $i = 1, \ldots, m$
  - Intermediates are numbered $m + 1, \ldots, r$
  - Outputs are numbered $r + 1, \ldots, n$
  - "Intermediates" are internal to the firm, $q_i = 0$ for these
- Previous single-product case is special case

Previous notation	New notation
$z_1, \ldots, z_m$	$q_1 \equiv -z_1, \ldots, q_m \equiv -z_m$
$q - \phi(z_1, \ldots, z_m)$	$\phi(q_1, \ldots, q_m, q_{m+1})$
$q \leq \phi(z_1, \ldots, z_m)$	$\phi(\mathbf{q}) \leq 0$
$w_1, \ldots, w_m$	$p_1 \equiv w_1, \ldots, p_m \equiv w_m$
Profits $pq - \Sigma_{i=1}^m w_i z_i$	Profits $\Sigma_{i=1}^{m+1} p_i q_i$

Marginal rate of transformation

- In general, production function is $\phi(q_1, \dots, q_n) \leq 0$
- Derivatives ϕ_i are ≥ 0 (assuming they exist)
- *Marginal rate of transformation* of output i into output j is

$$\text{MRT}_{ij} \equiv \frac{\phi_j(\mathbf{q})}{\phi_i(\mathbf{q})}$$

- Profit maximization formulated with Lagrangean:
  $\mathcal{L}(\mathbf{q}, \lambda, \mathbf{p}) \; = \Sigma_{i=1}^{n} p_i q_i - \lambda\phi(\mathbf{q})$
- First-order conditions (assuming optimal $q_i > 0$):

$$p_i - \lambda\phi_i(\mathbf{q}) = 0 \text{ for } i = 1, \dots, n$$

- For each pair of net outputs which are both $\neq 0$:

$$\frac{\phi_j(\mathbf{q})}{\phi_i(\mathbf{q})} = \frac{p_j}{p_i}$$

- When both q's > 0, p_j/p_i is slope of isoprofit lines

Convex profit function, Π

- $\Pi(\mathbf{p})$ defined as maximized profits

$$\Pi(p_1, \ldots, p_n) = \max_{q_1,\cdots,q_n} \sum_{i=1}^{n} p_i q_i \text{ s.t. } \phi(q_1, \ldots, q_n) \leq 0$$

- Error in Cowell, Theorem 2.7, pp. 44 and 610
- While C is concave, Π is *convex*
- Proof:
  - Let the vector $\mathbf{p}^{**}$ be a convex combination of $\mathbf{p}^{*}$ and $\mathbf{p}^{***}$, i.e., there exists a $t \in [0, 1]$ such that $\mathbf{p}^{**} = t\mathbf{p}^{*} + (1-t)\mathbf{p}^{***}$
  - Let $\mathbf{q}^{*}$ maximize profits at $\mathbf{p}^{*}$, $\mathbf{q}^{**}$ maximize profits at $\mathbf{p}^{**}$, and $\mathbf{q}^{***}$ maximize profits at $\mathbf{p}^{***}$
  - Then we have $\Pi(\mathbf{p}^{**}) = \Sigma_{i=1}^{n}(tp_i^{*} + (1-t)p_i^{***})q_i^{**}$ which equals $t\,\Sigma_{i=1}^{n}\, p_i^{*} q_i^{**} + (1-t)\,\Sigma_{i=1}^{n}\, p_i^{***} q_i^{**}$
  - But the vector $\mathbf{q}^{**}$ does not (necessarily) maximize profits at the other two price vectors, so we know that the first of these terms is less than or equal to $t\,\Sigma_{i=1}^{n}\, p_i^{*} q_i^{*}$, and likewise, the second is $\leq (1-t)\,\Sigma_{i=1}^{n}\, p_i^{***} q_i^{***}$
  - This implies that $\Pi(\mathbf{p}^{**}) \leq t\Pi(\mathbf{p}^{*}) + (1-t)\Pi(\mathbf{p}^{***})$

Hotelling's lemma

- Assume profit function is differentiable
- Similar to Shephard's lemma, but for outputs:

$$\text{Optimal } q_i = \frac{\partial \Pi(\mathbf{p})}{\partial p_i}$$

- An expression for the firm's (optimal) net output supply
- Follows from envelope theorem for constrained maximization
- Π is maximized value of constrained maximization problem

$$\Pi(p_1, \ldots, p_n) = \max_{q_1, \cdots, q_n} \sum_{i=1}^{n} p_i q_i \text{ s.t. } \phi(q_1, \ldots, q_n) \leq 0$$

- Then partial derivatives of Π can be found as partial derivatives of Lagrangean $\mathcal{L}(\mathbf{q}, \lambda, \mathbf{p}) = \Sigma_{i=1}^{n} p_i q_i - \lambda \phi(\mathbf{q})$
- Only one term since prices do not appear in constraint

Aggregate supply function, Cowell 3.2–3.3

- In sections 3.2–3.3: The number of firms is given
- Aggregate supply function is sum of each firm's supply
- By convention the functions' argument, p, is on vertical axis
- Summation is therefore horizontal
- Assume a falling market demand function
- Equilibrium when supply equals demand

No equilibrium due to jump in supply function?

- Cowell worries about jump in supply function, pp. 52–55
- Intersection of supply and demand function may not exist
- "Absence of market equilibrium" in figure 3.3
- Let jump occur at $\hat{p}$, from q_0 to q_1
- The quantity q_0 equals aggregate demand at a higher price
- The quantity q_1 equals aggregate demand at a lower price
- Demand function determines the quantity which must be supplied in order for $\hat{p}$ to be an equilibrium price

Equilibrium in spite of jump?

- Firms in this model may be different; assume at least two types
- Assume that supply from a subgroup of n firms jumps at $\hat{p}$
- For simplicity, assume these firms supply nothing when $p < \hat{p}$
- Each jumps to supplying $(q_1 - q_0)/n$ when price goes above $\hat{p}$
- Each has average cost equal $\hat{p}$ when it produces $(q_1 - q_0)/n$
- When $p = \hat{p}$, indifferent between producing $(q_1 - q_0)/n$ and 0
- Will show that equilibrium can occur in spite of jump

Equilibrium, but who will produce is not determined

- Competitive equilibrium in one market is a situation in which
  - All market participants behave as if the price is given
  - Each supplier maximizes profits (or utility)
  - Each demander maximizes utility (or profits)
  - (If indifferent between two q values, either can be chosen)
- In case with jump:
  - Exists equilibrium where only some "jump firms" produce
  - Only (exact) equilibrium if intersection with demand curve
  - Need $k \in (0, n)$ firms so that $q_0 + k(q_1 - q_0)/n$ equals demand at $\hat{p}$
  - If such number k exists, any group of k could produce in equilibrium, while the remaining $n - k$ produce nothing
  - If such number k does not exist, there is no equilibrium of the type described above; we return to mixed strategies in Cowell ch. 10

Size of industry Cowell, sect. 3.5

- Imagine process in which most efficient firms enter industry first
- New firms will enter as long as profits are positive
- Gradually less efficient firms are attracted
- Entry shifts aggregate supply to the left
- Moving down aggregate demand curve, reducing price
- Equilibrium number of firm when profits are zero for the marginal firm

Monopoly, Cowell, sect. 3.6

- Sect. 3.6.1: Monopoly without price discrimination
- Sect. 3.6.2: Monopoly with price discrimination
- Will assume these concepts are well known
- In both cases assume one homogeneous product only
- Optimal solution without price discrimination: Marginal revenue equals marginal cost
- Price discrimination:
  - Need ability to prevent resale between submarkets
  - Same (or coordinated) production for both markets; same marginal cost for both
  - May want to sell in only one market
  - But if selling in both: Marginal revenue in both are equal, and equal to marginal cost

Entry fee, Cowell, sect. 3.6.3 (example: Disneyland)

- Suppose the monopoly could charge an entry fee
- Assuming that all customer's are equal, the easy solution is to extract an entry fee equal to the area between the demand curve and the price line
- Area can be seen as the *consumer's surplus* (more in ch. 4)
- Monopoly chooses price p and entry fee F_0 to maximize

  $$p(q)q - C(\mathbf{w}, q) + \left[\int_0^q p(x)dx - p(q)q\right]$$

  where expression in brackets is entry fee; simplifies to

  $$= \int_0^q p(x)dx - C(\mathbf{w}, q)$$

  which is maximized by setting $p = C'_q(\mathbf{w}, q)$
- The ability to charge fixed fee lets monopolist
  - get around trade-off between price and quantity
  - charge a price equal to marginal cost, so that consumer surplus is maximized; but then, charge F_0 such that all the surplus ends up in the hands of the monpolist