UNIVERSITY OF OSLO
DEPARTMENT OF ECONOMICS

Exam: **ECON4240 – Game theory and economics of information**

Date of exam: Wednesday, May 23, 2007 **Grades will be given: Wednesday, June 13**

Time for exam: 2:30 p.m. – 5:30 p.m.

The problem set covers 3 pages

Note: You can give your answer in English or Norwegian!

Resources allowed:
- No resources allowed

The grades given: A-F, with A as the best and E as the weakest passing grade. F is fail.

The exam consists of *five* problems. They count as indicated. Start by reading through the whole exam, and make sure that you allocate time to answering questions you find easy. You can get a good grade even if there are parts of problems that you do not have time to solve.

Problem 1 (10 %)

In the following normal-form game, which strategy profiles survive iterated elimination of strictly dominated strategies? What are the pure-strategy Nash equilibria?

Player 2

		L	*C*	*R*
	U	2, 6	0, 4	6, 0
Player 1	*M*	6, 0	2, 2	7, 3
	D	6, 8	2, 4	4, 5

Problem 2 (10 %)

You and a friend are in a restaurant, and the owner offers both of you an 8-slice pizza under the following condition. Each of you must simultaneously announce how many slices you would like; that is, each player $i \in \{1, 2\}$ names his/her desired amount of pizza, $0 \le s_i \le 8$. If $s_1 + s_2 \le 8$, then the players get their demands (and the owner eat any leftover slices). If $s_1 + s_2 > 8$, then the players get nothing. Assume that you each care only about how much pizza you individually consume, preferring more pizza to less.

a) What is (are) each player's best response(s) for each of the possible demands for his/her opponent?
b) Find all the pure-strategy Nash equilibria.

Problem 3 (15 %)

Consider the situation of Problem 2, but assume now that player 1 makes her demand before player 2 makes his demand. Player 2 observes player 1's demand before making his choice.

a) Explain what a strategy is for player 2 in this game with sequential moves.
b) Find all the pure-strategy Nash equilibrium outcomes.
c) Find all the pure-strategy subgame perfect equilibria.

Problem 4 (15 %)

Consider the situation of Problem 3, but assume now in addition that the pizza comes in 5 different sizes, each with x slices, where $x \in \{4, 6, 8, 10, 12\}$. Player 1 observes x before making her demand, while players 2 only observes player 1's demand, but not x, before having to make his own demand. Before observing player 1's demand, player 2 thinks that the 5 different pizza sizes are equally likely, but he may infer something from her demand.

a) Explain what a strategy is for player 1 in this game of incomplete information.
b) Show that the following strategy for player 1 can be part of a perfect Bayesian equilibrium: $s_1(4) = 2$, $s_1(6) = 3$, $s_1(8) = 4$, $s_1(10) = 5$, $s_1(12) = 11$. Specify both player 2's strategy and player 2's beliefs.
c) Are there other perfect Bayesian equilibria in this game?

Problem 5 (50 %)

Peter owns a car and considers whether to go to an insurance company to have it insured against accidents. Peter may be a safe or a reckless driver; the probability of an accident being $\pi_S = ⅓$ in the former case and $\pi_R = ½$ in the latter case. Peter knows whether he is safe or reckless, this being an intrinsic property of the person, not something he can decide about.

There are many insurance companies in the market, and they are all risk neutral. Unless otherwise stated, information is asymmetric. That is, the companies cannot observe whether Peter is safe or reckless; they only knows that the fraction of safe drivers in the population is λ, $0 < \lambda < 1$.

All insurance companies offer contracts of the form (p, q), where p is the premium and q is the coverage; that is, p is paid ahead of time and q is received in case of an accident. Peter decides whether to buy insurance, and if so which contract to choose, in order to maximize expected utility. His utility function is

$$u(x) = \sqrt{x}$$

where x is his final wealth. His initial wealth is $x_0 = 100$ and an accident implies a loss of $c = 36$.

(a) What is Peter's expected utility if he is a safe driver and signs the contract (p, q)? Formulate the participation constraint, that is, the restriction guaranteeing that Peter really will be interested in signing the contract (p, q). Answer the same questions for a reckless driver.

(b) Calculate the expected profits the company makes by offering Peter the contract (p, q) when the company does not know Peter's type and
  (i) Peter never accepts the offer;
  (ii) Peter accepts the offer if and only if he is a safe driver;
  (iii) Peter accepts the offer if and only if he is a reckless driver;
  (iv) Peter accepts the offer in any case.

(c) Assume that the company can observe whether Peter is a safe or a reckless driver. Calculate the contracts that will exist in the market in this case, given that there are many insurance companies. Will Peter be fully insured if he is a safe driver? Will he be fully insured if he is a reckless safe driver?

(d) Then assume that the companies cannot observe whether Peter is safe or reckless but know that the fraction of safe drivers in the population is λ. Explain why the contracts of (c) will not appear in the market in this case.

(e) Suppose that there exists a separating equilibrium in the market under the assumptions of (d). Describe the contracts that will be offered in such a market. (If possible, describe the contracts explicitly; alternatively, write down conditions they must satisfy. A figure may be helpful.)

(f) Now assume that the insurance company knows whether Peter is safe or reckless, but this cannot be proven in a court of law (for example, because of privacy legislation). Try to construct a game between Peter and the company through which the contract menu of (c) can be implemented.

(g) (If time permits.) Return to the conditions of (e). Does there actually exist a separating equilibrium as described there? How does the answer to this question depend on λ.