UNIVERSITY OF OSLO
DEPARTMENT OF ECONOMICS

Postponed exam: **ECON4240 – Game theory and economics of information**

Date of exam: Wednesday, August 6, 2008

Time for exam: 09:00 a.m. – 12:00 noon

The problem set covers 3 pages

Resources allowed:

- No resources allowed

The grades given: A-F, with A as the best and E as the weakest passing grade. F is fail.

The exam consists of 3 problems. They count as indicated. Start by reading through the whole exam, and make sure that you allocate time to answering questions you find easy. You can get a good grade even if there are parts of problems that you do not have time to solve.

Problem 1 (20 %)

Define and briefly explain the following concepts:

(a) Subgame perfect equilibrium.
(b) Perfect Bayesian equilibrium.

Are the following statements true or false? For each statement, if true, try to explain why; if false, provide a counter-example.

(c) In any game of perfect information, every Nash equilibrium is also a subgame perfect equilibrium.
(d) In any 2-player game of incomplete information where each player plays only once, there is no subgame other than the whole game.

Problem 2 (40 %)

Consider the following normal form game

		Player 2		
		L	*C*	*R*
Player 1	*T*	10, 10	2, 8	0, 13
	M	8, 2	5, 5	0, 0
	B	13, 0	0, 0	1, 1

(a) Explain how you can determine the set of rationalizable strategies for each player. What strategies are rationalizable for each player?

(b) Determine all pure and mixed Nash equilibrium/a in this game.

Suppose now that the game is played twice and that the players can observe the first period outcome prior to the second period play. Assume that the payoffs of this two-period game are not discounted; hence, the payoff of the two-period game is the sum of the payoffs attained in the two periods.

(c) How many pure strategies does each player have in this two-period game?

(d) Determine a subgame perfect equilibrium where the players play (T, L) in the first round. Specify what the players play in the second round, for each possible outcome in the first round.

Finally, suppose the game is repeated infinitely many times, and that payoffs are discounted by the discount factor δ.

(e) Show that there exists a subgame perfect equilibrium where the players play (T, L) in every round if $\delta \geq 1/4$.

Problem 3 (40 %)

Consider the standard principal-agent model (the rent extraction-efficiency trade-off): An agent, who may be efficient or inefficient, shall do a job for a risk-neutral and profit-maximizing principal. The job consists in producing an amount of some good. The agent's type (efficient or inefficient) is known to the agent, but cannot be observed by the principal.

When answering the questions below, you may use the following notation:

- The amount produced is denoted q. The gross (monetary) value to the principal of q units of the good is $S(q)$, where $S(0)=0$, $S'>0$ and $S''<0$.

- The cost to the agent of producing q units is $\underline{\theta}q$ for an efficient agent and $\overline{\theta}q$ for an inefficient one, $0 < \underline{\theta} < \overline{\theta}$. From the point of view of the principal, the probability that the agent is efficient is ν.
- The principal offers one or more contracts of the form (q, t), where t is a transfer paid by the principal to the agent, provided that q units of the good are produced. If more than one contract is offered, the agent chooses among them, or can refuse to accept any of them.

(a) What is, in this connection, meant by the term "participation constraint"?

(b) Now suppose – contrary to the assumption made above – that the principal can observe the agent's type. Which contract(s) should the principal then offer?

(c) Return to the assumption that the principal cannot observe the agent's type. What will happen if the principal offers the contract(s) found in (b)?

(d) Suppose that the principal wants both types of agent to accept a contract.

  (i) Describe the contracts the principal will then offer. You can, for example, illustrate this in a figure, or derive formal expressions.

  (ii) What is, in this connection, meant by the term "incentive compatibility constraint"?

  (iii) Which participation constraints and incentive compatibility constraint are binding?

  (iv) What is meant by "information rent"? Who receives such rent? Find an expression for its size.

(e) Is it possible that the principal does not want both types of agent to accept a contract? If so, what should the principal do?

(f) Describe briefly what will change if the cost to the agent of producing q units of the good takes the more general form $C(q, \theta)$, where $\theta = \underline{\theta}$ for an efficient agent and $\theta = \overline{\theta}$ for an inefficient one.