ECON 4240 EXAM: SPRING 2014

SOLUTION SKETCH

Problem 1. (20 points)

(i) Suppose that all worker types submit to the test. If the firms see a worker who *does not*, then offering a wage no higher than $\underline{\theta}$ to that worker is optimal (note, this subgame is not reached in the proposed play). In the subgame where the worker takes the test, he will be paid the wage equal to his revealed type. Now given this behavior of the firms, consider a deviation by any worker. Clearly, he cannot gain by deviating since his true type is no lower than $\underline{\theta}$. Hence, this is an SPNE.

(ii) Consider an SPNE where at least some workers choose not to take the test. Then the firms must offer a wage equal to the mean of the types of the workers *not* submitting to the test. But then the worker with the highest type among the set which does not submit can gain by deviating and taking the test, since he will now be paid a wage equal to his type (which is higher than the mean of this set). So in every SPNE, all workers take the test.

Now consider an SPNE where all workers submit to the test *and* the firms offer a wage higher than $\underline{\theta}$ to any worker not taking the test. Call this wage $\hat{\theta}$. Then all workers with type lower than $\hat{\theta}$ can gain by *not* taking the test. Hence, contradiction. This establishes the claim.

Problem 2. (30 points)

(i) *Observable effort case:*

To induce effort e, the principal will pay $g(e)$. Hence, check the payoffs for the principal for the three effort levels. Then choose the effort corresponding to the highest one.

$$\pi(e_1) = \frac{2}{3}.10 - \frac{25}{9} = \frac{35}{9}.$$

$$\pi(e_2) = \frac{1}{2}.10 - \frac{64}{25} = \frac{61}{25}.$$

$$\pi(e_3) = \frac{1}{3}.10 - \frac{16}{9} = \frac{14}{9}.$$

Hence, optimal contract is $(w = \frac{25}{9}, e = e_1)$.

(ii) *Unobservable effort case:*

To implement e_1, consider the contract which pays w_i for $\pi = \pi_i$ where $i \in \{L, H\}$. Note, the following have to be met:

Participation Constraint (PC): $\frac{2}{3}\sqrt{w_H} + \frac{1}{3}\sqrt{w_L} \geq \frac{5}{3}$.

Incentive Compabitility $IC(e_2)$: $\frac{2}{3}\sqrt{w_H} + \frac{1}{3}\sqrt{w_L} - \frac{5}{3} \geq \frac{1}{2}\sqrt{w_H} + \frac{1}{2}\sqrt{w_L} - \frac{8}{5}$.

Incentive Compabitility $IC(e_3)$: $\frac{2}{3}\sqrt{w_H} + \frac{1}{3}\sqrt{w_L} - \frac{5}{3} \geq \frac{1}{3}\sqrt{w_H} + \frac{2}{3}\sqrt{w_L} - \frac{4}{3}$.

Check that $IC(e_2)$ does not bind. $IC(e_3)$ and PC bind. This yields $w_L = 1, w_H = 4$. Hence, the principal's expected profit is $\pi(e_1) = \frac{11}{3}$.

Conduct the same exercise for e_2. Here, however we get that e_2 is *not* implementable since the two incentive compatibility contsraints cannot be simultaneously satisfied.

To implement e_3, it is easily checked that the principal can do better by offering the first–best contract (flat wage of $\frac{16}{9}$) than by offering $w_H > w_L$. Hence, the principal's expected profit is $\pi(e_3) = \frac{14}{9}$.

So the principal will choose to implement e_1 by offering $\{w_L = 1, w_H = 4\}$. This the optimal contract when effort is *not* observable.

Problem 3. (25 points)

i. The maximization program is:

$$\max_{g_i \geq 0} \omega_i - g_i + a_i \log(g_i + G_{-i})$$

and the FOC is:

$$\frac{a_i}{G} \leq 1$$

with equality if $g_i > 0$.

ii. The equilibrium amount is:

$$G^* = g_n^* = a_n$$

Only consumer n will contribute, whereas all the other will free ride.

iii The equilibrium amount is:

$$G^* = \sum_{i=1}^{n} g_i^* = \sum_{i=1}^{n} \frac{a}{n} = a$$

iv. The efficient amount is obtained from:

$$\max_{G} \sum_{i=1}^{n} a_i \log(G) - G$$

Then, the optimal provision of public good is:

$$G^o = \sum_{i=1}^{n} a_i > G^* = a_n$$

The private provision of public good creates a situation in which externalities are present. The inefficiency is then related to the absence of a market for public good (incomplete market).

v. Discussion about Lindahl prices.

Problem 4. (25 points)

i. The Walrasian equilibrium is the equilibrium labor and consumption good, such that:
   - Consumer max utility:

$$\max_{x_1,x_2} a \log x_1 + \log x_2$$

$$\begin{aligned} s.t &: \\ px_2 &\leq w(1-x_1) \end{aligned}$$

   - Firm max profits:

$$\max_z pAz^\beta - wz$$

   - Markets clear:

$$\begin{aligned} x_2^* &= q^* \\ 1-x_1^* &= z^* \end{aligned}$$

ii. By applying the equilibrium definition, we obtain:

$$\begin{aligned} z^* &= 1-x_1^* = \frac{\beta}{a+\beta} \\ q^* &= x_2^* = A\left(\frac{\beta}{a+\beta}\right)^\beta \\ p^* &= 1 \text{ (numeraire)} \\ w^* &= A\beta\left(\frac{\beta}{a+\beta}\right)^{\beta-1} \end{aligned}$$

   and the equilibrium profits are:

$$\pi^* = A(1-\beta)\left(\frac{\beta}{a+\beta}\right)^\beta$$

iii. omitted (available in the textbook)

iv. The equilibrium is efficient for the first welfare theorem.