

***UNIVERSITY OF OSLO***  
***DEPARTMENT OF ECONOMICS***

Postponed exam: **ECON4240 – Equilibrium, welfare and information**

Date of exam: Wednesday, June 3, 2015

Time for exam: 09:00 a.m. – 12:00 noon

The problem set covers 2 text pages (not incl. cover sheet)

Resources allowed:

- No resources allowed (except if you have been granted use of a dictionary from the Faculty of Social Sciences)

The grades given: A-F, with A as the best and E as the weakest passing grade. F is fail.

## ECON 4240 Exam: Spring 2015

### PART I: ECONOMICS OF INFORMATION

#### Problem 1. (30 points)

Air Wizard is the only airline connecting Rohan and Gondor. There are two types of passengers: tourist and business. Business travelers are willing to pay more than tourists. The airline, however, *cannot* tell directly whether a ticket purchaser is a tourist or a business traveler. The two types do differ in how much they are willing to pay to avoid having to purchase their tickets in advance. (Passengers do not like to commit themselves in advance to traveling at a particular time.)

The utility levels of each of the two types net of the price of the ticket  $P$ , for any given amount of time  $W$  prior to the flight that the ticket is purchased are given by

$$\text{Business: } v - \theta_B P - W,$$

$$\text{Tourist: } v - \theta_T P - W,$$

where  $0 < \theta_B < \theta_T$ . (Note, that for any given level of  $W$ , the business traveler is willing to pay more for his ticket. Also, he is willing to pay more for any given reduction in  $W$ .) The proportion of travelers who are tourists is  $\lambda$ . Assume that the cost of transporting a passenger is  $c$ . Also assume that Air Wizard *wants* to carry both types of passengers.

- (i) Formulate the optimal (profit-maximizing) price discrimination problem that Air Wizard would want to solve. Note, non-negativity of prices has to be imposed as a constraint.
- (ii) Show that in the optimal solution, tourists are indifferent between buying a ticket and not traveling.
- (iii) Show that in the optimal solution, business travelers never buy their ticket prior to the flight and are just indifferent between doing this and buying when tourists buy.

#### Problem 2. (20 points)

Consider a risk-neutral principal and a risk-neutral agent in an asymmetric information setup. The principal cares about the level of output  $q$ . The realization of  $q$  is stochastic: either high ( $q_H$ ) or low ( $q_L$ ) where  $q_L < q_H$ .

The agent may perform a task requiring effort which is not observable by anyone other than the agent. Exerting effort is costly for the agent: call this cost  $C$  (which is a fixed positive number). If the agent undertakes effort, the probability that  $q = q_H$  is  $\pi_1$ . If the agent does not undertake effort, the probability that  $q = q_H$  is  $\pi_0$  where  $\pi_0 < \pi_1$ . Assume that the agent has a reservation utility of 0. The principal must design an incentive contract for the agent. Assume that  $C$  is low enough, so that the expected output when effort is exerted less the cost of effort  $C$  exceeds expected output when no effort is exerted.

- (i) Write down the principal's (constrained optimization) problem.

(ii) Is moral hazard an issue here, in the sense of deviation from the first-best? Illustrate by constructing a set of optimal transfers.

(iii) Now suppose, there is limited liability in the sense that transfers to the agent cannot be negative. Can first-best be achieved under this additional constraint?

## PART II: GENERAL EQUILIBRIUM ANALYSIS

### Problem 3. (20 points)

Suppose there are  $n$  individuals. There is a private good called money and a public good. Providing an amount  $g$  of the public good costs  $k(g)$  units of money, where  $k$  is an increasing and convex cost function. Individual  $i$  has a utility function  $U_i = v_i(g) - p_i$  where  $p_i$  is  $i$ 's contribution to the cost of public good provision. For each individual  $i$ ,  $v_i$  is a strictly increasing and strictly concave function.

(i) Show that an allocation is Pareto-efficient if and only if  $g$  is chosen such that the surplus from public good provision  $\sum_{i=1}^n v_i(g) - k(g)$  is maximized.

(ii) Show that contributions are such that  $\sum_{i=1}^n p_i = k(g)$ .

### Problem 4. (30 points)

Consider a two-good quasilinear model with one consumer and one firm. The initial endowment of the numeraire good,  $m$ , is  $w_m$ , and the initial endowment of the other good, denoted  $\ell$ , is 0. Let the consumer quasilinear utility function be  $\phi(x) + m$  where  $\phi(x) = \alpha + \beta \log x$  for some  $\alpha$  and  $\beta$  positive. Also let the firm's cost function be  $c(q) = \sigma q$  with  $\sigma$  positive. Assume that the consumer receives all the profits of the firm. Both the firm and the consumer act as price takers. Normalize the price of the numeraire to be equal to one, and denote the price of good  $\ell$  by  $p$ .

(i) Derive the consumer's and the firm's first-order conditions.

(ii) Derive the competitive equilibrium price and output of good  $\ell$ .

(iii) How do the equilibrium price and output vary with  $\alpha$ ,  $\beta$ , and  $\sigma$ ?