UNIVERSITY OF OSLO
DEPARTMENT OF ECONOMICS

EXAM ECON 4240 EQUILIBRIUM, WELFARE AND INFORMATION, SPRING 2016

Total: 80 points.

PART 1 (20 POINTS)

A bank raises funds $k \geq 0$ at the exogenous rate R, and lends them to a firm, who has no other access to capital markets. The firm uses k to produce a good (whose price is normalized to 1) according to the production function

$$Y(k,\theta) = \theta ln(1+k)$$

where $\theta > 0$ is the firm-specific productivity. After producing and selling the good, the firm pays back t to the bank. Both firm and bank are profit-maximizing, and their profit functions are:

$$U_{firm} = Y - t$$
$$U_{bank} = t - Rk$$

The timing is as follows: 1) the bank offers a contract to the firm; 2) the firm takes it or leaves it; 3) the contract (if signed at 2)) is executed. The outside option of the firm is normalized to 0.
The productivity of the firm θ is a random variable:

$$\theta = \begin{cases} \underline{\theta}, & \text{with probability } p \\ \overline{\theta}, & \text{with probability } 1-p. \end{cases}$$

where $\Delta\theta = \overline{\theta} - \underline{\theta} > 0$ and $\underline{\theta} > R$.

Question 1.1 (4 points)

Assume θ is observable by both bank and firm. Derive the equilibrium contract.

Question 1.2 (10 points)

Assume θ is observable only by the firm. Characterize the optimal contract (no need to solve explicitly for the contract here, just derive the First-Order conditions of the constrained maximization problem).

Question 1.3 (6 points)

Does the optimal contract characterized in Question 1.2 prescribe always giving a loan to both types of the firm for all parameter values? Explain.

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A firm would like to invest in a risky project that requires a fixed initial investment of size k. The only way the firm can finance the initial investment is to ask for a loan to a bank. The return on the project $W \in \{\underline{W}, \overline{W}\}$, where $\overline{W} > \underline{W}$, is stochastic and depends on the effort $e \in \{0, 1\}$ exerted by the firm and on a random component, such that

$$Pr(W = \overline{W}|e = 0) = p_0 < Pr(W = \overline{W}|e = 1) = p_1.$$

The utility cost of effort to the firm is ψe, where $\psi > 0$.

In exchange for the loan, the bank requires a payment z to be paid after W is realized. The payment z can be made contingent only on observable variables. Expected utilities of firm and bank are as follows:

$$U_{firm} = E[u(W - z)] - \psi e$$
$$U_{bank} = E[z] - k$$

where $u' > 0$, and $u'' \leq 0$.

The timing is as follows: 1) the bank offers a contract to the firm; 2) the firm takes it or leaves it; 3) the firm chooses effort; 4) the contract (if signed at 2)) is executed. The outside option of the firm is normalized to 0.

The firm is NOT subject to limited liability, unless otherwise explicitly stated.

Assume also that inducing high effort is always optimal for the bank.

Question 2.1 (4 points)

Assume that the bank can observe the effort exerted by the firm, and $u'' < 0$.

Does the optimal contract provide full insurance to the firm? Explain why in math or words.

Question 2.2 (8 points)

(For this question you are not required to prove your answers in math, but you must motivate your answers in words.)

Assume that the bank CANNOT observe the firm's effort, and $u'' < 0$.

a) Is the optimal contract providing full insurance to the firm?

b) Is the ex-ante expected cost sustained by the bank in order to induce high effort different compared to Question 2.1?

Question 2.3 (8 points)

Assume that the bank CANNOT observe the firm's effort, and that $u(W - z) = W - z$. Assume that the firm is protected by limited liability, such that $z \leq W$ in both states of the world.

a) Show that the optimal contract does not provide full insurance to the firm.

b) Show also that the ex-ante expected cost sustained by the bank in order to induce high effort is higher than in Question 2.1.

PART 3. (20 POINTS)

Consider a setting with 3 states, $\{\theta_1, \theta_2, \theta_3\}$, and 3 individuals, $\{A, B, C\}$. You can denote the individuals' respective preferences as $\succeq^h$, for $h \in \{A, B, C\}$. Suppose individuals have complete and transitive preferences. Moreover, assume individuals have strict preferences among any two alternative states.

Question 3.1 (4 points)

What is a constitution in this setting?
Consider the Borda-count rule: for each individual h, rank states from individual h's least favorite to individual h's most favorite and assign to each state θ_s a number $n_h(\theta_s)$ equal to the state's position in the ranking. Then, for any two states θ and θ': $\theta \succeq \theta'$ if and only if

$$n(\theta) \geq n(\theta'), \quad \text{where } n(x) := n_A(x) + n_B(x) + n_C(x).$$

Question 3.2 (5 points)

Does the Borda-count rule ensure transitive social preferences? Would your answer be different if there where $n > 3$ instead of 3 states? Justify your answer.

Question 3.3 (5 points)

Does the Borda-count rule satisfy the No Dictatorship axiom? Justify your answer.

Question 3.4 (6 points)

Construct an example to show that the Borda-count rule does not satisfy the Independence of Irrelevant Alternatives axiom.

PART 4. (20 POINTS)

Consider an exchange economy with 2 individuals, $\{A, B\}$, and two goods, $\{1, 2\}$. Utility of individual $h \in \{A, B\}$ is denoted as:

$$U^h(x^h) = \sqrt{x_1^h} + \sqrt{x_2^h},$$

where x_i^h denotes individual h's consumption of good i. Let endowments be $R^A = \{2, 1\}$ and $R^B = \{1, 2\}$.

Question 4.1 (5 points)

What is an allocation in this setting? What is market allocation in this setting?

Question 4.2 (5 points)

Find the contract curve for the economy described.

Question 4.3 (5 points)

Find the competitive equilibrium for the economy described.

Question 4.4 (5 points)

Assume now that there are 2 "type A" individuals and 2 "type B" individuals. Every individual's endowment and utility function is identical to the one in the economy described above (so the overall endowment is 6 units of good 1, and 6 units of good 2). Find one allocation that belongs to the core of this new economy.