

Seminar III

*Problem 1.*

Paul is an entrepreneur with a business idea he wants to develop: buying a property near the Geilo skiing resort and turning it into an area with mountain cabins, in Norwegian a *hytteby*.

The project is risky. In particular, we assume that there is probability  $p$  that it is successful and a corresponding probability  $(1 - p)$  that it fails. A success means that an investment of an amount  $I$  returns  $RI$ . Failure means a zero return on the project.

The probability  $p$  of success depends on Paul's own efforts in making it a success. In particular, if Paul works hard, there will be a probability  $p = p_H$  of success, while if he does not work hard, this probability is  $p = p_L$ , where  $0 < p_L < p_H < 1$ , and  $p_H R > 1 > p_L R$ . By working hard, he will, however, suffer a loss  $BI$  proportional to the size of the project, where  $0 < B < 1 - p_L R$ .

Paul has available own funds of size  $A$  for this project. If he wants to invest more than  $A$ , he will need funding from outside investors. We assume that the capital market is competitive, and that limited liability prevails. Because efforts are not observable, it is not possible to contract upon effort.

a)

- i. Define the concept of *borrowing capacity*, or *debt capacity*, and find an expression for it in the case of Paul described above. Discuss how the borrowing capacity is affected by the extent of the moral-hazard problem.
- ii. What would change if, instead of the above constant-returns-to-scale investment technology, the project featured decreasing returns to scale?

- iii. Also, find an expression for Paul's *shadow value of equity*, and discuss how it is affected by the extent of the moral-hazard problem.
- b) Due to difficulties with getting the funding he wants, Paul seeks the advice of a friend, who recommends splitting the development in two, with a first-phase project whose returns  $RI_1$  is obtained from an investment  $I_1$ , in case of success, before the decision to launch phase two, with returns  $RI_2$  obtained from an investment  $I_2$  in case of success. Each of the two phases have the same success probabilities and the same moral-hazard problem as detailed above. Let us also assume that the two projects are statistically independent, and that long-term contracts are not available.
- i. Do you agree with Paul's friend that this sequential development could help on the funding?
  - ii. What would change if long-term contracts were available?
  - iii. How does the result hinge on the projects being statistically independent? To what extent is it reasonable to assume imperfect correlation in the particular case of Paul's property development?
- c) Paul realizes that there is a risk for a cost over-run in the project. In particular, there is a need, before the project is completed, for a reinvestment equal to  $\rho I$ , where  $I$  is the initial investment, and  $\rho$  is distributed according to the probability distribution  $F(\rho)$  on  $[0, \infty)$ , with density  $f(\rho)$ . The moral-hazard problem, in case of a reinvestment and completion of the project, is as detailed above.
- i. Discuss how this need for intermediate funds, in order to complete the project, can be dealt with in the initial contract. Explain, in particular, how the risk of a cost over-run calls for a smaller project than otherwise called for.

*Problem 2.*

Review Problem 4, parts (i)-(iii), in Tirole, pp. 627-628.