Further determinants of borrowing capacity: Boosting pledgeable income

- Diversification: more than one project
- Collateral: pledging real assets
- Liquidity: a first look
- Human capital

Diversification

- It may be beneficial for a firm, in terms of getting hold of external funds, to have several projects.
- Equivalently, it may be beneficial for multiple project owners to merge into one firm.
- Previous analysis: constant returns to scale in investment technology
- Expansion in investment project equivalent to an increase in the number of projects whose outcomes are perfectly correlated.
- Consider the opposite extreme: Several projects are available, and they are statistically independent.
- *Cross pledging*: Incomes on one successful project can be offered as "collateral" for other projects.
- Model: Two identical projects. Otherwise: as in the fixed-investment model
- Entrepreneur's initial wealth per project: A; *i.e.*, total wealth: $2A$.

- A benchmark: project financing. For each of the two projects:
  - Borrower receives R_b if success, 0 otherwise.
  - Incentive constraint: $R_b \geq \frac{B}{\Delta p}$
  - Breakeven constraint: $p_H\left(R - \frac{B}{\Delta p}\right) \geq I - A$, or: $A \geq \overline{A}$.
  - Project financing not viable if $A < \overline{A}$.
- *Cross pledging*
  - The two projects financed in combination
  - Contract: Borrower receives R_0, R_1, or R_2 when 0, 1, or 2 projects are successful.
  - Expected return to borrower:
$$p_H^2 R_2 + 2p_H(1-p_H)R_1 + (1-p_H)^2 R_0$$
  - Two incentive constraints:
    - Working on two projects preferred to working on only one
$$p_H^2 R_2 + 2p_H(1-p_H)R_1 + (1-p_H)^2 R_0 \geq$$
$$p_H p_L R_2 + [p_H(1-p_L) + p_L(1-p_H)]R_1 + (1-p_H)(1-p_L)R_0 + B$$
    - Working on two projects preferred to working on none
$$p_H^2 R_2 + 2p_H(1-p_H)R_1 + (1-p_H)^2 R_0 \geq$$
$$p_L^2 R_2 + 2p_L(1-p_L)R_1 + (1-p_L)^2 R_0 + 2B$$
  - Clearly, $R_0 = 0$ in equilibrium, as before.

- *Full cross pledging*: We also have $R_1 = 0$ in equilibrium.
  - In order to increase the borrowing capacity, the borrower offers all returns that are available in those cases where only one project succeeds.
  - We can simplify the incentive constraints.
  - Working on both projects better than on none:

$$p_H^2 R_2 \geq p_L^2 R_2 + 2B \iff$$

$$(p_H^2 - p_L^2) R_2 \geq 2B \iff$$

$$(p_H + p_L) R_2 \geq 2\frac{B}{\Delta p} \iff$$

$$\frac{p_H + p_L}{2} R_2 \geq \frac{B}{\Delta p}$$

  - Working on both projects better than on a single one:

$$p_H^2 R_2 \geq p_H p_L R_2 + B \iff$$

$$p_H R_2 \geq \frac{B}{\Delta p}$$

  - This one is always satisfied when the previous one is.
  - It follows that, in equilibrium, $R_2 \geq \dfrac{2B}{(p_H + p_L)\Delta p}$

- Minimum expected payoff to borrower:

$$p_H^2 R_2 \geq \frac{2 p_H^2 B}{(p_H + p_L)\Delta p} = 2(1 - d_2) \frac{p_H B}{\Delta p},$$

where $d_2 = \dfrac{p_L}{p_H + p_L} \in \left(0, \dfrac{1}{2}\right)$ is an agency-based measure of the *economies of diversification* into two independent projects.

- The breakeven constraint:
  - Expected pledgeable income ≥ investors' expenses

$$2p_HR - 2(1 - d_2)\,\frac{p_HB}{\Delta p} \geq 2I - 2A \Leftrightarrow$$

$$p_HR - (1 - d_2)\,\frac{p_HB}{\Delta p} \geq I - A \Leftrightarrow$$

$$A \geq \overline{\overline{A}},\text{ where } \overline{\overline{A}} = I - p_H\left[R - (1 - d_2)\frac{B}{\Delta p}\right] < \overline{A}$$

  - Recall: $\overline{A} = p_H\,\frac{B}{\Delta p} - (p_HR - I) = I - p_H\left[R - \frac{B}{\Delta p}\right]$

- *Diversification and cross pledging* facilitates financing: $\overline{\overline{A}} < \overline{A}$

- *Statistical independence* of projects similarly facilitates financing.

- *Variable investment*: Diversification increases the borrowing capacity, rather than giving better access to financing.

- Extension to n independent projects: Let borrower have net worth nA. Breakeven constraint for investors now becomes:

$$p_HR - (1 - d_n)\,\frac{p_HB}{\Delta p} \geq I - A,$$

where $d_n = \dfrac{p_L\left(p_H^{n-1} - p_L^{n-1}\right)}{p_H^n - p_L^n}$ increases with n.

- Limits to diversification
  - Endogenous correlation: The borrower has an incentive to choose correlated projects, if she can. This decreases the value of cross pledging. → *Asset substitution*.
  - Limited expertise.
  - Limited attention.

- Sequential projects
    - Supplementary section 4.7
    - Variable investment in two projects.
    - Benchmark: simultaneous projects
        - Investment I_i in project $i \in \{1, 2\}$.
        - Return RI_i if success in project i, 0 otherwise
        - Probability of success p_H (p_L) if the borrower behaves (misbehaves)
        - Private benefit from misbehaving in project i: BI_i.
        - Total investment: $I = I_1 + I_2$.
    - Optimal with reward only when both projects succeed: R_b.
    - Binding incentive constraint: misbehavior on both projects

$$p_H^2 R_b \geq p_L^2 R_b + BI$$

        - We disregard misbehavior on one project for now
    - Total net present value: $(p_H R - 1)I$
    - Investors' breakeven constraint:

$$p_H RI - p_H \frac{BI}{p_H^2 - p_L^2} = I - A$$

    - In equilibrium,

$$I = \frac{A}{1-\hat{\rho}_0}, \text{ where}$$

$$\hat{\rho}_0 = p_H \left(R - \frac{p_H}{p_H + p_L} \frac{B}{\Delta p} \right) = p_H \left[R - (1 - d_2) \frac{B}{\Delta p} \right], \text{ and}$$

$$U_b = (p_H R - 1)I = \frac{\rho_1 - 1}{1 - \hat{\rho}_0} A$$

- Checking the other incentive constraint: misbehavior on project *i*:

$$p_H^2 R_b \geq p_H p_L R_b + BI_i$$

- Combining with the other incentive constraint:

$$\frac{I_i}{I} \leq \frac{p_H}{p_H + p_L}$$

  - This constraint does not bind if total investment is split relatively equally among the two projects

- Sequential projects: Short-term loan agreements
  - Financing one project at the time.
  - Increased incentives early on: success at the first project provides the borrower with extra funds for the second project.
  - Think ahead and reason back.
  - Project 2: the single-project variable-investment case, with the borrower entering date 2 with assets A_2.
  - Expected payoff per unit of investment: $\rho_1 = p_H R$
  - Expected pledgeable income per unit of investment:

$$\rho_0 = p_H\left(R - \frac{B}{\Delta p}\right)$$

  - Borrower's gross utility from project 2:

$$vA_2 = \frac{\rho_1 - \rho_0}{1 - \rho_0} A_2$$

  - $v > 1$ is the *shadow value of equity*: If you can increase your assets at the start of date 2 with 1 unit, then you increase your utility with v.

- Project 1: Borrower's initial assets A. Return if success: $RI_1 = R_b + R_l$
- Investors' breakeven constraint

$$P_H R_l \geq I_1 + A$$

- Borrower's incentive constraint: $vR_b \geq \frac{BI_1}{\Delta p}$
- Expected pledgeable income per unit of investment

$$\tilde{\rho}_0 = p_H\left(R - \frac{B}{v\Delta p}\right) = \rho_1 - \frac{\rho_1 - \rho_0}{v} = \rho_1 + \rho_0 - 1.$$

- *Debt capacity* at date 1: $I_1 = k_1 A$, where

$$k_1 = \frac{1}{1-\tilde{\rho}_0} = \frac{1}{2-\rho_0-\rho_1} > \frac{1}{1-\rho_0} = k$$

- Assume $\frac{\rho_0+\rho_1}{2} < 1$; otherwise, debt capacity is infinite.
  - Recall earlier assumption: $\rho_1 > 1 > \rho_0$.
- The borrower invests in project 2 if and only if project 1 is successful. She then invests:

$$I_2 = kA_2 = kR_b = \frac{kB}{v(\Delta p)} I_1 =$$

$$\frac{\frac{1}{1-\rho_0}B}{\frac{\rho_1-\rho_0}{1-\rho_0}\Delta p} I_1 = \frac{B}{p_H \frac{B}{\Delta p}\Delta p} I_1 = \frac{1}{p_H} I_1$$

- Expected investments in the projects are the same:

$$p_H I_2 = I_1$$

- *Stakes increase over time*: $I_2 > I_1$

- Sequential vs simultaneous projects

$$U_b^{seq} = p_H v A_2 - A = (p_H v \frac{B}{v(\Delta p)} k_1 - 1)A$$

$$U_b^{seq} = \frac{2(\rho_1 - 1)}{2 - \rho_0 - \rho_1} A > \frac{\rho_1 - 1}{1 - \hat{\rho}_0} A = U_b^{sim}$$

$$\Leftrightarrow \hat{\rho}_0 < \frac{\rho_0 + \rho_1}{2} \Leftrightarrow d_2 = \frac{p_L}{p_H + p_L} < \frac{1}{2}$$

  - Note error in Tirole, p. 186.

- Sequentiality is better: The borrower has no chance to misbehave on project 2 if project 1 fails, so the moral hazard problem is less serious.
- Long-term loan agreements
  - One agreement for both projects
  - Risk neutrality and constant returns to scale imply that short-term agreements fair equally well.

Collateral

- Assets = cash + productive assets
- Productive assets = quasi-cash, since they may be *pledged as collateral* to lenders
- *Redeployability* of productive assets
  - Fixed-investment model, with one new feature.
  - Suppose, after investment is made but before effort is put in, it becomes publicly known whether the project is *viable*
  - With probability x, the project is viable and the model proceeds as before
  - With probability $(1-x)$, the project is not viable, and assets can be sold at a given price $P \leq I$.
  - *Economic distress*, as opposed to financial distress.
  - New assumption on NPV: $xp_H R + (1-x)P > I$.
  - The entrepreneur chooses to pledge the resale price in full.
  - Breakeven constraint for investors:

$$xp_H\left(R - \frac{B}{\Delta p}\right) + (1-x)P \geq I - A$$

  - Threshold level of net worth:

$$\overline{A} = xp_H \frac{B}{\Delta p} - \left[xp_H R + (1-x)P - I\right]$$

    - Decreases with asset redeployability
  - *Borrowing patterns across industries*: The more liquid assets, the easier it is for firms borrow.
  - Endogenous redeployability: *fire sale externalities* – further aggravating credit rationing.

Collateral is costly

- A deadweight loss associated with collateralization: assets may have lower value for lenders than for the borrower
  - Transaction costs
  - Borrower's private benefit from ownership: sentimental values, specific skills
  - Prospects of future credit rationing makes the asset of higher value to the borrower than to investors
  - Risk aversion
  - Collateralized assets may receive poor maintenance

Costly collateral and contingent pledging

- Suppose first collateral would not exist without the investment.
- Borrower has no cash initially, needs to borrow I.
- Asset has residual value
  - A to the entrepreneur
  - $A' \leq A$ to the lenders
  - Deadweight loss if asset is seized: $A - A'$
- Contract: $\{R_b, R_l, y_S, y_F\}$
  - y_S – probability that the borrower keeps the asset if success
  - y_F – … if failure
  - *stochastic pledging*: needed in a simple model
- Otherwise, fixed-investment model.

- The equilibrium contract is the one that maximizes borrower's utility, subject to borrower's incentive-compatibility constraint and lenders' breakeven constraint.

  Max $U_b = p_H(R_b + y_S A) + (1 - p_H)y_F A$

  subject to

  $\Delta p[R_b + (y_S - y_F)A] \geq B$, and

  $p_H[R_l + (1 - y_S)A'] + (1 - p_H)(1 - y_F)A' \geq I$

- Borrower wants to pledge as little collateral as possible
- The outcome depends on *the strength of the balance sheet* of the borrower
  - o Strength of balance sheet depends on
    - ▪ Investment level I (–)
    - ▪ Agency costs, measured by $p_H \dfrac{B}{\Delta p}$ (–)
    - ▪ Any initial cash, $\tilde{A}$ (+)
  - o *Strong balance sheet* – no collateral

    $y_S = y_F = 1;\ R_b > 0.$
  - o *Intermediate balance sheet* – collateral if failure:

    $y_S = 1,\ y_F \leq 1;\ R_b \geq 0.$
  - o *Weak balance sheet* – borrower gets a share of the asset if success:

    $y_S \leq 1,\ y_F = 0;\ R_b = 0.$
    - ▪ *Contingent pledging*: borrower gets a contingent share of the asset rather than of income.

Solution: derivative of the Lagrangian with respect to y_S is positive if that with respect to R_b or that with respect to y_F is. Some of the three regimes may not exist.

- *Weak borrowers pledge more collateral than strong borrowers*
  - Pledging collateral in lack of cash
  - Opposite prediction from adverse-selection theories, where strong firms pledge collateral to show strength.

Pledging existing assets

- Suppose next that the entrepreneur has existing wealth
- *Contingent pledging*
  - If success, the entrepreneur keeps the asset.
  - If failure, the investors receive the collateral.
- Continuous collateral: the entrepreneur chooses an amount $C \in [0, C^{max}]$ to pledge as collateral in case of failure.
  - We need an upper limit on C^{max}; see below.
- Costly collateral: Value βC to investors, where $\beta < 1$.
- Borrower's net utility: Project's NPV without collateral minus expected deadweight loss from pledging collateral.

$$U_b = p_H R - I - (1 - p_H)(1 - \beta)C$$

  - To ensure that $U_b \geq 0$ for any feasible C, we assume

$$C^{max} \leq \frac{p_H R - I}{(1 - p_H)(1 - \beta)}$$

- Collateral costly $\Rightarrow C = 0$ if $A \geq \overline{A}$.

- The borrower's incentive compatibility constraint

$$p_H R_b - (1 - p_H)C \geq p_L R_b - (1 - p_L)C + B \Leftrightarrow$$
$$R_b + C \geq \frac{B}{\Delta p}$$

  - The borrower loses both the reward and the collateral when she fails
  - *Limited liability*: In order to ensure that $R_b \geq 0$ for any feasible C, we assume:

$$C^{max} \leq \frac{B}{\Delta p}$$

- The investors' breakeven constraint

$$p_H (R - R_b) + (1 - p_H)\beta C \geq I - A \Leftrightarrow$$

$$p_H \left(R - \frac{B}{\Delta p}\right) + p_H C + (1 - p_H)\beta C \geq I - A$$

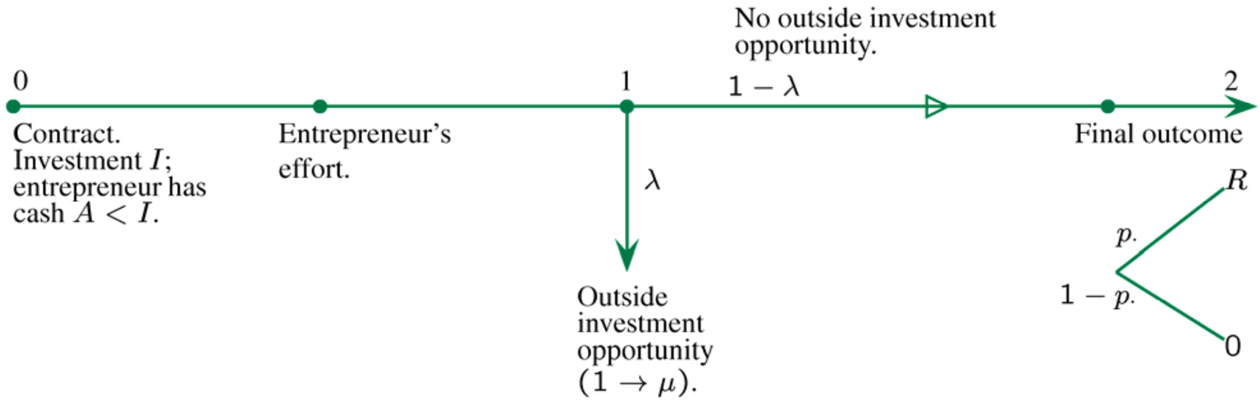
- Collateral has two ways of affecting pledgeable income
  - Directly: $+ (1 - p_H)\beta C$
  - Indirectly through a lower reward to borrower: $+ p_H C$
- Borrower pledges the minimum collateral necessary to satisfy the investors' breakeven constraint:

$$C = \frac{I - A - p_H\left(R - B/\Delta p\right)}{p_H + (1 - p_H)\beta}$$

  - … except if this expression gets too big, in which case collateral cannot solve the funding problem.
- Weaker firms pledge more collateral: $\frac{dC}{dA} < 0$.
- Conditional collateral preferable to unconditional.
- More abstract forms of collateral: Putting one's job at stake.

The liquidity-accountability tradeoff

- When should the borrower receive her compensation?
  - Towards the end: good for accountability, because more information about the project is available
  - Along the way, because of her need for liquidity
    - Consumption
    - New projects
- Outside investment opportunities not observable for investors
  - A scope for "strategic exit", escaping sanctions following poor performance
- The other side of the coin: the liquidity of investors
  - The more control you have, the less liquid your assets are
- Model: an extension of the fixed-investment one



- New feature: A new, fleeting investment opportunity at an intermediate date
- Initial investment I, entrepreneur's assets $A < I$.

- Moral hazard: misbehavior means a lower success probability ($p_L < p_H$) but also a private benefit B.
- Project returns at final date: R or 0 (whether or not an intermediate investment opportunity shows up).
- Limited liability, risk neutrality.
- Project would have been financed in the absence of the intermediate liquidity needs:

$$A > \overline{A}$$

- *Liquidity shock*: With probability λ, a new investment opportunity arises.
    - Investing x returns μx, where $\mu > 1$.
- Contract: $\{r_b, R_b\}$. Borrower receives
    - r_b on the intermediate date and nothing on the final date, in the case of a liquidity shock.
    - R_b on the final date if success (0 if failure) and nothing on the intermediate date, in the case of no liquidity shock.
- What if the liquidity shock is not verifiable?
- *Exit vs vesting*: what about *partial vesting*? – Some cash at the intermediate date and some payment at the final date (if success).
- Implementation: where does r_b come from? – Needs to be subtracted from pledgeable income.

- Benchmark case: *Verifiable liquidity shock*
- Borrower's incentive compatibility constraint

$$\lambda\mu r_b + (1-\lambda)p_H R_b \geq \lambda\mu r_b + (1-\lambda)p_L R_b + B \Leftrightarrow$$

$$(1-\lambda)(\Delta p)R_b \geq B \Leftrightarrow$$

$$R_b \geq \frac{1}{1-\lambda}\frac{B}{\Delta p}$$

  - No incentive effect from r_b.
  - Only effect of the liquidity shock is that the borrower's stake must be increased, since final date is reached only with probability $(1-\lambda)$.

- Borrower receives r_b with probability λ. So this is similar to no liquidity shock, but the entrepreneur having available $A - \lambda r_b$.
- Expected pledgeable income:

$$p_H R - \{\lambda r_b + (1-\lambda)\frac{1}{1-\lambda}\frac{B}{\Delta p}\} = p_H\left(R - \frac{B}{\Delta p}\right) - \lambda r_b.$$

- Competition among investors ensures that the borrower gets the NPV from the project. So her total expected net utility is

$$U_b = p_H R - I + \lambda(\mu - 1)r_b.$$

- It is optimal to have r_b as high as possible subject to incentive compatibility:

$$p_H\left(R - \frac{B}{\Delta p}\right) - \lambda r_b = I - A$$

- In equilibrium: $r_b = \frac{1}{\lambda}\left[p_H\left(R - \frac{B}{\Delta p}\right) - (I - A)\right]$; $R_b = \frac{1}{1-\lambda}\frac{B}{\Delta p}$.

- *Non-verifiable liquidity shock*
- A *two-dimensional moral-hazard problem*. Incentives needed for borrower
  - to behave in carrying out the project, and
  - to report truthfully about the liquidity shock
- The two forms of moral hazard interact
  - *Strategic exit*: A misbehaving borrower may want to exit even without a liquidity stock before the consequences are disclosed.
- Simplifying assumption: $p_L = 0 \;\Rightarrow\; \Delta p = p_H$
  - A misbehaving borrower would indeed want to cash out early, since there is nothing to be had later: $p_L R_b = 0$.
- Borrower's incentive constraint

$$\lambda\mu r_b + (1-\lambda)p_H R_b \geq [\lambda\mu + (1-\lambda)]r_b + B \Leftrightarrow$$

$$(1-\lambda)[p_H R_b - r_b] \geq B \Leftrightarrow$$

$$(1-\lambda)[(\Delta p)R_b - r_b] \geq B \Leftrightarrow$$

$$R_b \geq \frac{r_b}{\Delta p} + \frac{1}{1-\lambda}\frac{B}{\Delta p}$$

- Compare with the case of verifiable liquidity shock: the possibility of a strategic exit makes the incentive constraint stricter (for a given $r_b > 0$).
- When there is no liquidity shock, the borrower strictly prefers to continue: $p_H R_b > r_b$.
- But would the borrower want to cash out when there *is* a liquidity shock? Is $\mu r_b \geq p_H R_b$? – Suppose first that it is.

- Again, competition among investors ensures that all NPV of the project accrues to the borrower. So, given r_b, her expected net utility is:

$$U_b = p_H R - I + \lambda(\mu - 1)r_b.$$

- But the incentive constraint is stricter, so pledgeable income is smaller. Therefore r_b is lower when liquidity shock is nonverifiable.

- Expected pledgeable income for a given r_b:

$$p_H R - \left\{ \lambda r_b + (1-\lambda) p_H \left[\frac{r_b}{\Delta p} + \frac{1}{1-\lambda} \frac{B}{\Delta p} \right] \right\} = p_H \left(R - \frac{B}{\Delta p} \right) - r_b$$

- In equilibrium:

$$r_b = p_H \left(R - \frac{B}{\Delta p} \right) - (I - A); \; R_b = \frac{1}{1-\lambda} \frac{B + (1-\lambda) r_b}{\Delta p}$$

- Compared to the case of verifiable liquidity shock:

  r_b is lower, R_b is higher.

  - The possibility of strategic exit hurts the borrower, since she is allowed less liquidity.

- If the above contract does not obey $\mu r_b \geq p_H R_b$:
  - Happens when A is low.
  - Solution: *partial vesting.* Only implementation changes.
    - Total compensation has two components: One, a basis compensation, R_b^0, payed out in case of success.
    - At the intermediate date, the borrower receives cash r_b. She can choose to buy shares for this, which would pay ΔR_b in case of success, where

$$R_b^0 + \Delta R_b = R_b$$

Inalienability of human capital

- Is there a scope for the loan contract to be *renegotiated* as the project proceeds?
- A *renegotiation* must mean that the existing contract is not efficient for the parties involved – that a new contract exists that is weakly better for both borrower and lender, and strictly better for at least one of them.
- *Hold-up*: Suppose the entrepreneur is *indispensable* – the project cannot be completed without her. The entrepreneur may want to renegotiate the initial contract in order to obtain a better deal.
  - The *inalienability of human capital*.
- Model: no moral hazard: $B = 0$; no cash: $A = 0$.
- Otherwise, fixed-investment model.
- The act of "completing the project" cannot be contracted upon until after investment has been made: Renegotiation is needed.
  - Renegotiation replaces effort as the source of the incentive problem.
- Incomplete project returns 0.
- Complete project returns R [prob p_H] or 0 [prob $(1 - p_H)$].
- Disregarding renegotiation, the project can be financed by a debt contract: borrower pays investors D in case of success, such that $p_H D = I$.
  - $R_l = D$, $R_b = R - D$, and $U_b = p_H(R - D) = p_H R - I$.
- Renegotiation: Bargaining over $p_H R - I$.

- Who has *bargaining power*?
    - No longer competition among creditors: lender has b.p.
    - Entrepreneur is indispensable: borrower has b.p.
    - Both receive 0 in case of noncompletion of project
- Lender's bargaining power: θ
    - In the renegotiation, lender receives θR in case of success, and borrower receives $(1 - \theta)R$.
    - Lender willing to invest if $\theta p_H R \geq I$.
    - If $\theta > D/R$, then the borrower prefers to simply skip the renegotiation and complete the project.
    - If $\theta < D/R$, then $\theta p_H R < p_H D = I$: the project will not be financed.
    - If the borrower is too indispensable, the project is not carried out.
- Determinants of bargaining power
    - Reputations on both sides
    - Dispersion of lenders
    - Outside options
- If possible, the borrower may want to give the lenders the right to seize the firm's assets – in order to secure some external finance.
- A parallel to collateral – the value of the collateral may depend on how indispensable the entrepreneur is.