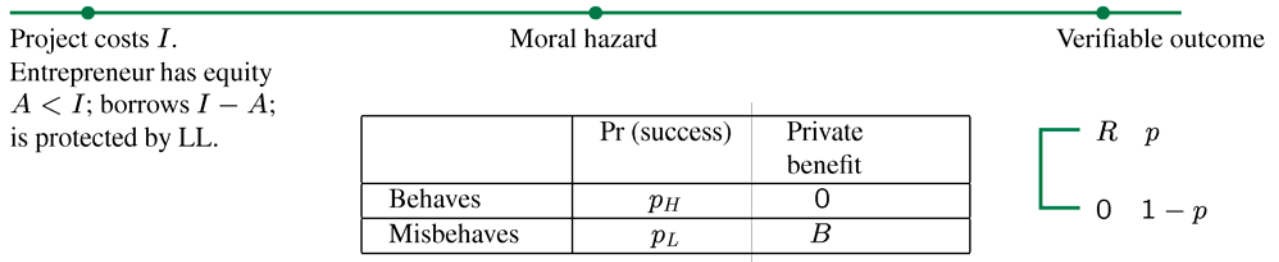


Credit rationing

- Loan markets are special
 - Personalized
 - Clearing through *both quantities and prices*
- This is because of private information among borrowers
 - Adverse selection: There are both good and bad firms out there, and banks cannot tell who is who.
 - Moral hazard: Banks cannot observe actions taken by firms.
- Increasing the interest rate makes the borrower care less about the project that is being financed.
 - Lower borrower's income in the absence of bankruptcy
 - No effect on her income in case of bankruptcy
- Moral hazard: a reduced stake reduces incentives
- Adverse selection: an increased interest rate attracts low-quality borrowers
- In equilibrium, borrowers may be *rationed*.
- In order to get outside financing, you may need own funds.

A simple model: Fixed investment



- A risk neutral entrepreneur has a project requiring a fixed investment I .
 - If success: project return is R . If failure, return is 0.
- The entrepreneur has own funds $A < I$.
 - $A = \text{net worth}$, or cash on hand.
- She needs to borrow $I - A$ to carry out the project.
- Project is risky, and success depends on entrepreneur's effort.
 - Misbehaving lowers the success probability of this project ($p_L < p_H$), but creates private benefits B to the entrepreneur.
 - $\Delta p = p_H - p_L$.
- Assume project is viable *if and only if* entrepreneur behaves
 - *Net present value* (NPV) if she behaves: $p_H R - I > 0$.
 - NPV if not: $p_L R - I + B < 0$.
 - In combination: $\frac{p_H R}{I} > 1 > \frac{p_L R}{I - B}$
 - No loan will be granted that induces misbehavior.
- Loan contract: If success, borrower gets R_b , lender $R_l = R - R_b$.
- *Limited liability*: If failure, both receive 0.

- Lenders are risk neutral and behave competitively.
- Competition among lenders implies $p_H R_l = I - A \Rightarrow R_l = \frac{I - A}{p_H}$.
- The interest rate is given by: $R_l = (1 + \iota)(I - A) \Rightarrow 1 + \iota = \frac{1}{p_H}$.
 - For $p_H < 1$, there is a default premium: $\iota > 0$.
- Are lenders interested at these terms? – Credit analysis.
 - Need to preserve borrower a sufficient stake in order to induce incentives
 - The *incentive compatibility constraint*

$$p_H R_b \geq p_L R_b + B \Rightarrow R_b \geq \frac{B}{\Delta p}$$

- What the borrower gets from behaving must be more than what she gets from misbehaving
- There is a lower limit on the borrower's return
 - Increasing in the private benefits B .
 - Decreasing in the effect of behaving Δp .
- The maximum income that can be *pledged to lenders* without inducing misbehavior is

$$R - \frac{B}{\Delta p}$$

- *Expected pledgeable income* is therefore

$$P = p_H \left(R - \frac{B}{\Delta p} \right)$$

- Lenders' *individual rationality constraint*

$$p_H \left(R - \frac{B}{\Delta p} \right) \geq I - A$$

- Expected pledgeable income must exceed lenders' initial expenses
- Other names
 - breakeven constraint
 - participation constraint
- A *necessary (and sufficient) condition* for financing of the entrepreneur's project
- Minimum level of own funds in order to get outside financing

$$A \geq \bar{A} = p_H \frac{B}{\Delta p} - (p_H R - I)$$

- Assumption:

$$\bar{A} > 0 \Leftrightarrow (0 <) p_H R - I < p_H \frac{B}{\Delta p} \quad (*)$$

- Otherwise, even a borrower without any wealth of her own would get credit
- NPV of project is less than the minimum that must be left to the borrower in order to ensure incentives.
- A project may have $NPV > 0$, and still not get funded
 - This happens in cases where $A < \bar{A}$.
 - "One only lends to the rich".

- The *agency rent*: what must be left to the borrower to ensure incentives

$$p_H \frac{B}{\Delta p}$$

- The condition $A \geq \bar{A}$ says that agency rent net of borrower's own input must be less than the project's NPV

$$p_H \frac{B}{\Delta p} - A \leq p_H R - I$$

- The borrower's net utility

$$U_b = 0, \quad \text{if } A < \bar{A};$$

$$= p_H R_b - A = p_H(R - R_l) - A = p_H R - I, \quad \text{if } A \geq \bar{A}.$$

- The borrower gets the entire net present value, if only she can get the project funded.

- Determinants of credit rationing

- Little cash on hand (low A)
- High agency costs (high $p_H \frac{B}{\Delta p}$).

- Moral hazard determined by two factors

- The extent of private benefits from misbehavior: B
- The extent to which the verifiable final outcome reveals misbehavior

- Informativeness measured by the *likelihood ratio*

$$\frac{\Delta p}{p_H} = \frac{p_H - p_L}{p_H}$$

- Is this debt or equity?
 - Debt: Entrepreneur owes R_l and must pay this or go bankrupt
 - Equity: Entrepreneur and investor own R_b/R and R_l/R each in the firm.
- A few dynamic considerations
 - A second investment (sec. 3.2.4)
 - Dilution of initial lenders' claim
 - Overinvestment
 - Argument for a negative *debt covenant* prohibiting further debt
 - Reputational capital (sec. 3.2.5)
 - The borrower would gain by a lowering of private benefits B .

$$b < B \Rightarrow \bar{A}(b) < \bar{A}(B)$$
 - A more reliable borrower is more likely to get loan
 - Two benefits of successful projects today
 - Increased retained earnings: A higher
 - Improved reputation: (lenders' perception of) B lower

Relative performance evaluation

- Making agents accountable for events they have no control over weakens incentives in general
- One should always try to make use of the most precise measurement of the agent's performance – the *sufficient statistic* (Holmström, 1979).
- Benchmarking
- Reinterpreting the model in terms of benchmarking
 - Three states of nature
 - Favorable state (probability p_L): Project will succeed whatever the entrepreneur does.
 - Unfavorable state (probability $1 - p_H$): Project will fail whatever the entrepreneur does.
 - Intermediate state (probability $\Delta p = p_H - p_L$): Success not guaranteed but will result if entrepreneur behaves.
 - No-one knows the true state. But lenders can – say, by looking at other firms in the same industry – learn whether or not the state is favorable.
 - Contract: Entrepreneur receives nothing in the favorable state; otherwise, she receives R_b if success.
 - Incentive compatibility constraint is the same: $R_b \geq \frac{B}{\Delta p}$
 - But *pledgeable income* is increased, since entrepreneur is not paid for being lucky: $p_H R - \Delta p \frac{B}{\Delta p} = p_H R - B$.

Debt overhang

- Project is profitable, but entrepreneur is unable to raise funds because of previously incurred debt
- Two interpretations
 - Previous investors have collateral claims that reduce net worth A to below the threshold level \bar{A} .
 - Previous debt needs to be renegotiated in order to enable new investments.

Previous debt reduces net worth

- Suppose the entrepreneur has A in cash but owes D to the initial investors.
- Initial investors insisted on a covenant specifying that further loans require their consent
- The assets A are pledged as collateral to initial investors in case of default.
- Let $A > \bar{A} > A - D \geq 0$.
- The new project would have been undertaken in absence of previous debt but is not undertaken, because the investors (old and new together) cannot recoup their expenses $(I - A)$ plus the previous debt (D), since $A - D < \bar{A}$, but they can get D by seizing the collateral, since $A \geq D$.

Lack of renegotiation with previous lenders

- Suppose the borrower has no cash: $A = 0$
- But $\bar{A} < 0$: the project would be able to attract funds even without any net worth for the borrower.
- The borrower has already a long-term debt D , which is due later.
- The problem cannot be overcome by the expected profitability of the new project: The slack in pledgeable income, $-\bar{A}$, is smaller than what has to be paid back to previous investors, $p_H D$, if the project is funded:

$$p_H D > -\bar{A} \Leftrightarrow \bar{A} + p_H D > 0$$

- Initial investors may want to put in more funds, since they get nothing in case of bankruptcy now ($A = 0$).
- But what if initial investors have no funds available? Are new investors willing? The problem is that old debt is senior, and that the borrower needs to keep a minimum stake in the project to ensure incentives; so expected pledgeable income is

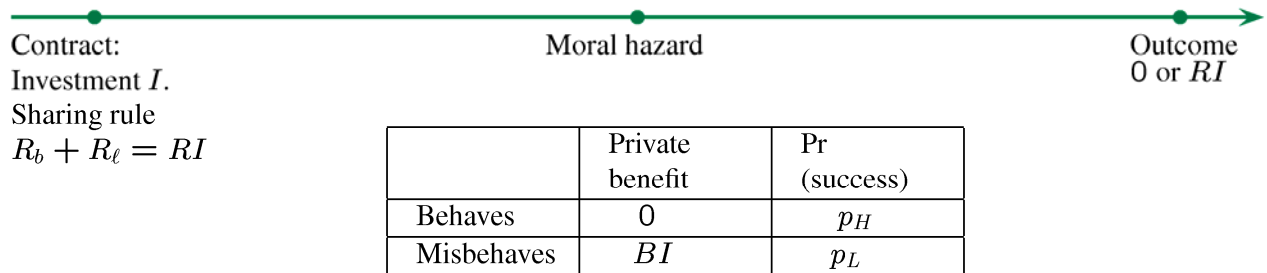
$$p_H \left(R - \frac{B}{\Delta p} - D \right)$$

- New investors are willing to fund if and only if:

$$p_H \left(R - \frac{B}{\Delta p} - D \right) \geq I \Leftrightarrow \bar{A} + p_H D \leq 0$$

- This contradicts the assumption above.
- It is impossible to raise funds from new investors unless some debt forgiveness is renegotiated with old investors.

Borrowing capacity: a variable-investment model



- Constant returns to scale in investment: Investing $I \geq 0$ yields a return RI if success, 0 if failure, with $R > 0$.
- Borrower's private benefit from misbehaving: BI , with $B > 0$.
- Borrower can choose to behave or not.
- Borrower's cash: A ; must borrow $I - A$ to invest I .
- Loan contract: $\{R_b, R_l\}$, where $R_b + R_l = RI$.
- Assume project is profitable if and only if borrower behaves

$$p_H R > 1 > p_L R + B$$

- ... but that NPV per unit of investment is less than agency costs per unit

$$p_H R - 1 < \frac{p_H B}{\Delta p}$$

- Equivalent to the $\bar{A} > 0$ assumption in the fixed-investment model
- Needed here to ensure equilibrium investment being finite, because of the constant-returns-to-scale technology.

- Lenders behave competitively
- Lenders' credit analysis
 - Incentive compatibility: $R_b \geq \frac{BI}{\Delta p}$
 - Breakeven: $p_H(RI - R_b) \geq I - A$
 - Borrower's net utility: $U_b = (p_H R - 1)I$
 - The borrower would like as much funding as possible.
- The equity multiplier
 - Determined by incentive compatibility and breakeven constraints. Combining them, we get

$I \leq kA$, where

$$k = \frac{1}{1 - p_H \left(R - \frac{B}{\Delta p} \right)} > 1.$$

- The borrower can lever her wealth, with the *equity multiplier* k .
- The *equity multiplier* is smaller, the higher is the private benefit B , and the lower is the likelihood ratio $\Delta p/p_H$ – our two measures of agency cost.

- The entrepreneur's *borrowing capacity*.
 - Outside financing capacity; debt capacity
 - It is possible for the borrower to invest k times her cash A , that is, to borrow $d = k - 1$ times her cash, where

$$d = \frac{p_H \left(R - \frac{B}{\Delta p} \right)}{1 - p_H \left(R - \frac{B}{\Delta p} \right)}.$$

- The maximum loan, dA , is the *borrowing capacity*.
 - The borrowing capacity
 - increases with per-unit return R
 - decreases with the extent of the agency problem
- The *shadow value of equity*
 - Borrower's gross utility: $U_b^g = A + U_b$
 - Combine $U_b = (p_H R - 1)I$ and $I = kA$ to get:

$$U_b^g = \nu A, \text{ where } \nu = \frac{p_H \frac{B}{\Delta p}}{1 - p_H \left(R - \frac{B}{\Delta p} \right)} > 1$$

- The shadow value of equity ν
 - increases in the per-unit return R
 - decreases in the extent of the agency problem

- Useful notation
 - *Expected payoff per unit of investment:* $\rho_1 = p_H R$
 - *Expected pledgeable income per unit of investment:*

$$\rho_0 = p_H \left(R - \frac{B}{\Delta p} \right)$$
 - Earlier assumptions imply: $\rho_1 > 1 > \rho_0$.
 - The equity multiplier: $k = \frac{1}{1 - \rho_0}$
 - The borrowing capacity per unit of net worth: $d = \frac{\rho_0}{1 - \rho_0}$
 - The shadow value of equity: $\nu = \frac{\rho_1 - \rho_0}{1 - \rho_0}$
 - Borrower's net utility: $U_b = (\nu - 1)A = (\rho_1 - 1)I$.
- Note: Firms with a low agency cost has a greater *sensitivity of investment to cash flow*.

$$\frac{\partial^2 I}{\partial A \partial \rho_0} = \frac{\partial k}{\partial \rho_0} = \frac{1}{(1 - \rho_0)^2} > 0.$$

The maximal incentives principle

- Resolving the debt vs equity question.
- *Salvage value* of assets
 - Investing $I \geq 0$ yields a return $R^S I$ if success, $R^F I$ if failure, with $R^S > R^F > 0$.
 - Define $RI = (R^S - R^F)I$ as the profit increase following success.
 - When secondary asset markets perform better, we should expect R^F to be higher.

- Generalizing $\rho_1 > 1 > \rho_0$:

$$p_H R + R^F > 1 > p_H \left(R - \frac{B}{\Delta p} \right) + R^F$$

- Contract: $\{R_b^S, R_b^F, I\}$ – how much to invest, and how much of the returns generated that the borrower should have following success and failure.
- The optimal contract maximizes the entrepreneur's net utility,

$$p_H R_b^S + (1 - p_H) R_b^F - A,$$

subject to two constraints:

- the entrepreneur's incentive compatibility constraint

$$R_b^S - R_b^F \geq \frac{BI}{\Delta p}$$

- the investors' breakeven constraint

$$p_H(R^S I - R_b^S) + (1 - p_H)(R^F I - R_b^F) \geq I - A$$

- In equilibrium, both constraints will be binding

- As before, the entrepreneur receives all the NPV:

$$U_b = (p_H R + R^F - 1)I$$

- In equilibrium, the entrepreneur receives nothing following failure: $R_b^F = 0$.

- Suppose instead $R_b^F > 0$. Then one can reduce it, and

increase R_b^S , at a rate $\frac{\Delta R_b^F}{\Delta R_b^S} = -\frac{p_H}{1-p_H}$, keeping the

breakeven constraint binding and the entrepreneur's utility unchanged; but this would make the incentive compatibility constraint slack – a contradiction.

- An all-equity firm is not optimal

- With no debt, the entrepreneur would, after a failure, receive her share of $R^F I$ corresponding to her share of the firm's stocks.

- Outside investors must hold debt $D \geq R^F I$.

- Borrowing capacity: $I = kA$, and so $D = I - A = dA = (k - 1)A$, where now

$$k = \frac{1}{1 - \left[p_H \left(R - \frac{B}{\Delta p} \right) + R^F \right]}$$

- Firms borrow more

- the lower agency costs are
 - the more liquid assets are

- *Incentives are maximized when outside investors hold a combination of debt and equity.*

Extensions of the analysis

- Supplementary sections to chapter 3
- A *continuum of effort levels*, disutility of effort $g(e)$
- A *continuum of outcomes*, probability of outcome R with effort level e is $p(R | e)$.
- Linking effort and outcome: higher effort tends to increase income – *the monotone likelihood ratio property* (MLRP)

$$\frac{\partial}{\partial R} \left[\frac{\frac{\partial p(R | e)}{\partial e}}{p(R | e)} \right] > 0$$

- Essentially same result: A *standard debt contract* – making entrepreneur a residual claimant for the marginal income above the debt repayment level
- Risk aversion – brings in another problem: the *insurance/incentives tradeoff*.
 - Providing incentives means making the risk averse entrepreneur take part in the lottery.
 - A solution exists if effort can be verified after contracts are signed, but before outcome is realized, so that contracts can be *renegotiated*. This makes it possible to separate the insurance and incentives problems.

- *Semi-verifiable* outcome
 - Outcome from investment not verifiable, unless outside investors incur an *audit cost*.
 - The incentive problem is related to hiding income, rather than to enjoying private benefits or reducing effort.
 - Outcome is reported by entrepreneur: \hat{R} .
 - The problem for outsiders is to induce truthful reporting
 - Contract now includes a probability $y(\hat{R})$ of no audit for each report \hat{R} .
 - Again, a standard debt contract.

- *Non-verifiable* outcome
 - Not even an audit can verify outcome.
 - Repayment is the result of threats of termination or nonfinancing of future projects.