

Further determinants of borrowing capacity:  
Boosting pledgeable income

- Diversification: more than one project
- Collateral: pledging real assets
- Liquidity: a first look
- Human capital

Diversification

- It may be beneficial for a firm, in terms of getting hold of external funds, to have several projects.
- Equivalently, it may be beneficial for multiple project owners to merge into one firm.
- Previous analysis: constant returns to scale in investment technology
- Expansion in investment project equivalent to an increase in the number of projects whose outcomes are perfectly correlated.
- Consider the opposite extreme: Several projects are available, and they are statistically independent.
- *Cross pledging*: Incomes on one successful project can be offered as “collateral” for other projects.
- Model: Two identical projects. Otherwise: as in the fixed-investment model
- Entrepreneur’s initial wealth per project:  $A$ ; *i.e.*, total wealth:  $2A$ .

- A benchmark: project financing. For each of the two projects:

- Borrower receives  $R_b$  if success, 0 otherwise.

- Incentive constraint:  $R_b \geq \frac{B}{\Delta p}$

- Breakeven constraint:  $p_H \left( R - \frac{B}{\Delta p} \right) \geq I - A$ , or:  $A \geq \bar{A}$ .

- Project financing not viable if  $A < \bar{A}$ .

- *Cross pledging*

- The two projects financed in combination

- Contract: Borrower receives  $R_0$ ,  $R_1$ , or  $R_2$  when 0, 1, or 2 projects are successful.

- Expected return to borrower:

$$p_H^2 R_2 + 2p_H(1 - p_H)R_1 + (1 - p_H)^2 R_0$$

- Two incentive constraints:

- Working on two projects preferred to working on only one

$$p_H^2 R_2 + 2p_H(1 - p_H)R_1 + (1 - p_H)^2 R_0 \geq$$

$$p_H p_L R_2 + [p_H(1 - p_L) + p_L(1 - p_H)]R_1 + (1 - p_H)(1 - p_L)R_0 + B$$

- Working on two projects preferred to working on none

$$p_H^2 R_2 + 2p_H(1 - p_H)R_1 + (1 - p_H)^2 R_0 \geq$$

$$p_L^2 R_2 + 2p_L(1 - p_L)R_1 + (1 - p_L)^2 R_0 + 2B$$

- Clearly,  $R_0 = 0$  in equilibrium, as before.

- *Full cross pledging*: We also have  $R_1 = 0$  in equilibrium.
  - In order to increase the borrowing capacity, the borrower offers all returns that are available in those cases where only one project succeeds.
  - We can simplify the incentive constraints.
  - Working on both projects better than on none:

$$p_H^2 R_2 \geq p_L^2 R_2 + 2B \Leftrightarrow$$

$$(p_H^2 - p_L^2)R_2 \geq 2B \Leftrightarrow$$

$$(p_H + p_L)R_2 \geq 2 \frac{B}{\Delta p} \Leftrightarrow$$

$$\frac{p_H + p_L}{2} R_2 \geq \frac{B}{\Delta p}$$

- Working on both projects better than on a single one:

$$p_H^2 R_2 \geq p_H p_L R_2 + B \Leftrightarrow$$

$$p_H R_2 \geq \frac{B}{\Delta p}$$

- This one is always satisfied when the previous one is.

- It follows that, in equilibrium,  $R_2 \geq \frac{2B}{(p_H + p_L)\Delta p}$

- Minimum expected payoff to borrower:

$$p_H^2 R_2 \geq \frac{2p_H^2 B}{(p_H + p_L)\Delta p} = 2(1 - d_2) \frac{p_H B}{\Delta p},$$

where  $d_2 = \frac{p_L}{p_H + p_L} \in \left(0, \frac{1}{2}\right)$  is an agency-based measure

of the *economies of diversification* into two independent projects.

- The breakeven constraint:
  - Expected pledgeable income  $\geq$  investors' expenses

$$2p_H R - 2(1 - d_2) \frac{p_H B}{\Delta p} \geq 2I - 2A \Leftrightarrow$$

$$p_H R - (1 - d_2) \frac{p_H B}{\Delta p} \geq I - A \Leftrightarrow$$

$$A \geq \bar{A}, \text{ where } \bar{A} = I - p_H \left[ R - (1 - d_2) \frac{B}{\Delta p} \right] < \bar{A}$$

$$\circ \text{ Recall: } \bar{A} = p_H \frac{B}{\Delta p} - (p_H R - I) = I - p_H \left[ R - \frac{B}{\Delta p} \right]$$

- *Diversification and cross pledging* facilitates financing:  $\bar{A} < \bar{A}$
- *Statistical independence* of projects similarly facilitates financing.
- *Variable investment*: Diversification increases the borrowing capacity, rather than giving better access to financing.
- Extension to  $n$  independent projects: Let borrower have net worth  $nA$ . Breakeven constraint for investors now becomes:

$$p_H R - (1 - d_n) \frac{p_H B}{\Delta p} \geq I - A,$$

$$\text{where } d_n = \frac{p_L (p_H^{n-1} - p_L^{n-1})}{p_H^n - p_L^n} \text{ increases with } n.$$

- Limits to diversification
  - Endogenous correlation: The borrower has an incentive to choose correlated projects, if she can. This decreases the value of cross pledging.  $\rightarrow$  *Asset substitution*.
  - Limited expertise.
  - Limited attention.

- Sequential projects
  - Supplementary section 4.7
  - Variable investment in two projects.
  - Benchmark: simultaneous projects
    - Investment  $I_i$  in project  $i \in \{1, 2\}$ .
    - Return  $RI_i$  if success in project  $i$ , 0 otherwise
    - Probability of success  $p_H$  ( $p_L$ ) if the borrower behaves (misbehaves)
    - Private benefit from misbehaving in project  $i$ :  $BI_i$ .
    - Total investment:  $I = I_1 + I_2$ .
  - Optimal with reward only when both projects succeed:  $R_b$ .
  - Binding incentive constraint: misbehavior on both projects

$$p_H^2 R_b \geq p_L^2 R_b + BI$$

- We disregard misbehavior on one project for now
- Total net present value:  $(p_H R - 1)I$
- Investors' breakeven constraint:

$$p_H RI - p_H \frac{BI}{p_H^2 - p_L^2} = I - A$$

- In equilibrium,

$$I = \frac{A}{1 - \hat{\rho}_0}, \text{ where}$$

$$\hat{\rho}_0 = p_H \left( R - \frac{p_H}{p_H + p_L} \frac{B}{\Delta p} \right) = p_H \left[ R - (1 - d_2) \frac{B}{\Delta p} \right], \text{ and}$$

$$U_b = (p_H R - 1)I = \frac{\rho_1 - 1}{1 - \hat{\rho}_0} A$$

- Checking the other incentive constraint: misbehavior on project  $i$ :

$$p_H^2 R_b \geq p_H p_L R_b + B I_i$$

- Combining with the other incentive constraint:

$$\frac{I_i}{I} \leq \frac{p_H}{p_H + p_L}$$

- This constraint does not bind if total investment is split relatively equally among the two projects

- Sequential projects: Short-term loan agreements

- Financing one project at the time.
- Increased incentives early on: success at the first project provides the borrower with extra funds for the second project.
- Think ahead and reason back.
- Project 2: the single-project variable-investment case, with the borrower entering date 2 with assets  $A_2$ .
- Expected payoff per unit of investment:  $\rho_1 = p_H R$
- Expected pledgeable income per unit of investment:

$$\rho_0 = p_H \left( R - \frac{B}{\Delta p} \right)$$

- Borrower's gross utility from project 2:

$$v A_2 = \frac{\rho_1 - \rho_0}{1 - \rho_0} A_2$$

- $v > 1$  is the *shadow value of equity*: If you can increase your assets at the start of date 2 with 1 unit, then you increase your utility with  $v$ .

- Project 1: Borrower's initial assets  $A$ . Return if success:  $RI_1 = R_b + R_l$

- Investors' breakeven constraint

$$P_H R_l \geq I_1 + A$$

- Borrower's incentive constraint:  $vR_b \geq \frac{BI_1}{\Delta p}$

- Expected pledgeable income per unit of investment

$$\tilde{\rho}_0 = p_H \left( R - \frac{B}{v\Delta p} \right) = \rho_1 - \frac{\rho_1 - \rho_0}{v} = \rho_1 + \rho_0 - 1.$$

- *Debt capacity* at date 1:  $I_1 = k_1 A$ , where

$$k_1 = \frac{1}{1 - \tilde{\rho}_0} = \frac{1}{2 - \rho_0 - \rho_1} > \frac{1}{1 - \rho_0} = k$$

- Assume  $\frac{\rho_0 + \rho_1}{2} < 1$ ; otherwise, debt capacity is infinite.

- Recall earlier assumption:  $\rho_1 > 1 > \rho_0$ .

- The borrower invests in project 2 if and only if project 1 is successful. She then invests:

$$I_2 = kA_2 = kR_b = \frac{kB}{v(\Delta p)} I_1 = \frac{\frac{1}{1 - \rho_0} B}{\frac{\rho_1 - \rho_0}{1 - \rho_0} \Delta p} I_1 = \frac{B}{p_H \frac{B}{\Delta p} \Delta p} I_1 = \frac{1}{p_H} I_1$$

- Expected investments in the projects are the same:

$$p_H I_2 = I_1$$

- *Stakes increase over time*:  $I_2 > I_1$

- Sequential vs simultaneous projects

$$U_b^{seq} = p_H v A_2 - A = (p_H v \frac{B}{v(\Delta p)} k_1 - 1) A$$

$$U_b^{seq} = \frac{2(\rho_1 - 1)}{2 - \rho_0 - \rho_1} A > \frac{\rho_1 - 1}{1 - \hat{\rho}_0} A = U_b^{sim}$$

$$\Leftrightarrow \hat{\rho}_0 < \frac{\rho_0 + \rho_1}{2} \Leftrightarrow d_2 = \frac{p_L}{p_H + p_L} < \frac{1}{2}$$

- Note error in Tirole, p. 186.

- Sequentiality is better: The borrower has no chance to misbehave on project 2 if project 1 fails, so the moral hazard problem is less serious.

- Long-term loan agreements

- One agreement for both projects
- Risk neutrality and constant returns to scale imply that short-term agreements fair equally well.



## Collateral

- Assets = cash + productive assets
- Productive assets = quasi-cash, since they may be *pledged as collateral* to lenders
- *Redeployability* of productive assets
  - Fixed-investment model, with one new feature.
  - Suppose, after investment is made but before effort is put in, it becomes publicly known whether the project is *viable*
  - With probability  $x$ , the project is viable and the model proceeds as before
  - With probability  $(1 - x)$ , the project is not viable, and assets can be sold at a given price  $P \leq I$ .
  - *Economic distress*, as opposed to financial distress.
  - New assumption on NPV:  $x p_H R + (1 - x)P > I$ .
  - The entrepreneur chooses to pledge the resale price in full.
  - Breakeven constraint for investors:
$$x p_H \left( R - \frac{B}{\Delta p} \right) + (1 - x)P \geq I - A$$
  - Threshold level of net worth:
$$\bar{A} = x p_H \frac{B}{\Delta p} - [x p_H R + (1 - x)P - I]$$
    - Decreases with asset redeployability
  - *Borrowing patterns across industries*: The more liquid assets, the easier it is for firms borrow.
  - Endogenous redeployability: *fire sale externalities* – further aggravating credit rationing.

## Collateral is costly

- A deadweight loss associated with collateralization: assets may have lower value for lenders than for the borrower
  - Transaction costs
  - Borrower's private benefit from ownership: sentimental values, specific skills
  - Prospects of future credit rationing makes the asset of higher value to the borrower than to investors
  - Risk aversion
  - Collateralized assets may receive poor maintenance

## Costly collateral and contingent pledging

- Suppose first collateral would not exist without the investment.
- Borrower has no cash initially, needs to borrow  $I$ .
- Asset has residual value
  - $A$  to the entrepreneur
  - $A' \leq A$  to the lenders
  - Deadweight loss if asset is seized:  $A - A'$
- Contract:  $\{R_b, R_l, y_S, y_F\}$ 
  - $y_S$  – probability that the borrower keeps the asset if success
  - $y_F$  – ... if failure
  - *stochastic pledging*: needed in a simple model
- Otherwise, fixed-investment model.

- The equilibrium contract is the one that maximizes borrower's utility, subject to borrower's incentive-compatibility constraint and lenders' breakeven constraint.

$$\text{Max } U_b = p_H(R_b + y_S A) + (1 - p_H)y_F A$$

subject to

$$\Delta p[R_b + (y_S - y_F)A] \geq B, \text{ and}$$

$$p_H[R_l + (1 - y_S)A'] + (1 - p_H)(1 - y_F)A' \geq I$$

- Borrower wants to pledge as little collateral as possible
- The outcome depends on *the strength of the balance sheet* of the borrower

- Strength of balance sheet depends on

- Investment level  $I$  (−)

- Agency costs, measured by  $p_H \frac{B}{\Delta p}$  (−)

- Any initial cash,  $\tilde{A}$  (+)

- *Strong balance sheet* – no collateral

$$y_S = y_F = 1; R_b > 0.$$

- *Intermediate balance sheet* – collateral if failure:

$$y_S = 1, y_F \leq 1; R_b \geq 0.$$

- *Weak balance sheet* – borrower gets a share of the asset if success:

$$y_S \leq 1, y_F = 0; R_b = 0.$$

- *Contingent pledging*: borrower gets a contingent share of the asset rather than of income.

Solution: derivative of the Lagrangian with respect to  $y_S$  is positive if that with respect to  $R_b$  or that with respect to  $y_F$  is. Some of the three regimes may not exist.

- *Weak borrowers pledge more collateral than strong borrowers*
  - Pledging collateral in lack of cash
  - Opposite prediction from adverse-selection theories, where strong firms pledge collateral to show strength.

### Pledging existing assets

- Suppose next that the entrepreneur has existing wealth
- *Contingent pledging*
  - If success, the entrepreneur keeps the asset.
  - If failure, the investors receive the collateral.
- Continuous collateral: the entrepreneur chooses an amount  $C \in [0, C^{max}]$  to pledge as collateral in case of failure.
  - We need an upper limit on  $C^{max}$ ; see below.
- Costly collateral: Value  $\beta C$  to investors, where  $\beta < 1$ .
- Borrower's net utility: Project's NPV without collateral minus expected deadweight loss from pledging collateral.

$$U_b = p_H R - I - (1 - p_H)(1 - \beta)C$$

- To ensure that  $U_b \geq 0$  for any feasible  $C$ , we assume

$$C^{max} \leq \frac{p_H R - I}{(1 - p_H)(1 - \beta)}$$

- Collateral costly  $\Rightarrow C = 0$  if  $A \geq \bar{A}$ .

- The borrower's incentive compatibility constraint

$$p_H R_b - (1 - p_H)C \geq p_L R_b - (1 - p_L)C + B \Leftrightarrow$$

$$R_b + C \geq \frac{B}{\Delta p}$$

- The borrower loses both the reward and the collateral when she fails
- *Limited liability*: In order to ensure that  $R_b \geq 0$  for any feasible  $C$ , we assume:

$$C^{max} \leq \frac{B}{\Delta p}$$

- The investors' breakeven constraint

$$p_H (R - R_b) + (1 - p_H)\beta C \geq I - A \Leftrightarrow$$

$$p_H \left( R - \frac{B}{\Delta p} \right) + p_H C + (1 - p_H)\beta C \geq I - A$$

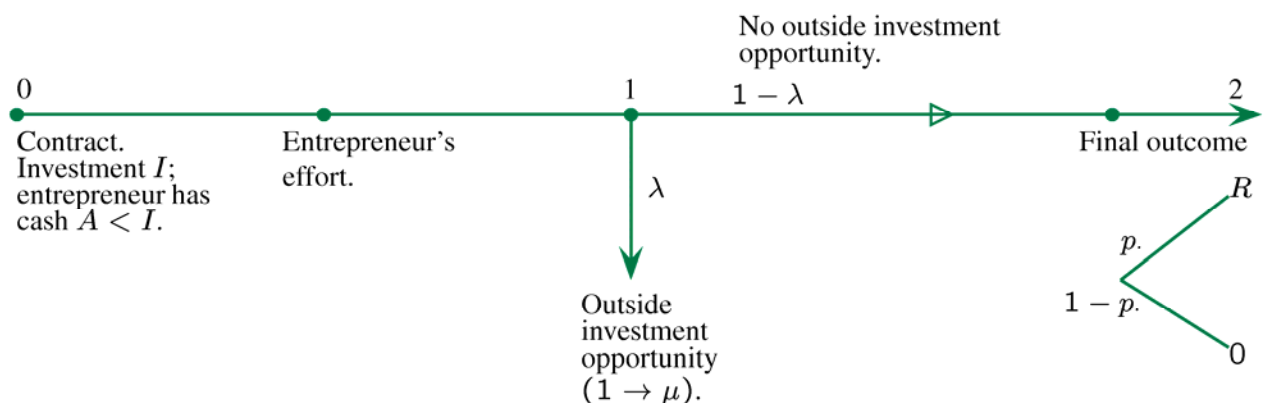
- Collateral has two ways of affecting pledgeable income
  - Directly:  $+(1 - p_H)\beta C$
  - Indirectly through a lower reward to borrower:  $+ p_H C$
- Borrower pledges the minimum collateral necessary to satisfy the investors' breakeven constraint:

$$C = \frac{I - A - p_H \left( R - \frac{B}{\Delta p} \right)}{p_H + (1 - p_H)\beta}$$

- ... except if this expression gets too big, in which case collateral cannot solve the funding problem.
- Weaker firms pledge more collateral:  $\frac{dC}{dA} < 0$ .
- Conditional collateral preferable to unconditional.
- More abstract forms of collateral: Putting one's job at stake.

## The liquidity-accountability tradeoff

- When should the borrower receive her compensation?
  - Towards the end: good for accountability, because more information about the project is available
  - Along the way, because of her need for liquidity
    - Consumption
    - New projects
- Outside investment opportunities not observable for investors
  - A scope for “strategic exit”, escaping sanctions following poor performance
- The other side of the coin: the liquidity of investors
  - The more control you have, the less liquid your assets are
- Model: an extension of the fixed-investment one



- New feature: A new, fleeting investment opportunity at an intermediate date
- Initial investment  $I$ , entrepreneur's assets  $A < I$ .

- Moral hazard: misbehavior means a lower success probability ( $p_L < p_H$ ) but also a private benefit  $B$ .
- Project returns at final date:  $R$  or  $0$  (whether or not an intermediate investment opportunity shows up).
- Limited liability, risk neutrality.
- Project would have been financed in the absence of the intermediate liquidity needs:

$$A > \bar{A}$$

- *Liquidity shock*: With probability  $\lambda$ , a new investment opportunity arises.
  - Investing  $x$  returns  $\mu x$ , where  $\mu > 1$ .
- Contract:  $\{r_b, R_b\}$ . Borrower receives
  - $r_b$  on the intermediate date and nothing on the final date, in the case of a liquidity shock.
  - $R_b$  on the final date if success ( $0$  if failure) and nothing on the intermediate date, in the case of no liquidity shock.
- What if the liquidity shock is not verifiable?
- *Exit vs vesting*: what about *partial vesting*? – Some cash at the intermediate date and some payment at the final date (if success).
- Implementation: where does  $r_b$  come from? – Needs to be subtracted from pledgeable income.

- Benchmark case: *Verifiable liquidity shock*
- Borrower's incentive compatibility constraint

$$\lambda\mu r_b + (1 - \lambda)p_H R_b \geq \lambda\mu r_b + (1 - \lambda)p_L R_b + B \Leftrightarrow$$

$$(1 - \lambda)(\Delta p)R_b \geq B \Leftrightarrow$$

$$R_b \geq \frac{1}{1 - \lambda} \frac{B}{\Delta p}$$

- No incentive effect from  $r_b$ .
- Only effect of the liquidity shock is that the borrower's stake must be increased, since final date is reached only with probability  $(1 - \lambda)$ .
- Borrower receives  $r_b$  with probability  $\lambda$ . So this is similar to no liquidity shock, but the entrepreneur having available  $A - \lambda r_b$ .
- Expected pledgeable income:

$$p_H R - \left\{ \lambda r_b + (1 - \lambda) \frac{1}{1 - \lambda} \frac{B}{\Delta p} \right\} = p_H \left( R - \frac{B}{\Delta p} \right) - \lambda r_b.$$

- Competition among investors ensures that the borrower gets the NPV from the project. So her total expected net utility is

$$U_b = p_H R - I + \lambda(\mu - 1)r_b.$$

- It is optimal to have  $r_b$  as high as possible subject to incentive compatibility:

$$p_H \left( R - \frac{B}{\Delta p} \right) - \lambda r_b = I - A$$

- In equilibrium:  $r_b = \frac{1}{\lambda} \left[ p_H \left( R - \frac{B}{\Delta p} \right) - (I - A) \right]$ ;  $R_b = \frac{1}{1 - \lambda} \frac{B}{\Delta p}$ .



- *Non-verifiable liquidity shock*
- *A two-dimensional moral-hazard problem.* Incentives needed for borrower
  - to behave in carrying out the project, and
  - to report truthfully about the liquidity shock
- The two forms of moral hazard interact
  - *Strategic exit:* A misbehaving borrower may want to exit even without a liquidity shock before the consequences are disclosed.
- Simplifying assumption:  $p_L = 0 \Rightarrow \Delta p = p_H$ 
  - A misbehaving borrower would indeed want to cash out early, since there is nothing to be had later:  $p_L R_b = 0$ .
- Borrower's incentive constraint

$$\lambda \mu r_b + (1 - \lambda) p_H R_b \geq [\lambda \mu + (1 - \lambda)] r_b + B \Leftrightarrow$$

$$(1 - \lambda) [p_H R_b - r_b] \geq B \Leftrightarrow$$

$$(1 - \lambda) [(\Delta p) R_b - r_b] \geq B \Leftrightarrow$$

$$R_b \geq \frac{r_b}{\Delta p} + \frac{1}{1 - \lambda} \frac{B}{\Delta p}$$

- Compare with the case of verifiable liquidity shock: the possibility of a strategic exit makes the incentive constraint stricter (for a given  $r_b > 0$ ).
- When there is no liquidity shock, the borrower strictly prefers to continue:  $p_H R_b > r_b$ .
- But would the borrower want to cash out when there *is* a liquidity shock? Is  $\mu r_b \geq p_H R_b$ ? – Suppose first that it is.

- Again, competition among investors ensures that all NPV of the project accrues to the borrower. So, given  $r_b$ , her expected net utility is:

$$U_b = p_H R - I + \lambda(\mu - 1)r_b.$$

- But the incentive constraint is stricter, so pledgeable income is smaller. Therefore  $r_b$  is lower when liquidity shock is nonverifiable.

- Expected pledgeable income for a given  $r_b$ :

$$p_H R - \left\{ \lambda r_b + (1 - \lambda) p_H \left[ \frac{r_b}{\Delta p} + \frac{1}{1 - \lambda} \frac{B}{\Delta p} \right] \right\} = p_H \left( R - \frac{B}{\Delta p} \right) - r_b$$

- In equilibrium:

$$r_b = p_H \left( R - \frac{B}{\Delta p} \right) - (I - A); \quad R_b = \frac{1}{1 - \lambda} \frac{B + (1 - \lambda)r_b}{\Delta p}$$

- Compared to the case of verifiable liquidity shock:

$r_b$  is lower,  $R_b$  is higher.

- The possibility of strategic exit hurts the borrower, since she is allowed less liquidity.

- If the above contract does not obey  $\mu r_b \geq p_H R_b$ :

- Happens when  $A$  is low.

- Solution: *partial vesting*. Only implementation changes.

- Total compensation has two components: One, a basis compensation,  $R_b^0$ , payed out in case of success.

- At the intermediate date, the borrower receives cash  $r_b$ . She can choose to buy shares for this, which would pay  $\Delta R_b$  in case of success, where

$$R_b^0 + \Delta R_b = R_b$$

## Inalienability of human capital

- Is there a scope for the loan contract to be *renegotiated* as the project proceeds?
- A *renegotiation* must mean that the existing contract is not efficient for the parties involved – that a new contract exists that is weakly better for both borrower and lender, and strictly better for at least one of them.
- *Hold-up*: Suppose the entrepreneur is *indispensable* – the project cannot be completed without her. The entrepreneur may want to renegotiate the initial contract in order to obtain a better deal.
  - The *inalienability of human capital*.
- Model: no moral hazard:  $B = 0$ ; no cash:  $A = 0$ .
- Otherwise, fixed-investment model.
- The act of “completing the project” cannot be contracted upon until after investment has been made: Renegotiation is needed.
  - Renegotiation replaces effort as the source of the incentive problem.
- Incomplete project returns 0.
- Complete project returns  $R$  [prob  $p_H$ ] or 0 [prob  $(1 - p_H)$ ].
- Disregarding renegotiation, the project can be financed by a debt contract: borrower pays investors  $D$  in case of success, such that  $p_H D = I$ .
  - $R_l = D$ ,  $R_b = R - D$ , and  $U_b = p_H(R - D) = p_H R - I$ .
- Renegotiation: Bargaining over  $p_H R - I$ .

- Who has *bargaining power*?
  - No longer competition among creditors: lender has b.p.
  - Entrepreneur is indispensable: borrower has b.p.
  - Both receive 0 in case of noncompletion of project
- Lender's bargaining power:  $\theta$ 
  - In the renegotiation, lender receives  $\theta R$  in case of success, and borrower receives  $(1 - \theta)R$ .
  - Lender willing to invest if  $\theta p_H R \geq I$ .
  - If  $\theta > D/R$ , then the borrower prefers to simply skip the renegotiation and complete the project.
  - If  $\theta < D/R$ , then  $\theta p_H R < p_H D = I$ : the project will not be financed.
  - If the borrower is too indispensable, the project is not carried out.
- Determinants of bargaining power
  - Reputations on both sides
  - Dispersion of lenders
  - Outside options
- If possible, the borrower may want to give the lenders the right to seize the firm's assets – in order to secure some external finance.
- A parallel to collateral – the value of the collateral may depend on how indispensable the entrepreneur is.