Lecture 1 - 26-01-15 by P.Piacquadio

Introduction to distributive justice and economic inequality.

- The role of distributive justice in economics
  - Economics as a branch of ethics? read "Why economics needs ethical theory" written by John Broome for the Meeting of the British Association for the Advancement of Science.
  - normative vs positive;
- Utilitarianism
  - J. Bentham (1748-1832) "An introduction to the principles of morals and legislation" 1789
    - socially approve/disapprove collective decisions based on the total amount of pain or pleasure these bring.
    - "Felicific calculus".
  - J.S. Mill suggested to interpret utilitarianism as to promote the largest happiness for the largest number of people
  - *consequentialism*: acts should be judged strictly by their effects on individuals, rather than by some other intrinsic merit or demerit of those acts.
- Example of utilitarianism in 1 dimension
  - computation of the optimum;
  - the role of comparability;
  - "Of two people having unequal fortunes, he who has most wealth must by a legislator be regarded as having most happiness. But the quantity of happiness will not go on increasing in anything near the same proportion as the quantity of wealth: ten thousand times the quantity of wealth will not bring with it ten thousand times the quantity of happiness. It will even be matter of doubt, whether ten thousand times the wealth will in general bring with it twice the happiness. The effect of wealth in the production of happiness goes on diminishing, as the quantity by which the wealth of one man exceeds that of another goes on increasing: In other words, the quantity of happiness produced by a particle of wealth (each particle being of the same magnitude) will be less at every particle; the second will produce less than the first, the third than the second, and so on." From *The Works of Jeremy Bentham* edited by Bowring, Vol.I, p.305, 1843.

  - Comparability based on assigned resources (wealth);
  - the role of diminishing marginal utilities.

- Example of utilitarianism in 2 dimensions;
  - computation of the optimum;
  - illustration through the Edgeworth box;
  - the utility possibility frontier;
  - issue of comparability.

Lecture 2 - 02-02-15

Arrow's impossibility result: setting, axioms, and result

- Ordinal and non-comparable information about preferences
  - The "new welfare economics" and the Pareto school (Robbins, 1932);
  - Bergson-Samuelson social welfare function;
  - representation in the utility space;
  - "It is the great merit of Bergson's 1938 paper to have carried the same [ordinalist] principle into the analysis of social welfare. The social welfare function was to depend only on indifference maps; in other words, welfare judgments were to be based only on interpersonally observable behavior." Arrow, 1963, p.109

- Arrowian framework
  - X is the set of all possible social states with $|X| \geq 3$.
  - The society consists of a finite set of individuals $N \equiv \{1, ..., n\}$ with preferences R_i over X.
  - The n-tuple of preferences, a preference profile, is denoted by $\{R_i\}$.
  - Social preference are denoted by R (complete and transitive).
  - The Arrowian problem is to define the mapping f that assigns social preferences for each preference profile, i.e. $R = f(\{R_i\})$. f is an Arrowian social welfare function.
  - $R = f(\{R_i\})$ does not mean that only preference information is to be used: f can depend on many more arguments.

- Axioms
  - Unrestricted Domain (**U**). The domain of the Arrowian social welfare function f should include ALL conceivable preference profiles $\{R_i\}$;

- Weak Pareto (**P**). If all individuals prefer x to y, then also society does.
- Nondictatorship (**D**). R cannot be such that x is preferred to y whenever xR_jy and independently of the preference profile.
- Independence of Irrelevant Alternatives (**I**). R is such that xRy depends only on individual preferences over x and y.

- Proof
  - A subset of individuals $G \subseteq N$ is **decisive over** $\{\mathbf{x}, \mathbf{y}\}$, denoted by $D_G(x, y)$, if and only if xR_jy for each $j \in G$ implies that xRy. It is **decisive**, denoted by D_G, if it is decisive over each pair $\{x, y\}$.
  - **Lemma 1**. (Field expansion lemma) $D_G(x, y)$ implies D_G.
  - **Lemma 2.** (Group contraction lemma) Assume D_G for some G. If $G = G_1 \bigcup G_2$ with $G_1 \bigcap G_2 = \emptyset$, then either D_{G_1} or D_{G_2}.
  - $P \Rightarrow D_N(x, y) \Rightarrow D_N \Rightarrow D_j$.

- Exercise for the seminar:
  - Each group of students selects one of the axioms (U, P, D, I, transitivity of R, completeness of R). The task is to present to the others: *(i)* the role of the selected axiom for establishing Arrow's result; and *(ii)* discuss a social welfare function that satisfies all other axioms except the one selected.

Lecture 3 - 09-02-15

- Interpretation of Arrow's impossibility result
  - the role of IIA
  - The conclusion: from truncating preference information to the need of further preference information

Sen's social welfare functional approach

- The approach
  - The Arrowian problem is to define the mapping f that assigns social preferences for each **UTILITY** profile, i.e. $R = f(\{u_i\})$. f is a social welfare function**AL**.
  - One can study in detail the consequence of adding utility information.
  - When the information added by U is disregarded, the framework is "Arrowian."

- Let each preference relation R_i be represented by a utility function U_i. Possible informational assumptions:
  - Ordinality and non-comparability. Invariance to transformations $V_i = \varphi_i \circ U_i$
  - co-ordinality (common ordinal scale). $V_i = \varphi \circ U_i$
  - co-cardinality (cardinal scale and full comparability). $V_i = a + bU_i$
  - (ordinal scale and full comparability). $V_i = a_i + b_i U_i$
  - (cardinal scale and unit comparability). $V_i = a_i + bU_i$
  - (ratio-scale and full comparability). $V_i = bU_i$
  - (ratio-scale and no comparability).$V_i = b_i U_i$
  - GRAPH with implications
- Formal welfarism
  - definition of a social welfare ordering SWO R^*
  - Formal welfarism and the equivalence with jointly binary independence and Pareto indifference (to be further discussed).
- Assignment for the next seminar (26-02-2015)
  - Write a less-than-2-pages essay and send it to me by email before Tuesday evening (24-02-2015) at 23:00 pm.
  - The essay should discuss distributive justice for the following situation.
  - There are 4 agents: Andrea, Barbara, Carlo, and Davide.
  - Andrea and Carlo have preferences $u = \ln c + \ln \ell$ over consumption and leisure; Barbara and Davide prefer leisure relatively more and have preferences $v = \ln c + 2 \ln \ell$.
  - Assume that Andrea and Barbara can work at an hourly wage of 200 dollars; Carlo and Davide have an hourly wage of 100 dollars.
  - What matters for distributive justice? Should the society compensate those that get a lower wage?
  - Should the society care about differences in incomes or consumptions?
  - Can you suggest and defend a specific measure of social welfare? (optional)

Lecture 4. 23-02-15

- Formal welfarism
  - definition of a social welfare ordering SWO R^*
  - Binary independence. for each V and each x,y; xR_vy if there exists U such that $V_x = U_x$and „$V_y = U_y$ and xR_Uy.
  - Formal welfarism and the equivalence with jointly binary independence and Pareto indifference
- Characterization of (pure) utilitarianism SWO R^*
  - Weak Pareto*.
  - Anonoymity*.
  - Inv*$(a_i + bu_i)$.
  - Proof. (theorem 4.4. in the reference chapter)
- Characterization of leximin SWO R^*
  - Strict Pareto*.
  - Minimal Individual Symmetry*. For any 2 individuals $i, j \in N$, there exists uI^*v such that $u_i > v_i$, $u_j < v_j$ and $u_k = v_k$ for all $k \neq i, j$.
  - Minimal equity*. If $v_i < u_i < u_j < v_j$ and $u_k = v_k$ for all $k \neq i, j$, then uR^*v.
  - Inv*$(\phi(u_i))$.
  - Separability*.
  - Proof. with anonymity (theorem 4.16. in the reference chapter)

Lecture 5. 09-03-15

- Back to the Arrowian setting:
  - which information about preferences is relevant?
  - interpretation of the independence axiom.
- Characterization of ordinalist utilitarian criterion
  - Domain specification
  - Weak Pareto. Continuity. Separability.
  - Transfer among equals. Anonymous opportunities.
  - Utilitarianism with ordinal non-comparable information about individual preferences (no proof).