Exercise 1: Alternative presentation of the Lorenz curve

- (i) Explain how the information content of the Lorenz curve can be given an alternative presentation which is consistent with the notion of inequality captured by the Lorenz curve.
- (ii) The alternative representation of the Lorenz curve, called the scaled conditional mean curve (the M-curve), shows to possess several attractive properties. Explain these properties.

Exercise 2: Measures of inequality for empirical applications

- (i) Provide a justification for why the moments of the M-curve can be considered to form a family of inequality measures that uniquely determines the M-curve and the Lorenz curve.
- (ii) The standard approach in empirical work is to employ the Gini coefficient in combination with measures from the Atkinson family and/or the Theil family. Discuss whether this is a suitable approach
- (iii) Provide arguments for why it might be attractive to employ moments of the M-curve to summarize inequality exhibited by an income distribution.
- (iv) Let C₁, C₂ and C₃ denote the three first moments of the M-curve. Show that these three measures can be given the following expression in terms of the distribution function F:

$$C_1 = \frac{1}{\mu} \int_0^\infty F(x) \log F(x) dx \,, \quad C_2 = G = \frac{1}{\mu} \int_0^\infty F(x) (1 - F(x)) dx \,, \quad C_3 = \frac{1}{2\mu} \int F(x) (1 - F^2(x)) dx \,.$$

- (v) Show that C₁, C₂ and C₃ can be given explicit expressions in terms of social welfare and that the associated social welfare functions can be decomposed with regard to the mean and inequality.
- (vi) The (empirical) Gini coefficient can be estimated as
 - a. Twice the area between the Lorenz curve and the 45° line

b.
$$G^b = \frac{\sum_{j=1}^{n} \sum_{k=1}^{n} |y_j - y_k|}{2n^2 \mu}$$

c. $G^c = 2 \frac{\sum_{i=1}^{n} iy_i}{n^2 \mu} - \frac{n+1}{n}$

c.
$$G^{c} = 2 \frac{\sum_{i=1}^{n} i y_{i}}{n^{2} \mu} - \frac{n+1}{n}$$

where $y_1, ..., y_n$ are the incomes in society, given in increasing order.

Exercise 3 Inequality and poverty

- (i) Explain what we mean by poverty and how it can be measured.
- (ii) Compare and contrast measures of poverty, inequality, and miserliness
- (iii) To study each of the following societal issues, which of the above mentioned measure would be most suitable:
 - a. Misery in a poor country
 - b. Is it necessary that the global community helps a country to alleviate poverty
 - c. Political instability due to tensions among rich and poor
 - d. The possibility of the poor to benefit from redistribution
 - e. The prevalence of poverty traps