Seminar 4
ECON4271
Spring 2015

Exercise 1: Primal and dual social welfare functions

(i) Give an account of the major difference between the primal and the dual social welfare functions, and present and discuss the basic conditions that have been used to justify these two families of social welfare functions.

(ii) What conditions ensure that respectively the primal and dual social welfare functions exhibit inequality aversion? What is the underlying normative principle that is used to justify inequality aversion?

(iii) Provide a sketch of the proof that justifies the condition associated with inequality aversion for the primal social welfare functions.

Exercise 2: Primal and dual inequality measures

(i) A summary measure of (relative) inequality has to satisfy two basic conditions. What are these conditions? Explain their impact on our perception and interpretation of inequality.

(ii) The Lorenz curve is normally considered as the basic device for measuring inequality in a distribution function. Explain why the Lorenz curve can be considered as a mathematical representation of the concept of inequality.

(iii) Explain why poverty measures (for example the proportion of poor), top income shares or ratios of quantiles (for example the ratio between the 10 per cent richest and poorest person) do not justify the conditions of being measures of inequality.

(iv) Let F be a cumulative distribution function with mean μ. Demonstrate why

$$\frac{\int_0^{\infty} u(x)dF(x)}{u(\mu)} \quad (1)$$

cannot be considered as an appropriate measure of inequality.

(v) Demonstrate how the primal social welfare functions can be used as basis for obtaining a family of inequality measures.

(vi) Provide a justification for why the dual social welfare functions, as opposed to the primal social welfare functions, can directly be used as a basis for defining a family of inequality measures.

(vii) Demonstrate whether or not primal and dual measures of inequality can be given an explicit expression in terms of the Lorenz curve. When this not the case show that there is an implicit relationship between the measures of inequality and the Lorenz curve.

(viii) Demonstrate why and how social welfare functions can be decomposed with respect to the mean and the inequality of a distribution function (the size and the division of the cake).