



Inequality along time, across space & over states

Lecture 3 on distributive intergenerational justice

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Endogenously varying population and uncertainty



Evaluation of climate policies, and other long-term policy issues, requires a variable population setting where

- population is endogenously determined
- uncertainty is taken into account.

How to evaluate such policies while treating people equally?

This raises the questions:

- What does it mean to treat individuals equally?
- How to combine equal treatm. w/ other desirable properties?



Priority for the worse off

In a purely consequentialist setting
equal treatment of individuals is compatible with
with letting the welfare weights assigned to
different individuals' marginal wellbeing depend on

- their absolute level of wellbeing
- their relative rank in wellbeing

Fleurbaey (2001) distinguishes an "absolute" Priority View from a "relative" one
See also Buchak (2015)



Weights depending on levels of wellbeing only: Undiscounted utilitarianism

x_i A comprehensive measure of consumption indicating the wellbeing of individual i . Assume that unequal distribution of consumption is deemed undesirable in social evaluation.
 $u(x_i)$ Utility derived from x_i , where u is an increasing and strictly concave function.

Under undiscounted utilitarianism, the weight on individual i 's wellbeing is proportional to $u'(x_i)$.

Since u is strictly concave, the weight on a worse off individual is higher than the weight on a better off individual.

Often $u(x_i) = \frac{x_i^{1-\eta}}{1-\eta}$, so that $u'(x_i) = x_i^{-\eta}$, where η is an inequality aversion parameter.

Welfare weights depending on rank only: Lexicographic maximin

- Maximize the wellbeing of the worst-off individual
- To break ties
(if the worst-off individual has the same wellbeing)
maximize the wellbeing of the second worst-off individual
- Etc.

Claim

Claim

When $\eta \rightarrow \infty$, then undisc. utilitarianism \rightarrow *leximin*

Corollary

No need to combine "absolute" and "relative" priority

In this lecture

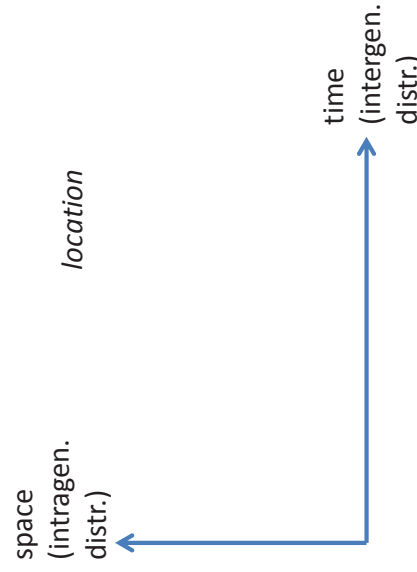
With an infinite number of (potential) people, the consequences of undisc. utilitarianism remain different from leximin even when $\eta \rightarrow \infty$

Conclusion

There is a need to combine "absolute" and "relative" priority

Location in time and space

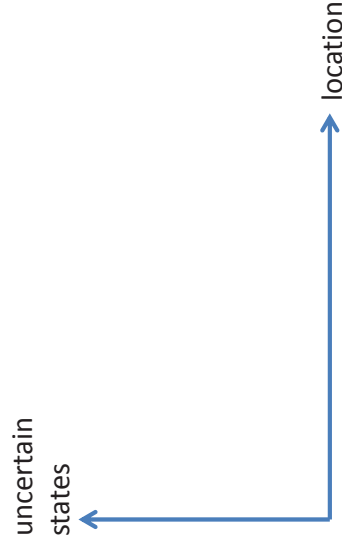
Assume that each location is inhabited by a different individual
 \rightarrow Intergenerational perspective (time period: a generation)



Uncertain states

At each location there is a resolved partition of the state space

Assume that each (location-partition element) pair is inhabited by a different individual \rightarrow No individual is subjected to risk



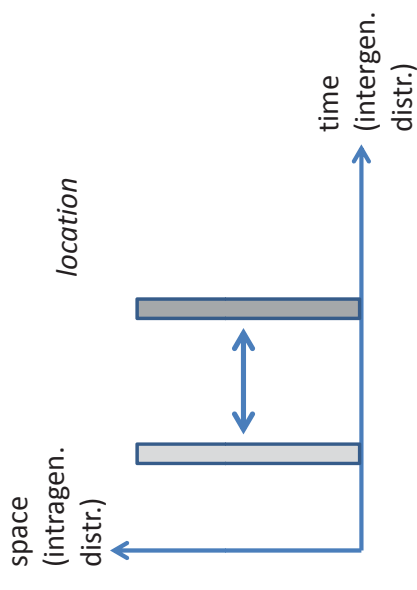
Inequality

- If alternatives have no inequality of wellbeing across location and states, then evaluation is straightforward
- Consider
 - ◇ **Inequality along time only** (intergenerational equity)
 - ◇ Inequality along time and across space only (inter- and intragenerational equity / no uncertainty)
 - ◇ Inequality along time and across space only but not all locations are inhabited (endogenous population)
 - ◇ Inequality along time, across space and over states (inter- and intragenerational equity + uncertainty)
 - ◇ **Inequality along time, across space and over states** but not all location-state pairs are inhabited (endogenous population + positive prob. of extinction)

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Intergenerational equity

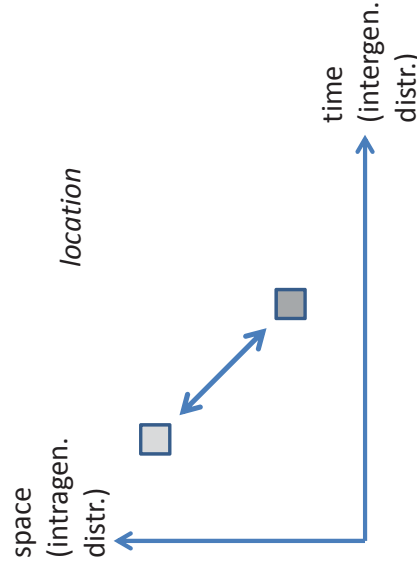
Axiom of equal treatment: Strong/Finite Anonymity



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Inter- and intragenerational equity

Axiom of equal treatment: Strong/Finite Anonymity



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Endogenous population

Motivation:

- Policy (e.g. concerning climate change) may lead to different people living in the future. So we need to be able to evaluate alternatives with different sets of potential people.

Key question posed in the population-ethics literature:

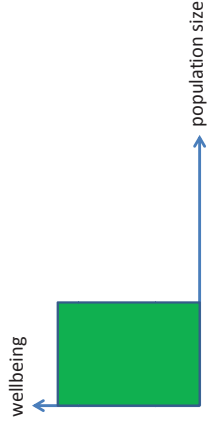
- How to avoid the repugnant conclusion

See Arrhenius (forthcoming) for an overview

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The Repugnant Conclusion (Parfit, 1984)

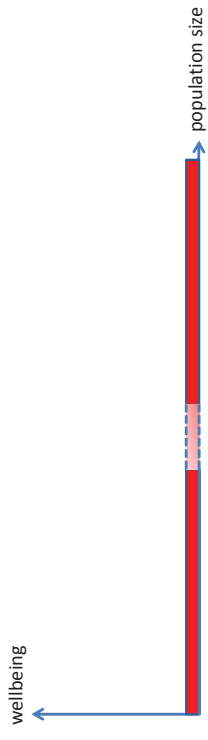
For any egalitarian allocation with very high positive wellbeing,



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The Repugnant Conclusion (Parfit, 1984)

For any egalitarian allocation with very high positive wellbeing, there is an allocation with very low positive wellbeing which is better.



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Weights depending on levels of wellbeing only: Total utilitarianism

Total utilitarianism (TU):

$$\mathbf{x} \succsim \mathbf{y} \iff \sum_{r=1}^{n(\mathbf{x})} (u(x_{[r]}) - u(0)) \geq \sum_{r=1}^{n(\mathbf{y})} (u(y_{[r]}) - u(0))$$

leads to the repugnant population for any value of η

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Weights depending on levels of wellbeing only: Critical-level utilitarianism

Critical-level utilitarianism (CLU): (Blackorby et al., 2005)

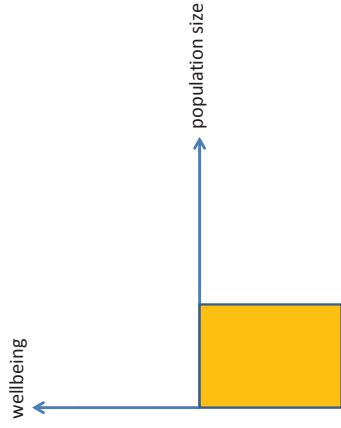
$$\mathbf{x} \succsim \mathbf{y} \iff \sum_{r=1}^{n(\mathbf{x})} (u(x_{[r]}) - u(c)) \geq \sum_{r=1}^{n(\mathbf{y})} (u(y_{[r]}) - u(c))$$

has another problem

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The Very Sadistic Conclusion (Arrhenius, forthcoming)

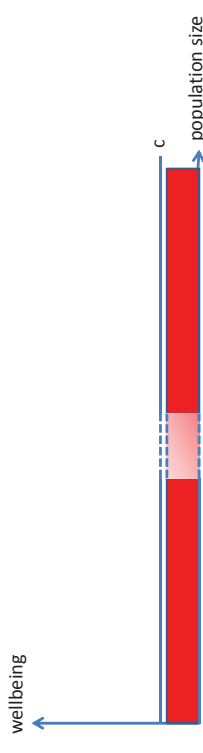
For any egalitarian allocation with negative wellbeing,



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The Very Sadistic Conclusion (Arrhenius, forthcoming)

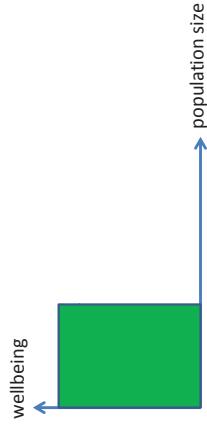
For any egalitarian allocation with negative wellbeing, there is an allocation with positive wellbeing which is worse.



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Welfare weights depending on rank only: (A version of) Critical-level leximin Problem

The Reverse Repugnant Conclusion



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Welfare weights depending on rank only: (A version of) Critical-level leximin Problem

The Reverse Repugnant Conclusion



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Rank-discounted critical-level utilitarianism

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Rank-discounted critical-level utilitarianism (RDCLU):

$$x \succsim y \iff \sum_{r=1}^{n(x)} \beta^r (u(x_{[r]}) - u(c)) \geq \sum_{r=1}^{n(y)} \beta^r (u(y_{[r]}) - u(c))$$

has none of these problems

Proposed and axiomatized in Asheim and Zuber (2014)

RDCLU fills out the space between

- ◇ critical-level utilitarianism and
- ◇ (a version of) critical-level leximin

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	RDCLU $c \geq 0$	CLU $c = 0$	CLU $c > 0$	AU	CLL $c \geq 0$
Avoiding the Repugnant Conclusion	+	-	+	+	+
Avoiding the Weak Repugnant Conclusion	+	-	-	+	+
Avoiding the Very Sadistic Conclusion	+	+	-	+	+
Weak Non-Sadism Condition	+ ^a	+	-	-	+
Negative Mere Addition Principle	+	+	+	-	+
Avoiding the Reverse Repugnant Conclusion	+	+	+	-	-
Strong Quality Addition Principle	+	-	+	-	+
Non-Anti-Egalitarianism Principle	+ ^b	+ ^c	+ ^c	+ ^c	+
Mere Addition Principle	-	+	-	-	-

^aIf $(u(z); z \in \mathbb{R})$ is bounded above.
^bIf $\beta \times c_{tr} \leq 1$.
^cIf u is concave.

Rank-discounted critical-level utilitarianism

Environmental effects \Rightarrow Intergenerational risks

Possible (even probable) that many future people will live miserable lives

ENDOGENOUS

- Emitting greenhouse gases
 - ◇ Risk of catastrophic climate change
- Overusing antibiotics
 - ◇ Serious health risks
- Exploiting soil and water resources
 - ◇ Risks of malnutrition and deceases
- Undermining biodiversity
 - ◇ Reduced resilience

EXOGENOUS

- Natural changes in conditions for human existence
 - ◇ Earth acceptable habitat for humans for 500 million years

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Environmental effects \Rightarrow Intergenerational risks

Possible (even probable) that many future people will live miserable lives

In this time frame, where people's lives are short compared to the time period for which the policies will have an effect, the objective social risk concerning

what kind of societies will future people be born into

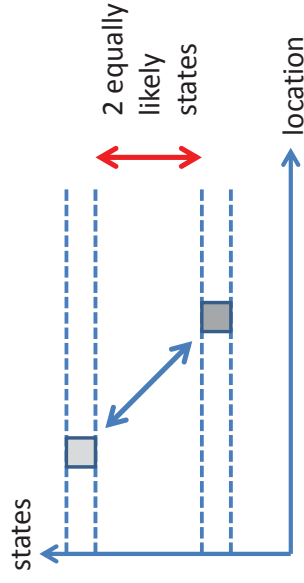
is more important than the subjective individual risk concerning

what will happen to people during their own lifetimes.

That is, it might be reasonable to be more concerned about reducing the probability that future people will lead miserable lives, rather than avoiding volatility in the living conditions that people experience within their own lifetimes.

Inter- and intragenerational equity + uncertainty

Consider equally likely states and impose equal treatment between locations in equally likely states



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Endogenous population + positive prob. of extinction

Motivation:

- There is a positive probability of human extinction.
- Stern Review's justification for time utility discounting.

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How to extend RDCLU to uncertainty (Asheim and Zuber, 2015)

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 - ◇ Rank-dependent expected utilitarianism
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Introduction: Focus on intergenerational equity

- Individuals live for one period only; not subjected to individual risk
- Social risk associated with the lifetime wellbeing of future individuals
- Intergenerational focus motivates abstracting from individual fluctuations & risk
- Fluctuation & risk aversion play no role; only inequality aversion

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Framework

Individuals are described by wellbeing & prob. of existence

$\mathbf{X} = \cup_{n \in \mathbb{N}} (\mathbb{R} \times (0, 1])^n$: Set of finite allocations

$\mathbf{x} = ((x_1^w, x_1^p), \dots, (x_n^w, x_n^p))$: Distr. of wellbeing & prob.

x_i^w is individual i 's wellbeing where $x_i^w = 0$ represents neutrality

x_i^p is individual i 's probability of existence

$\nu(\mathbf{x}) = \sum_{i=1}^n x_i^p$ is the probability adjusted population size

\succsim A social welfare relation (SWR) on \mathbf{X}

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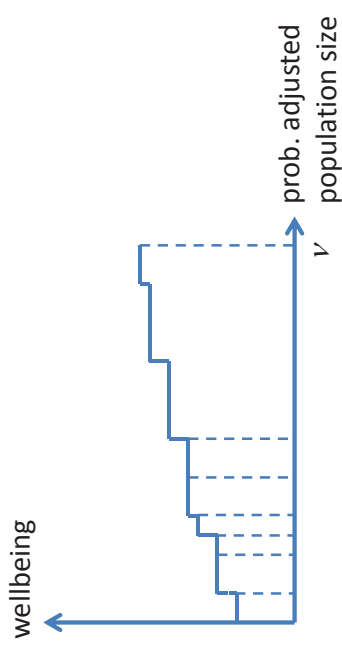
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Framework: Prob. adjusted rank-ordered allocation

$\mathbf{x}[\cdot] : (0, \nu(\mathbf{x})] \rightarrow \mathbb{R}$ the prob. adjusted rank-ordered allocation:
 $x_{[\rho]} = x_{\pi(r)}^w$ for $\rho_{r-1} < \rho \leq \rho_r$ and $1 \leq r \leq n(\mathbf{x})$



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Axioms

Axiom (O – Order)

The relation \succsim is complete, reflexive and transitive on \mathbf{X} .

Axiom (C – Continuity)

For all $\nu \in \mathbb{R}_{++}$ and $\mathbf{x} \in \mathbf{X}_\nu$, the sets $\{\mathbf{y} \in \mathbf{X}_\nu : \mathbf{y} \succsim \mathbf{x}\}$ and $\{\mathbf{y} \in \mathbf{X}_\nu : \mathbf{x} \succsim \mathbf{y}\}$ are closed.

Axiom (SS – Probability adjusted Suppes-Sen)

For all $\nu \in \mathbb{R}_{++}$ and $\mathbf{x}, \mathbf{y} \in \mathbf{X}_\nu$, if $\mathbf{x}[\cdot] > \mathbf{y}[\cdot]$, then $\mathbf{x} \succ \mathbf{y}$.

Axiom (IB – Existence independence of the best-off)

For all $\nu \in \mathbb{R}_{++}$, $\mathbf{x}, \mathbf{y} \in \mathbf{X}_\nu$, $p \in (0, 1]$ and $\mathbf{z} \in \mathbb{R}$ satisfying $\mathbf{z} \geq \max\{x_{[\rho]}, y_{[\rho]}\}$, $(\mathbf{x}, (\mathbf{z})_p) \succsim (\mathbf{y}, (\mathbf{z})_p)$ if and only if $\mathbf{x} \succsim \mathbf{y}$.

Axiom (IW – Existence independence of the worst-off)

For all $\mathbf{x}, \mathbf{y} \in \mathbf{X}$, $p \in (0, 1]$ and $\mathbf{z} \in \mathbb{R}$ satisfying $\mathbf{z} \leq \min\{x_{[0]}, y_{[0]}\}$, $(\mathbf{x}, (\mathbf{z})_p) \succsim (\mathbf{y}, (\mathbf{z})_p)$ if and only if $\mathbf{x} \succsim \mathbf{y}$.

Axiom (CL – Existence of a critical level)

There exist $c \in \mathbb{R}_+$ and $\nu \in \mathbb{R}_{++}$ such that, for all $p \in (0, 1]$ and $\mathbf{x} \in \mathbf{X}_\nu$ satisfying $x_{[\rho]} \leq c$, $(\mathbf{x}, (c)_p) \sim \mathbf{x}$.

Axiom (EE – Existence of egalitarian equivalence)

For all $\mathbf{x}, \mathbf{y} \in \mathbf{X}$ and $p \in (0, 1]$, if $\mathbf{x} \succ \mathbf{y}$, then there exists $\mathbf{z} \in \mathbb{R}$ such that, for all $N \in \mathbb{N}$, $\mathbf{x} \succ (\mathbf{z})_{np} \succ \mathbf{y}$ for some $n \geq N$.

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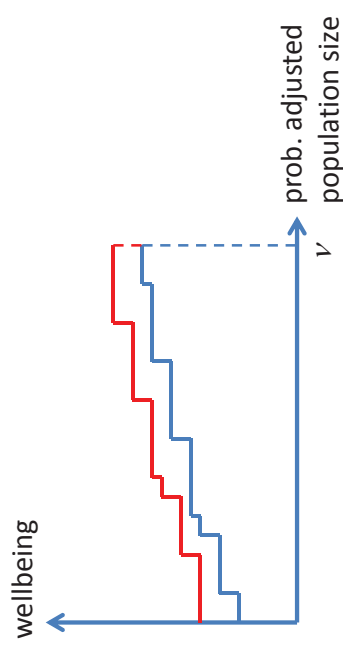
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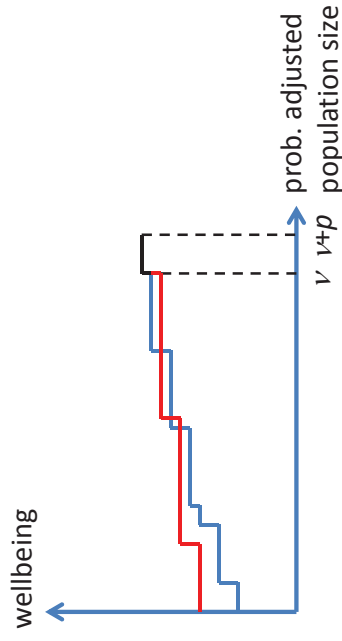
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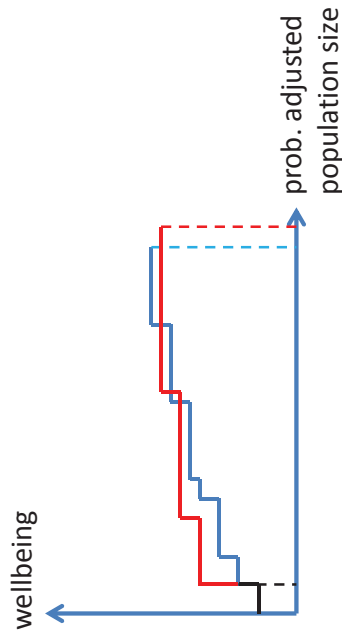
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Existence independence of the best-off

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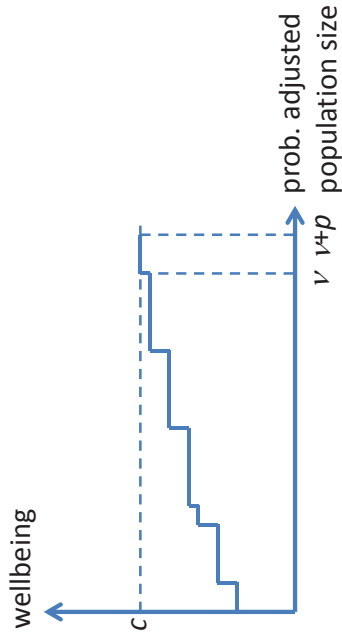


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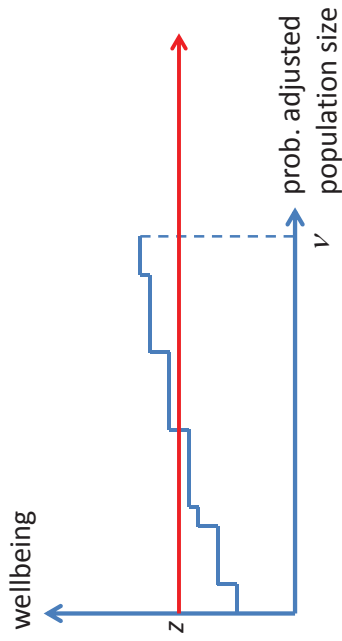


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Existence of a critical level

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Existence of egalitarian equivalence

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Characterization

Definition (Probability adjusted RDCLL)

An SWR on \mathbf{X} is a probability adjusted rank-discounted critical-level generalized utilitarian SWO (PARDCLU SWO) if it is represented by an SWF $W : \mathbf{X} \rightarrow \mathbb{R}$ defined by:

$$W(\mathbf{x}) = \int_0^{r'(\mathbf{x})} e^{-\delta\rho} (u(\mathbf{x}_{[\rho]}) - u(c)) d\rho,$$

where $\delta > 0$ is a rank utility discount rate and $u : \mathbb{R} \rightarrow \mathbb{R}$ is a continuous and increasing utility function.

Theorem

Consider an SWR \succsim on \mathbf{X} . These two statements are equivalent:

- (1) \succsim satisfies Axioms **O**, **C**, **SS**, **IB**, **IW**, **CL** and **EE**.
- (2) \succsim is an PARDCLU SWO.

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Sketch of proof

In the case with no uncertainty (i.e., if $\mathbf{X} = \cup_{n \in \mathbb{N}} (\mathbb{R} \times \{1\})^n$), the axioms collapse to Axioms 1–7 of Asheim and Zuber (2014)

Hence, in this case, by Theorem 1 of Asheim and Zuber (2014) there exist $0 < \beta_1 < 1$ and a continuous increasing function $u_1 : \mathbb{R} \rightarrow \mathbb{R}$ such that, for all $\mathbf{x}, \mathbf{y} \in \mathbf{X}$, $\mathbf{x} \succsim \mathbf{y}$ if and only if

$$\begin{aligned} & (1 - \beta_1) \sum_{r=1}^{n(\mathbf{x})} \beta_1^{r-1} (u_1(\mathbf{x}(\pi(r))) - u_1(c)) \\ & \geq (1 - \beta_1) \sum_{r=1}^{n(\mathbf{y})} \beta_1^{r-1} (u_1(\mathbf{y}(\pi(r))) - u_1(c)), \end{aligned}$$

where the factor $1 - \beta_1$ ensures that the weights sum up to $1 - \beta_1^{n(\mathbf{x})}$ and $1 - \beta_1^{n(\mathbf{y})}$

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Characterization

Definition (Probability adjusted RDCLL)

An SWR on \mathbf{X} is a probability adjusted rank-discounted critical-level generalized utilitarian SWO (PARDCLU SWO) if it is represented by an SWF $W : \mathbf{X} \rightarrow \mathbb{R}$ defined by:

$$W(\mathbf{x}) = \int_0^{r'(\mathbf{x})} e^{-\delta\rho} (u(\mathbf{x}_{[\rho]}) - u(c)) d\rho,$$

where $\delta > 0$ is a rank utility discount rate and $u : \mathbb{R} \rightarrow \mathbb{R}$ is a continuous and increasing utility function.

Theorem

Consider an SWR \succsim on \mathbf{X} . These two statements are equivalent:

- (1) \succsim satisfies Axioms **O**, **C**, **SS**, **IB**, **IW**, **CL** and **EE**.
- (2) \succsim is an PARDCLU SWO.

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The exact same arguments apply to the case with uncertainty if all individuals exist with probability $p = 1/k$, where $k \in \mathbb{N}$

In this case, there exist $0 < \beta_p < 1$ and a continuous increasing function $u_p : \mathbb{R} \rightarrow \mathbb{R}$ such that, for all $\mathbf{x}, \mathbf{y} \in \mathbf{X}$, $\mathbf{x} \succsim \mathbf{y}$ if and only if

$$\begin{aligned} & (1 - \beta_p) \sum_{r=1}^{n(\mathbf{x})} \beta_p^{r-1} (u_p(\mathbf{x}(\pi(r))) - u_p(c)) \\ & \geq (1 - \beta_p) \sum_{r=1}^{n(\mathbf{y})} \beta_p^{r-1} (u_p(\mathbf{y}(\pi(r))) - u_p(c)), \end{aligned}$$

where the factor $1 - \beta_p$ ensures that the weights sum up to $1 - \beta_p^{n(\mathbf{x})}$ and $1 - \beta_p^{n(\mathbf{y})}$

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The consistency requirement imposed by axioms **C** and **SS** across equivalent representations of the alternatives (where individuals are split, but total probability remains the same) implies that $u := u_1 = u_{1/k}$ and $\beta_1 = (\beta_{1/k})^k$ for all $k \in \mathbb{N}$

Hence, if all individuals exist with probability $p = 1/k$, then $\beta_p = e^{-\delta p}$, where $\delta := -\ln \beta_1$. Also:

$$\begin{aligned} & (1 - \beta_p) \beta_p^{r-1} = (1 - e^{-\delta p}) e^{-\delta p(r-1)} \\ & = e^{-\delta p(r-1)} - e^{-\delta pr} = \delta \int_{p(r-1)}^{pr} e^{-\delta \rho} d\rho \end{aligned}$$

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This leads to the Theorem in the case where all individuals exist with probability $1/k$

Extends to the case

where each individual exists with rational probability

Since the rational numbers are dense in the real numbers, the result can be extended to $\mathbf{X} \equiv \cup_{n \in \mathbb{N}} (\mathbb{R} \times (0, 1]^n)$ by again applying axioms **C** and **SS**.

Sketch of proof

$$\mathbf{x} \succ \mathbf{y} \Rightarrow \int_0^\nu e^{-\delta\rho} u(\mathbf{x}_{[\rho]}) d\rho > \int_0^\nu e^{-\delta\rho} u(\mathbf{y}_{[\rho]}) d\rho$$

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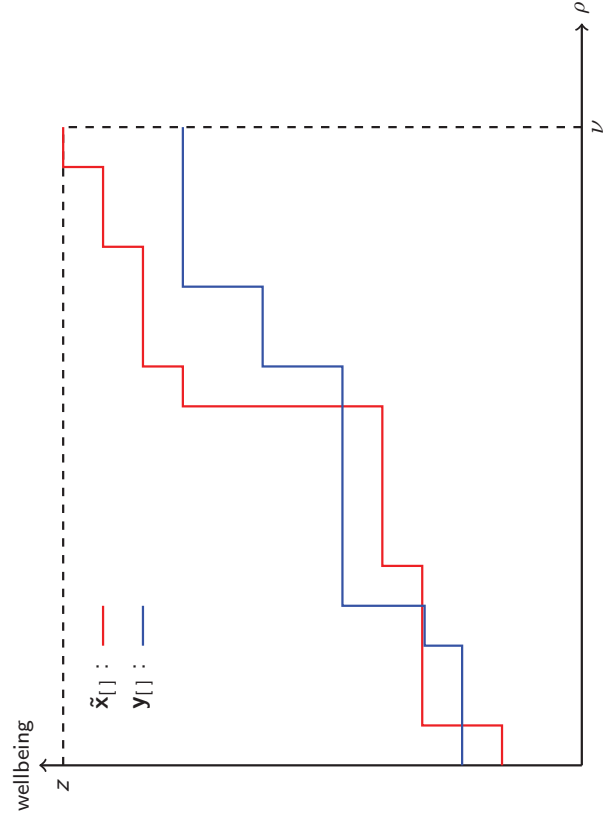
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Sketch of proof

$$\mathbf{x} \succ \mathbf{y} \Rightarrow \int_0^\nu e^{-\delta\rho} u(\mathbf{x}_{[\rho]}) d\rho > \int_0^\nu e^{-\delta\rho} u(\mathbf{y}_{[\rho]}) d\rho$$

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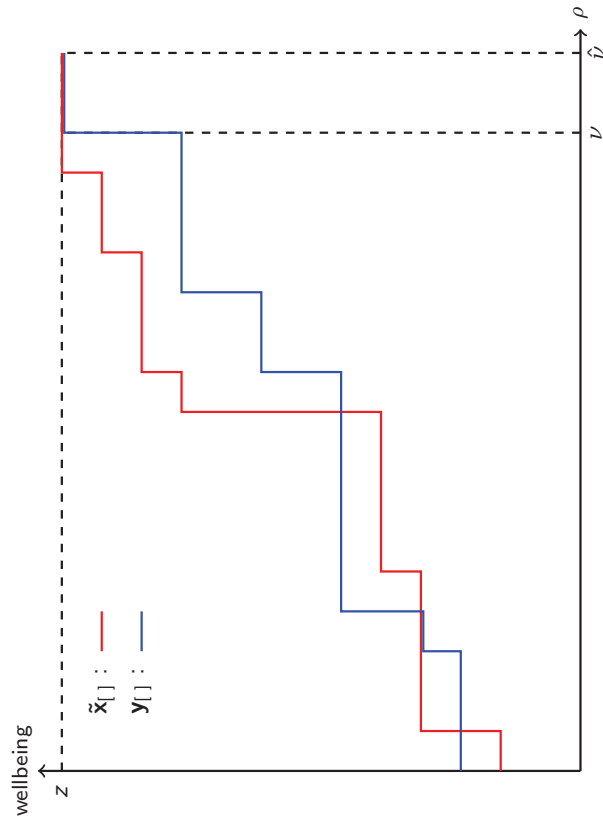
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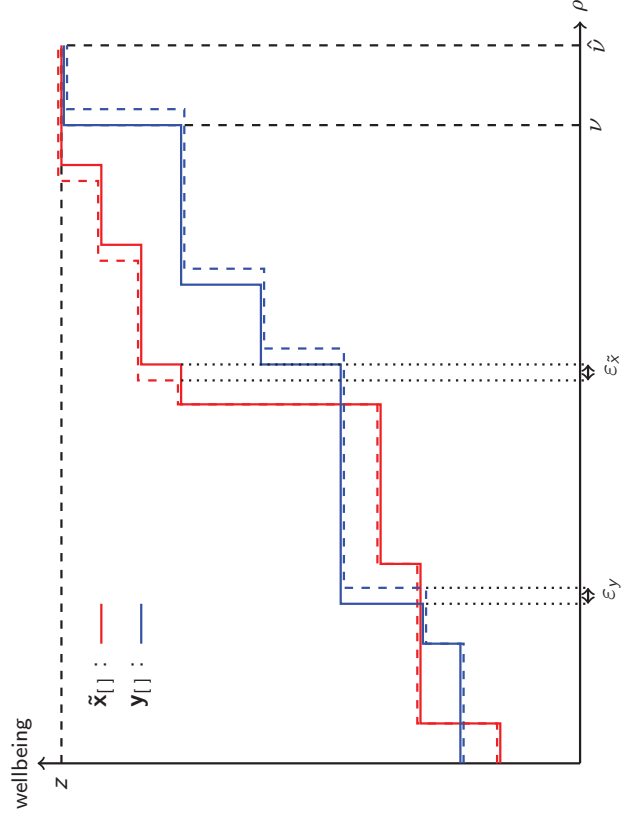
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Sketch of proof

$$\mathbf{x} \succ \mathbf{y} \Rightarrow \int_0^\nu e^{-\delta\rho} u(\mathbf{x}_{[\rho]}) d\rho > \int_0^\nu e^{-\delta\rho} u(\mathbf{y}_{[\rho]}) d\rho$$



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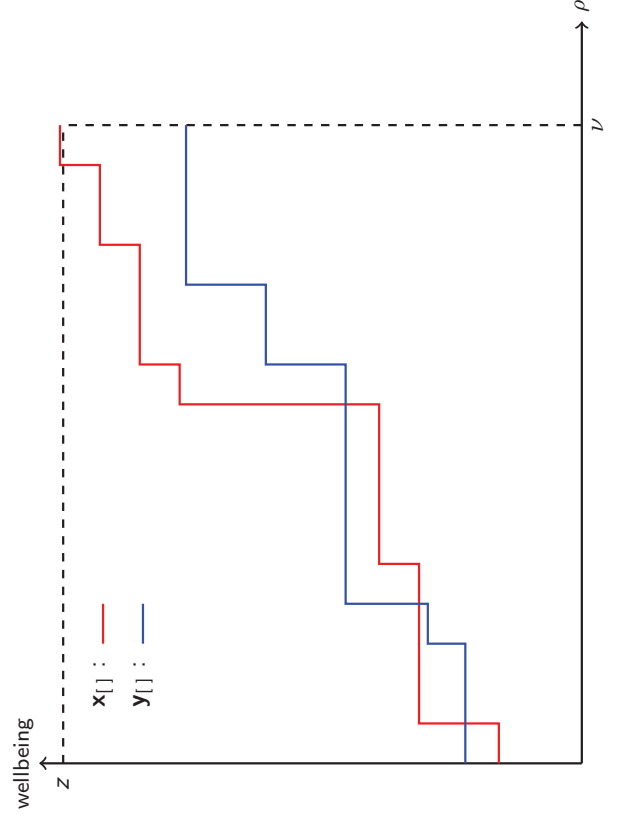
$$\int_0^\nu e^{-\delta\rho} u(\mathbf{x}_{[\rho]}) d\rho > \int_0^\nu e^{-\delta\rho} u(\mathbf{y}_{[\rho]}) d\rho \Rightarrow \mathbf{x} \succ \mathbf{y}$$

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Sketch of proof

$$\int_0^\nu e^{-\delta\rho} u(\mathbf{x}_{[\rho]}) d\rho > \int_0^\nu e^{-\delta\rho} u(\mathbf{y}_{[\rho]}) d\rho \Rightarrow \mathbf{x} \succ \mathbf{y}$$



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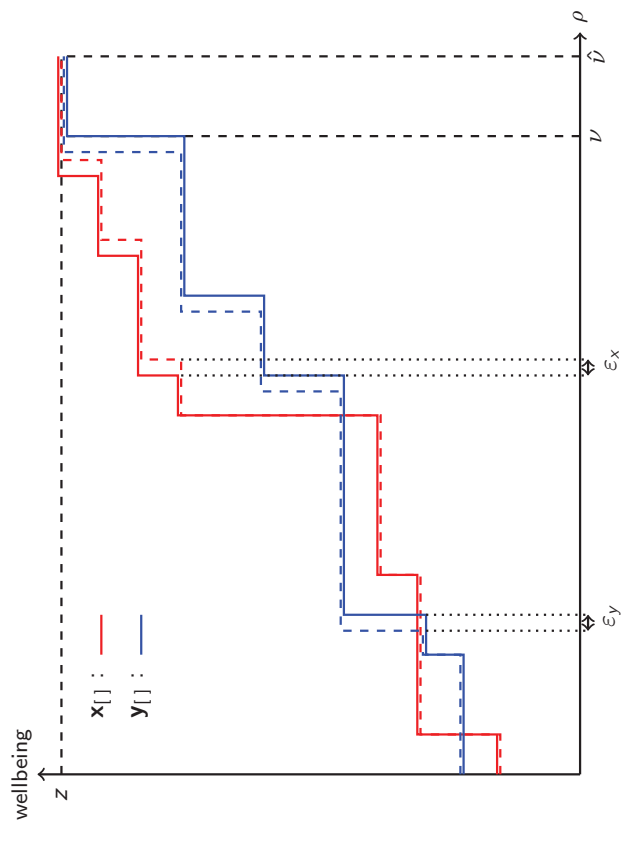
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Sketch of proof

$$\int_0^\nu e^{-\delta\rho} u(\mathbf{x}_{[\rho]}) d\rho > \int_0^\nu e^{-\delta\rho} u(\mathbf{y}_{[\rho]}) d\rho \Rightarrow \mathbf{x} \succ \mathbf{y}$$



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Special cases

- Rank-dependent expected utilitarianism
 - ◇ With prob. adjusted population size $\nu(\mathbf{x}) = \sum_{i=1}^n x_i^p = 1$, Theorem 1 leads to *rank-dependent expected utilitarianism* where the decision maker substitutes 'decision weights' for probability (Quiggin, 1982), but with additional structure.
- Human extinction
 - ◇ To what extent does PARDCLU provide a foundation for discounting according to the prob. of human extinction, as applied in, e.g., the Stern Review?

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Rank-dependent expected utilitarianism

Interpretation: 1 person being subject to a lottery where prizes (w_1, \dots, w_S) are won w/prob. (p_1, \dots, p_S)

Let $(w_{[1]}, \dots, w_{[S]})$ denote a reordering of (w_1, \dots, w_S) : $w_{[r]} \leq w_{[r+1]}$ for all ranks $r = 1, \dots, S - 1$

Write $\mathbf{p} = (p_{[1]}, \dots, p_{[S]})$

PARDCLU implies preferences for lotteries represented by:

$$\sum_{r=1}^S h_r(\mathbf{p}) u(w_{[r]}),$$

$$h_r(\mathbf{p}) = f\left(\sum_{r'=1}^r p_{[r']}\right) - f\left(\sum_{r'=1}^{r-1} p_{[r']}\right),$$

where the prob. weighting fcn. $h_r : [0, 1]^S \rightarrow [0, 1]$ are defined by

with $f : [0, 1] \rightarrow [0, 1]$ given by $f(\rho) = (1 - e^{-\delta\rho}) / (1 - e^{-\delta})$

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Human extinction

Assume that the probability of generation t is π^t for $t = 1, \dots, T$

Assume that population is constant (and normalized to 1) up to the time of extinction.

If well-being is correlated with time so that $w_t \leq w_{t+1}$ for all $t = 1, \dots, T - 1$,

then PARDCLU implies preferences represented by:

$$\sum_{t=1}^T \left[f\left(\frac{\pi(1-\pi^t)}{1-\pi}\right) - f\left(\frac{\pi(1-\pi^{t-1})}{1-\pi}\right) \right] u(w_t),$$

where $f : \mathbb{R}_+ \rightarrow \mathbb{R}_+$ is given by $f(\rho) = (1 - e^{-\delta\rho}) / (1 - e^{-\delta})$

As $\delta \downarrow 0$ and, thus, f approaches the identity function:

$$f\left(\frac{\pi(1-\pi^t)}{1-\pi}\right) - f\left(\frac{\pi(1-\pi^{t-1})}{1-\pi}\right) \rightarrow \frac{\pi}{1-\pi} (\pi^{t-1} - \pi^t) = \pi^t$$

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Concluding remarks: PARDCLU . . .

- . . . generalizes **rank-discounted utilitarianism** (Zuber and Asheim, 2012) by analyzing individuals rather than generations
- . . . allows for analysis of a situation
 - ◇ where population changes endogenously
 - ◇ with risk
- . . . thereby prepares **rank-discounted utilitarianism** for practical application in numerical models
 - ◇ however, one can question the relevance of doing so

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Concluding remarks

- + **Rank-discounted criteria** are needed to fill out the space
 - ◇ between utilitarianism
 - ◇ and leximin
- **Rank-discounted criteria** lead to time-inconsistency unless
 - ◇ wellbeing is correlated with time, or
 - ◇ evaluation is dependent what has been or could have been

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Voluntary Human Extinction Movement

- Under **PARDCLU** important to avoid lives with low wellbeing to increase the value of lives with high wellbeing
- Therefore, important to reduce the size of (or eliminate entirely) the population that might potentially live when future environmental effects become grave
- Leads to antinatalism
 - ◇ *Voluntary Human Extinction Movement*
 - Undermines support for sustainability
 - ◇ Suppose per capita wellbeing is decreasing over time. Under **PARDCLU** it might be social preferable to increase the per-period probability of extinction, as this increases the utility weight on the better-off earlier generations.

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Climate change

- Population control a means for avoiding climate change
- ◇ Less need to burn fossil fuels with small population size
- Population control as an ultimate adaptation policy
- ◇ No-one will experience catastrophic climate change

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A solution to take into account social context & time inconsistency?

- Peoples' living situation will be influenced by
- Restricted reproduction
 - Severely downsized future generations

Should this be included in the notion of wellbeing? Include

- ◇ altruism (people care about future generations)
- ◇ existence value (people care that future generations exist)
- ◇ reproductive value (people care about having children)

Take into account time inconsistency:

- ◇ Protecting future generations with low wellbeing becomes more important as their time of potential existence becomes nearer

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