Distributive Justice and Economic Inequality

P. G. Piacquadio

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Overview

- Arrow's impossibility has been as important as traumatic for social choice.

  "If we exclude the possibility of interpersonal comparisons of utility, then the only methods of passing from individual tastes to social preferences which will be satisfactory and which will be defined for a wide range of sets of individual orderings are either imposed or dictatorial." [Arrow, 1963, p.59]

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- This statement seems to suggest that a “satisfactory” social ranking that is neither imposed nor dictatorial requires comparable information about individual’s utilities, such as subjective well-beings or happiness.
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- While some economists never accepted this conclusion (i.e. Samuelson, Atkinson, etc.), others have tried to escape from subjective well-beings or happiness in other ways.

- The fair allocation theory (see Thomson, 2011, Chapter 22 in the *Handbook of Social Choice and Welfare*):
  - Steinhaus, Foley, Varian, Dworkin, Moulin, Thomson, etc.
  - The idea is to give up a complete ranking of alternatives and only determine how society ought to choose.

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The canonical model

- **Society** consists of a finite set of individuals $N \equiv \{1, ..., n\}$.
- The question is how to share a **social endowment** $\Omega \in \mathbb{R}^{\ell}_{++}$ to individuals. An **allocation** $x \equiv (x_1, ..., x_n)$ assigns a bundle $x_i \in X \equiv \mathbb{R}^{\ell}_{+}$ to each $i \in N$. An allocation is **feasible** if $\sum_i x_i \leq \Omega$.
- Each individual has strongly monotone, continuous, and convex preferences R_i over $X \equiv \mathbb{R}^{\ell}_{+}$. A **preference profile** is $R \equiv (R_1, ..., R_n)$.
- An economy $E \equiv (\Omega, R)$ specifies the endowment and the preference profile. The domain of economies is $\mathscr{E}$.
- So, we are after the mapping $\phi : \mathscr{E} \rightrightarrows X^n$, named **allocation rule**, that associates to each $E \in \mathscr{E}$ a non-empty subset of feasible allocations.

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Axioms

- An allocation rule ϕ satisfies **Pareto efficiency** if for each $E \in \mathscr{E}$ and each $x \in \phi(E)$, there exists no feasible allocation x' such that for each $i \in N$, $x'_i R_i x_i$ and for some $i \in N$, $x'_i P_i x_i$.
- An allocation rule ϕ satisfies **equal-split guarantee** if for each $E \in \mathscr{E}$ and each $x \in \phi(E)$, no individual $i \in N$ prefers the equal share of resources to her assignment: for each $i \in N$, $x_i R_i \frac{\Omega}{n}$.
- An allocation rule ϕ satisfies **no-domination** if for each $E \in \mathscr{E}$ and each $x \in \phi(E)$, no individual $i \in N$ is assigned less than any other individual: for each pair $i, j \in N$, $x_i \not\ll x_j$.

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- An allocation rule ϕ satisfies **equal-treatment of equals** if for each $E \in \mathscr{E}$, each pair $i, j \in N$ such that $R_i = R_j \equiv R_0$, and each $x \in \phi(E)$, $x_i I_0 x_j$.
- An allocation rule ϕ satisfies **envy freeness (or no-envy)** if for each $E \in \mathscr{E}$,each pair $i, j \in N$, and each $x \in \phi(E)$, $x_i R_i x_j$.
- [Note that *envy freeness* implies both *no-domination* and *equal-treatment of equals*.]

Theorem

The **equal-split Walrasian** allocation rule satisfies all the above axioms.

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The equal-split Walrasian allocation rule

- The idea is very simple.
- As a thought experiment, assume you assigned to each individual an equal share of resources Ω/n.
- Now, let them trade at Walrasian equilibrium prices.
- Assign to each the bundles they would choose.
- Since the Walrasian equilibrium is *Pareto efficient*, this allocation rule is as well. Since the equal share of resources could be chosen, it cannot be preferred to their choice, so *equal-split guarantee* holds. Since all individuals choose from the same budget set, the allocation rules satisfies *no-envy* (and thus, *no-domination* and *equal-treatment of equals*).

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Discussion

- Drawbacks:
  - What if the first-best chosen allocations are not feasible?
  - There are no results that hold for all type of problems;
  - Each domain has its specific axioms and solutions;
  - Thus, the field moved towards more and more specialization.
- An allocation rule as a ranking.
  - An allocation rule can be thought of as a very special ranking: the allocations that are chosen are top ranked; all others are bottom-ranked;
  - What axiom among Arrow's are violated?

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"Provisional utilitarianism"

- "My own attitude to problems of political action has always been one of what I may call provisional utilitarianism." (Robbins, 1938)
- This approach is well described by Lucas (2003) in his Presidential Address to the AEA. He discusses "the general logic of quantitative welfare analysis:"
  - "To evaluate the effects of policy change on many different consumers, we can calculate welfare gains (perhaps losses, for some) for all of them, one at a time, and add the needed compensations to obtain the welfare gain for the group."

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Utilitarianism

- Utilitarianism measures the desirability of a social decision by the sum of the utilities enjoyed by individuals in society. Formally:

$$W(\cdot) \equiv \sum_{i \in N} U_i(\cdot)$$

- Utilitarianism faces two major difficulties:
  - it requires that the functions U_i be cardinally measurable and unit comparable;
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Fairness and Utilitarianism

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Fairness and Utilitarianism: wealth or happiness?

Of two people having unequal fortunes, he who has most wealth must by a legislator be regarded as having most happiness. But the quantity of happiness will not go on increasing in anything near the same proportion as the quantity of wealth: ten thousand times the quantity of wealth will not bring with it ten thousand times the quantity of happiness. [Pannomial Fragments, Bentham J., 1843]

Utilitarianism: income distributions

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where the functions $(U_i)_{i \in N}$ are:

- equal across individuals;
- increasing; and
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	Utilitarianism	Fairness
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- increasing; and	non satiation	efficiency
- concave.	law of decreasing marginal utilities	Pigou-Dalton transfer principle

Two approaches to utilitarianism

Income distributions	Utility based	Fairness
	$W = \sum_{i \in N} U(y_i)$	$W = \sum_{i \in N} U(y_i)$
	Based on: -interpersonal comparability; -cardinal measurability.	Based on: -anonymity; -Pareto; -Dalton's transfer principle.

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Income distributions	$W = \sum_{i \in N} U(y_i)$ Based on: -interpersonal comparability; -cardinal measurability.	$W = \sum_{i \in N} U(y_i)$ Based on: -anonymity; -Pareto; -Dalton's transfer principle.
Multi-commodity allocations	$W = \sum_{i \in N} U_i(x_i)$ Based on: -interpersonal comparability; -cardinal measurability.	

Model

- Society consists of $n \geq 3$ individuals: $N = \{1, ..., n\}$.
- Each individual $i \in N$ has well-behaved preferences R_i *(continuous, strongly monotone, convex, + ...)* over the own consumption set $X = \mathbb{R}_+^\ell$.
- An **allocation** is a list $x \equiv \left(\{x_i\}_{i \in N}\right) \in X^n$.
- A **social ranking**, denoted $\succsim$, is a weak ordering of all allocations.

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Continuity
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efficiency
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separability

$$\Rightarrow \quad W(x) = \sum U_i(x_i)$$

Possibility of trade-offs

Let $(0, \ldots, x_i, \ldots, 0)$.

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Then, for each $j \neq i$ there exists $x_j \in X$ *such that*

$$(0, \dots, x_i, \dots, 0, \dots, 0) \sim (0, \dots, 0, \dots, x_j, \dots, 0)$$

Continuity
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efficiency
+
separability
+
possibility of trade-offs

$\Rightarrow$

$$W(x) = \sum U_i(x_i),$$

where

each U_i has the same image.

Discrimination (income distributions)

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Discrimination (income distributions)

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there exists $j \neq i$ and $y_j \geq 0$ *such that*

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then **i is discriminated against at y_i.**

Discrimination (multi-commodities)

Let $(0, \dots, x_i, \dots, 0)$.

If

for each $x'_i \in X$ with $x'_i \; I_i \; x_i$

there exists $j \neq i$

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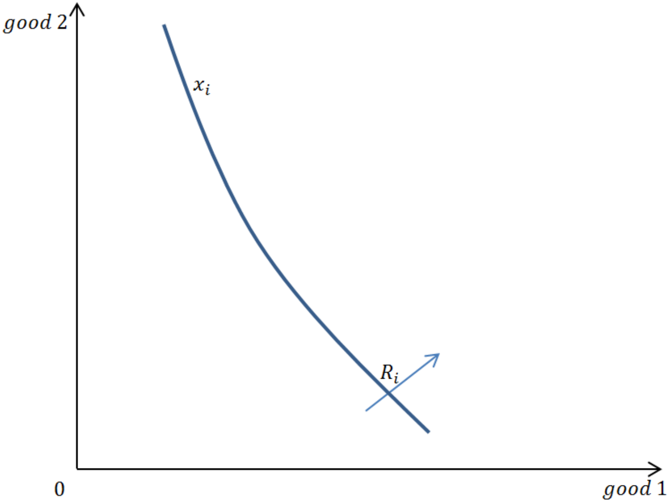
such that

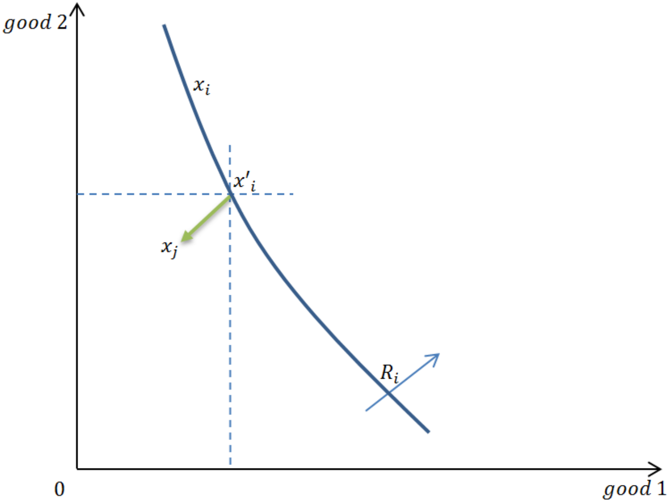
$$(0, \dots, 0, \dots, x_j, \dots, 0) \succ (0, \dots, x'_i, \dots, 0, \dots, 0)$$

and

$$x_j \ll x'_i,$$

then **i is discriminated against at x_i.**


good 2
x_i
R_i
0
good 1


good 2
x_i
x'_i
x_j
R_i
0
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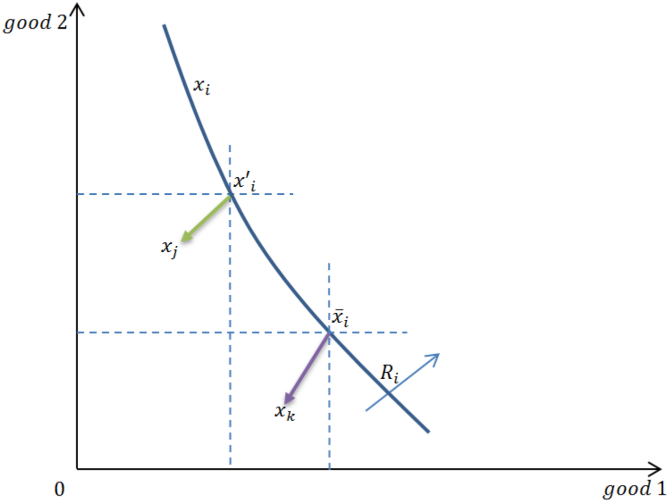

good 2
x_i
x'_i
x_j
$\bar{x}_i$
R_i
x_k
0
good 1

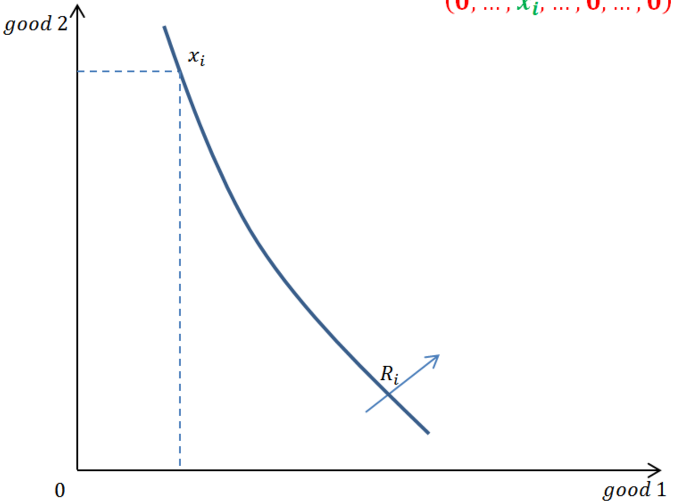
Non discrimination

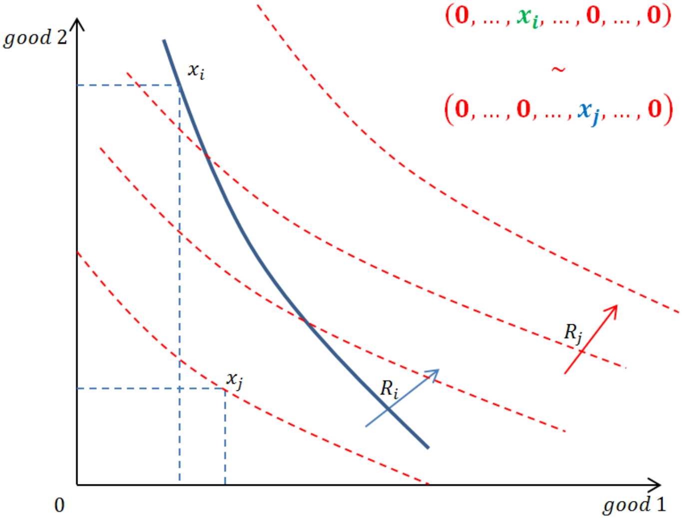
There exist no individual i and assignment x_i

such that

i is discriminated against at x_i.

$(0, \dots, x_i, \dots, 0, \dots, 0)$

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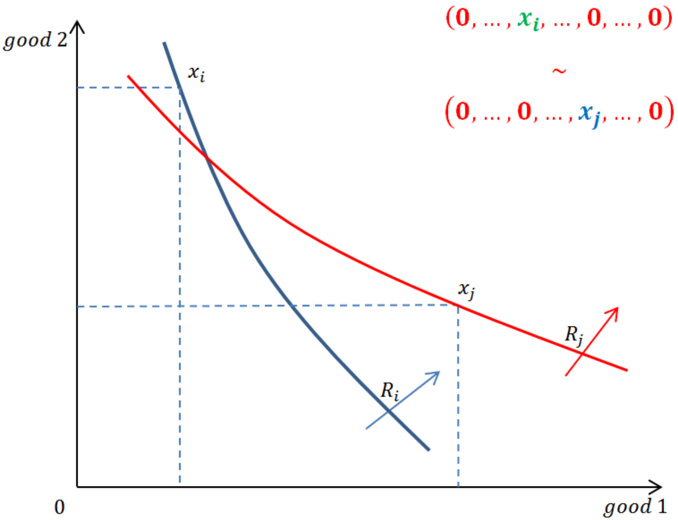
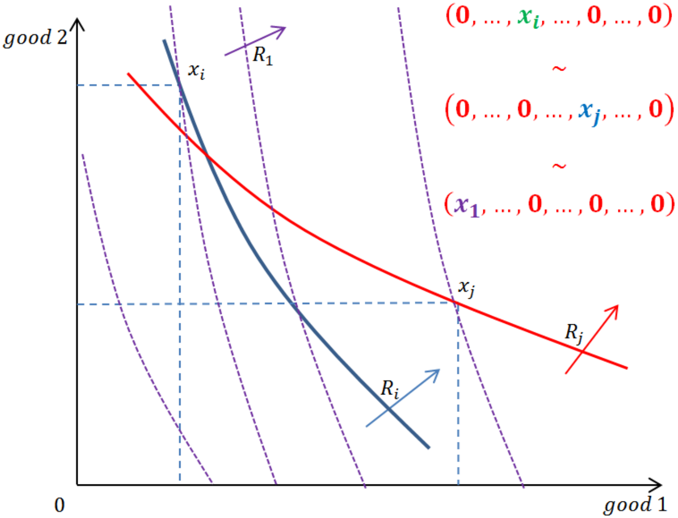
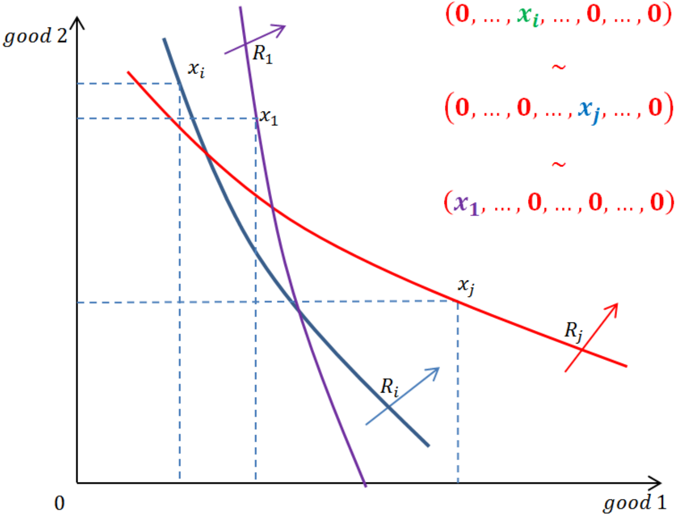
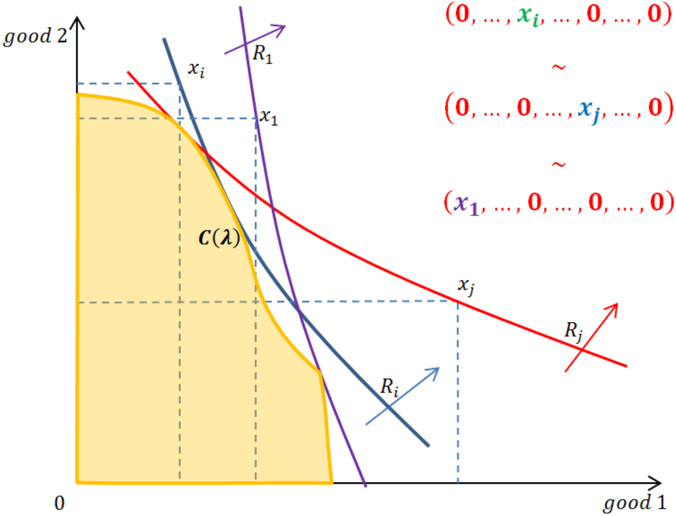

$(0, \ldots, x_i, \ldots, 0, \ldots, 0)$
$\sim$
$(0, \ldots, 0, \ldots, x_j, \ldots, 0)$
good 2
x_i
x_j
R_j
R_i
0
good 1


good 2
good 1
0
x_i
x_j
R_1
R_i
R_j
$(0, \dots, x_i, \dots, 0, \dots, 0)$
$\sim$
$(0, \dots, 0, \dots, x_j, \dots, 0)$
$\sim$
$(x_1, \dots, 0, \dots, 0, \dots, 0)$

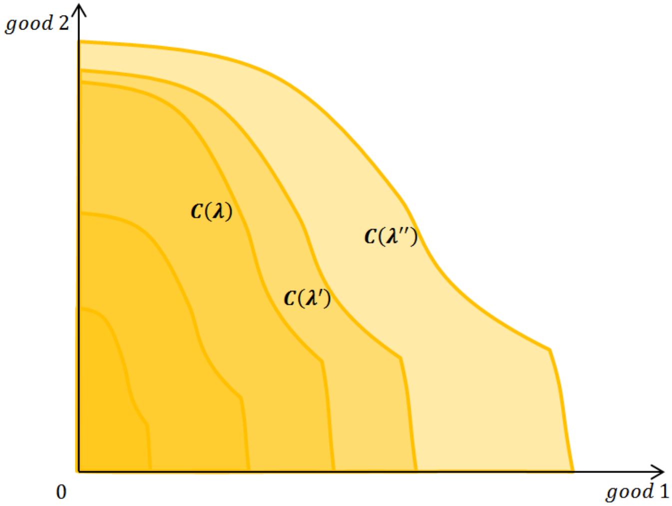


$$(0, \dots, x_i, \dots, 0, \dots, 0)$$
$$\sim$$
$$(0, \dots, 0, \dots, x_j, \dots, 0)$$
$$\sim$$
$$(x_1, \dots, 0, \dots, 0, \dots, 0)$$



$$(0, \dots, x_i, \dots, 0, \dots, 0)$$
$$\sim$$
$$(0, \dots, 0, \dots, x_j, \dots, 0)$$
$$\sim$$
$$(x_1, \dots, 0, \dots, 0, \dots, 0)$$

The "opportunity mapping"



The "opportunity-equivalent" representation

$$U_i^C(x_i) = \lambda \quad \Longleftrightarrow \quad \begin{array}{c} x_i \, I_i \, z_i \\ \text{and} \\ z_i \text{ maximizes } R_i \text{ on } C(\lambda) \end{array}$$

Continuity
+
efficiency
+
separability
+
possibility of trade-offs
+
non discrimination

$\Rightarrow$

$$W(x) = \sum g \circ U_i^C(x_i),$$

where

g is order-preserving.

Equal-preference transfer

Let $R_i = R_j = R$ and $x_i \; P \; x_j$.

Define $\Delta = x_i - x_j$.

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$$x_i - \alpha\Delta \;\; P \;\; x_j + \alpha\Delta$$

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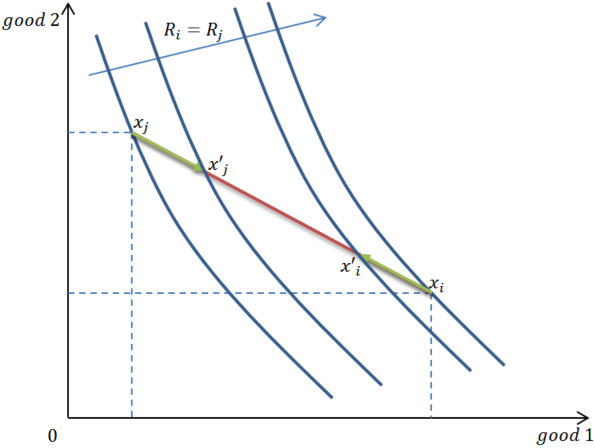
Define $\Delta = x_i - x_j$.

If

$x_i - \alpha\Delta \;\; P \;\; x_j + \alpha\Delta$

then

$$(x_1, \ldots, x_i - \alpha\Delta, \ldots, x_j + \alpha\Delta, \ldots, x_n) \succcurlyeq (x_1, \ldots, x_i, \ldots, x_j, \ldots, x_n)$$


good 2
$R_i = R_j$
x_j
x'_j
x'_i
x_i
0
good 1

Continuity
+
efficiency
+
separability
+
possibility of trade-offs
+
non discrimination
+
equal-preference transfer

$\Rightarrow$

Opportunity-equivalent utilitarian

$$W(x) = \sum f \circ \phi \circ U_i^C(x_i),$$

where

f reflects inequality aversion;

$\phi \circ U_i^C$ is concave for each i;

ϕ is "least joint concave".

Continuity
+
efficiency
+
separability
+
possibility of trade-offs
+
non discrimination
+
equal-preference transfer

$\Leftarrow$
$\Rightarrow$

Opportunity-equivalent utilitarian

$$W(x) = \sum f \circ \phi \circ U_i^C(x_i),$$

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A special case

- The money-metric representation of preferences measures each agent's well-being by the minimum income necessary, at some reference prices, to purchase a bundle that the agent considers as desirable as the assigned one (see McKenzie, 1957; Samuelson and Swamy, 1974).