ECON 4310
Labor supply elasticites

by
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This note is a supplement to the Labor supply note, section 1 Static model, single individuals, avaiable on the webside for the 4310 course.

Here we will derive the elasticites set out in the Labor supply note.

As before let

U= the utility level
C= disposable income=consumption
h= annual hours worked
L=leisure
w= hourly wage rate
k=non-wage income, hereafter called capital income.
T= taxes

The behavior of the individual follows from solving the following maximization problem:

$$\mathrm{Max}_h U(C, L)$$

subject to

$$(1) \quad C \le wh + k - T(wh, k)$$

$$(2) \quad L = 1 - \nu h$$

$$(3) \quad \nu = \frac{1}{8760}$$

$$(4) \quad h \ge 0$$

Eqs (2) and (3) imply that we have normalized annual leisure to vary between 0 and 1.

T(.) is the tax function. In practice this function is a stepwise linear function of wh. In Norway the tax function has the following structure:

$$(5) \begin{cases} T = t_k k \text{ for } wh \le b \\ T = t_k k + t_1(wh - b) \text{ for } b \le wh \le R_1 \\ T = t_k k + t_1(R_1 - b) + t_2(wh - R_1) \text{ for } R_1 \le wh \le R_2 \\ T = t_k k + t_1(R_1 - b) + t_2(R_2 - R_1) + t_3(wh - R_2) \text{ for } R_2 \le wh \end{cases}$$

The marginal tax rate on capital income is t_k and it is constant (proportional taxation), while taxation on wage income is progressive. Wage income below b is not taxed. Wage income in the interval (b,R_1) is taxed at the marginal tax rate t_1, wage income in the next interval (R_1, R_2) is taxed at the marginal tax rate rate t_2, and wage income above R_2 is taxed at the marginal tax rate t_3. In a strict progressive tax system, $t_1<t_2<t_3$. In the tax literature the income intervals are called tax brackets. We observe that (5) gives a step-wise linear representation of the tax function in eq (1). Note that the policy instrument of the government are the tax rates (t_k and t_i, i=1,2,3) and the bounds of the tax brackets (b, R_1 and R_2).

Combining (1) and (5) we get

$$(6) \quad C = m_i h + I_i \ ; i = 0,1,2,3.$$

where

$$(7)\left\langle\begin{array}{l}\{m_0 = w \ ; I_0 = k(1-t_k)\ \} \text{ for } wh \le b \\ \{m_1 = w(1-t_1)\ ; I_1 = k(1-t_k) + t_1 b\ \} \text{ for } b \le wh \le R_1 \\ \{m_2 = w(1-t_2)\ ; I_2 = k(1-t_k) + t_1 b + (t_2 - t_1)R_1\ \} \text{ for } R_1 \le wh \le R_2 \\ \{m_3 = w(1-t_3)\ ; I_3 = k(1-t_k) + t_1 b + (t_2 - t_1)R_1 + t_3 R_2\ \} \text{ for } R_2 \le wh\end{array}\right.$$

In the tax-labor supply literature m_i is called the marginal wage rate and I_i the virtual income.

We will assume that the utility function has the follwing structure:

$$(8) \quad U = \alpha\frac{C^\mu - 1}{\mu} + \beta\frac{L^\gamma - 1}{\gamma}$$

where

$$(9) \quad \{\mu, \gamma\} \le 1$$

Eq (9) is a sufficient condition for a quasi-concave utility function.

The maximization problem now is

$$(10\left\langle\begin{array}{l}\text{Max}_h\left[U = \alpha\frac{C^\mu - 1}{\mu} + \beta\frac{L^\gamma - 1}{\gamma}\right] \\ \text{given} \\ C = mh + I \\ L = 1 - \nu h\end{array}\right.$$

We should keep in mind that m and I vary with respect to the optimal income and hence optimal hours.

The first order conditions (necessary conditions, but if (9) is fulfilled they are also sufficient conditions) are:

$$(11)\quad \frac{\partial U/\partial L}{\partial U/\partial C}=\frac{m}{\nu}$$

or using (8) and $L=1-\nu h$

$$(12)\quad \frac{(1-\nu h)^{\gamma-1}}{C^{\mu-1}}=\frac{m}{\nu}$$

In addition

$$(13)\quad C=mh+I$$

Taking logs in (12) gives us the following representation of the first order conditions:

$$(14)\quad (\gamma-1)\ln(1-\nu h)-(\mu-1)\ln C=\ln m-\ln\nu$$

$$(13)\quad C=mh+I$$

We are now ready to derive the elasticities. These elastcities have to be interpreted as giving the marginal change in optimal hours around the optimal point.

1) **The uncompensated marginal wage elasticity (Cournot).**

Inserting (13) in (14) and taking the derivatives of h wrt m, and using the definition of an elasticity, which here says that $El(h:m)=\frac{\partial h}{\partial m}\frac{m}{h}$, we get immediately:

$$(15)\quad El(h:m)=\frac{1+(\mu-1)\frac{mh}{C}}{(1-\mu)\frac{mh}{C}+(1-\gamma)\frac{\nu h}{(1-\nu h)}}$$

The numerator can be negative or positive depending on the magnitudes of (μ, mh and C). The denominator is positive given that $(\mu,\gamma)<1$. (For those who are interested: Show that while $(\mu,\gamma)<1$ is a suffcient condition for a quasi-concave utility function, a necessary condition is that the denominator in (15) is positive.)

2) **The virtual income elasticity.**

Following the same procedure but now taking the derivative with respect to I and henceforth using the definition of an elasticity, we get

$$(16) \quad \mathrm{El}(h:I) = \frac{(\mu-1)\dfrac{I}{C}}{(1-\mu)\dfrac{mh}{C} + (1-\gamma)\dfrac{\nu h}{(1-\nu h)}}$$

3) The compensated (utility constant) marginal wage elasticity (Slutsky).

Now, we cannot use (13), because we have to replace it by the condition that utility is kept constant, or

$$(17) \quad \bar{U} = U(C, L)$$

Taking the derivatives in (17) wrt the marginal wage rate we get

$$(18) \quad \frac{\partial U}{\partial C}\frac{\partial C}{\partial m} + \frac{\partial U}{\partial L}\frac{\partial L}{\partial m} = 0$$

which implies

$$(19) \quad \frac{\partial C}{\partial m} = -\frac{\partial U/\partial L}{\partial U/\partial C}\frac{\partial L}{\partial m}$$

But since (11) holds at the optimal point (before the marginal wage is changed), we get

$$(20) \quad \frac{\partial C}{\partial m} = -\frac{m}{\nu}\frac{\partial L}{\partial m} = -\frac{m}{\nu}(-\nu\frac{\partial h}{\partial m}) = m\frac{\partial h}{\partial m}.$$

Taking the derivative in (14) wrt m and using (20) and the definition of an elasticity, we get

$$(21) \quad \mathrm{El}(h:m \mid U = \bar{U}) = \frac{1}{(1-\mu)\dfrac{mh}{C} + (1-\gamma)\dfrac{\nu h}{(1-\nu h)}}$$

4) The consumption constant marginal wage elasticity (Frisch).

Holding C constant in (14) we immediately get

$$(22) \quad \mathrm{El}(h:m \mid C = \bar{C}) = \frac{1}{1-\gamma}\frac{1-\nu h}{\nu h}.$$