ECON 4310
MCandless and Wallace, Chapter 1[1]

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NOTE: check course homepage on
folk.uio.no/∼kjstore/teaching/econ4310/

1 Introduction

- Aggregate behavior of the economy must be an outcome of individual behavior (study macroeconomics with theory from microeconomics)
- Intertemporal aspects of macroeconomics
- Study the whole time interval
- A dynamic game: Agents understand the rules of the game in the sense that they can (rationally) evaluate the consequences of all players' actions including their own. Each agent has an objective function and choose (rationally) their strategy in order to maximize it.
- Key question: how would the economy evolve under different policy rules?
- Government policy ≡ policy rule (commitment)
- Expectations about the future matters for the current outcomes
- Standard welfare analysis for evaluating different policies.

2 The model

- Description of the physical environment (e.g. population and preferences) and the kind of social organizations in place (e.g. market structure).

[1]The lecture notes of the first part of the class (first 7-8 lectures) are largely based on McCandless and Wallace. Correspondance to kjetil.storesletten@econ.uio.no

2.1 The environment

Time

- Time t is discrete and indexed by integers, $t \in \{-\infty, ..., 0, ..., +\infty\}$.
- $t = 1$ is the initial period
- $t \in \{-\infty, ..., 0\}$ is the history of the economy and exogenously given.

Population

- Overlapping generation model (alternative: infinitely lived dynasty model).
- At each time period t, a new generations is born. There are $N(t)$ members of this generation. All generations live for two periods; generation t is "young" at time t and "old" at time $t+1$.[2]
- Population size and longevity are exogenous.

Resources

- Each period t there is only one good available (the "t" good). Until lecture 5 (chapter 8 in M & W) this good will be perishable (apples, say).
- In each period the economy is endowed with $Y(t)$ units of the time good t. The sequence $\{Y(t)\}_{t=1}^{\infty}$ denotes all current and future endowments.

Feasible Consumption Allocations

- A "consumption allocation" describes who consumes what.
- Let $c_t^h(s)$ stand for the consumption of agent $\#h$ of generation t when she is s periods old, where $h \in \{1, 2, ..., N(t)\}$.
- Let c_t^h denote a vector (or ordered pair) of consumption over a lifetime,

$$c_t^h = \left\{c_t^h(t), c_t^h(t+1)\right\}.$$

[2] A generalization of the OLG model is an economy where agents survive with some probability. One such model is the "perpetual youth" model of Blanchard and Yaari.

- A time t consumption allocation denotes the set of consumptions for the young $\left\{c_t^h(t)\right\}_{h=1}^{N(t)}$ and the old $\left\{c_{t-1}^h(t)\right\}_{h=1}^{N(t-1)}$.
- A consumption allocation is the sequence of consumption allocations for $t \in \{1, 2, ..., \infty\}$.
- A feasible allocation is a consumption allocation that can be achieved given the total amount of resources in the economy.
- The total consumption at time t is denoted $C(t)$ and is given by

$$C(t) = \sum_{h=1}^{N(t)} c_t^h(t) + \sum_{h=1}^{N(t-1)} c_{t-1}^h(t)$$

For an "endowment" economy (without access to storage technology) we have that

Definition 1 *A consumption allocation is feasible if the path of total consumption satisfies* $C(t) \leq Y(t)$ *for all* $t \geq 1$.

Definition 2 *A feasible consumption allocation is efficient if there is no alternative feasible allocation with more total consumption of some good and no less of any other good.*

Symmetric Consumption Allocations

- A consumption allocation is symmetric is all members of all generations consume the same consumption pair (may have different consumption for young and old), i.e.

$$c_t^h(t) = c_s^j(s)$$

and

$$c_t^h(t+1) = c_s^j(s+1)$$

for all agents h and j born in periods t and s.

2.2 Preferences

The utility function of an individual h of generation t is written as

$$u_t^h = u_t^h(c_t^h(t), c_t^h(t+1))$$

- Note that the only arguments in the utility function are consumption when young and old. Thus, agents do not care for their children (although a bequest motive is introduced in chapter 4 of M & W), and their utility does not depend on the consumption or utility of others.
- Strictly increasing in each argument (agents prefer more to less).
- Differentiable.
- Convex (i.e. U-shaped indifference curves). Mathematically, convexity is defined as follows: The utility function u_t is said to be convex if for any two individual consumption bundles $\{c_t^1(t), c_t^1(t+1)\}$ and $\{c_t^2(t), c_t^2(t+1)\}$, the convex combination is preferred to at least one of the consumption bundles, i.e.

$$u_t(c_t^3(t), c_t^3(t+1)) \geq \min\{u_t(c_t^1(t), c_t^1(t+1)), u_t(c_t^2(t), c_t^2(t+1))\}$$

where

$$c_t^3(t) = \alpha c_t^1(t) + (1-\alpha)c_t^2(t)$$

and

$$c_t^3(t+1) = \alpha c_t^1(t+1) + (1-\alpha)c_t^2(t+1).$$

2.3 Pareto Optimality

Definition 3 *Consumption allocation A is Pareto Superior to consumption allocation B if (i) no one strictly prefers B to A, and (ii) at least one person strictly prefers A to B.*

Definition 4 *A consumption allocation is said to be Pareto Optimal if it is feasible and if there does not exist a feasible consumption allocation that is Pareto superior to it.*

Note: Pareto optimality is a quite weak concept and includes a large number of allocations, only a few of which are "socially desirable" (e.g. Nero burning Rome is Pareto optimal but probably not socially desirable).

Necessary conditions for Pareto Optimality

- Efficiency

- Equality of marginal rate of substitution (MRS) for all members of a given generation. MRS is defined as

$$\text{MRS} = \frac{\frac{\partial u_t^h}{\partial c_t^h(t)}}{\frac{\partial u_t^h}{\partial c_t^h(t+1)}}$$

  Note that the MRS evaluated at a given consumption pair is equal to minus the slope of the indifference curve at that point.

- The MRS must be sufficiently high.

Note that efficiency and equality of MRS are not alone sufficient to guarantee Pareto optimality.