ECON 4310
MCandless and Wallace, Chapter 2[1]

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2 Defining the equilibrium

Purpose of lecture: characterizing the social organization (allocation mechanism) of the economy.

2.1 Introduction

- Examples: (i) competitive market economy, and (ii) "social planner" (dictator), and (iii) elected body (democratic government)
- Macroeconomics: predict outcome of (i). Workhorse: Competitive equilibrium.
- Which social organization, (i) or (ii), give rise to the most desirable allocations?
  1. Central planning require enormous amounts of information about preferences and production possibilities.
  2. Asymmetric information (e.g. employee has more information about his/her own effort than then manager or firm owner) $\Rightarrow$ planner must devote massive resources for auditing (police state) or device ingenious incentive schemes (e.g. share in profits or output of firm).
- New field: political economy, predict outcome of economy with mixture of (i) and (iii).
  - Key positive question: can this model explain the political outcomes we see?
  - Key normative question: what constitution gives rise to the most desirable outcomes?

2.2 Private ownership

- "Private ownership"= full right of disposal.
- Real-world limitations to private ownership:
  1. Government taxes
  2. theft

[1] The lecture notes of the first part of the class (first 7-8 lectures) are largely based on McCandless and Wallace. Correspondance to kjetil.storesletten@econ.uio.no

3. social and legal restrictions on disposal of wealth
4. limited control (stock owners vs. management)
5. externalities and common resources (e.g. air, public capital)

- Our model: each individual has a claim to some quantities of the goods available during his/her lifetime. Denote the endowment of person h of generation t by the ordered pair

$$\omega_t^h = \{\omega_t^h(t), \omega_t^h(t+1)\}$$

where $\omega_t^h(s) \geq 0$ for $s \in \{t, t+1\}$.

- The stream of endowments are known at birth.

- All endowments are privately owned (save taxes) in this economy, and the total endowments are given by

$$Y(t) = \sum_{h=1}^{N(t)} \omega_t^h(t) + \sum_{h=1}^{N(t-1)} \omega_{t-1}^h(t)$$

2.3 Trade and Budget Constraints

Trade

- Trade is only possible if there is private ownership (and is generally welfare improving).

- Trade = exchange of goods. Thus, with only one good in each period, the trade can only be intertemporal trade, i.e. borrowing and lending.

- Borrowing and lending will only take place within each generation (why?), and contracts are assumed to be enforceable by law.

Lending

- Let lending of individual h in generation t be denoted $l^h(t)$, measured in terms of time t goods. In period $t+1$, individual h receives a repayment of $r(t) * l^h(t)$ of the time $t+1$ good. The gross real interest rate is denoted $r(t)$ and is defined in terms of time $t+1$ good per unit of time t good (and is known at time t).

- Markets are competitive $\Rightarrow$ individuals view prices as given and unaffected by their own actions. The only prices in the economy are the gross real interest rates.

Budget constraints

- Individuals face the following budget constraints; constraint when young:

$$c_t^h(t) \leq \omega_t^h(t) - l_t^h(t) \tag{1}$$

and constraint when old:

$$c_t^h(t+1) \leq \omega_t^h(t+1) + r(t)\, l_t^h(t) \tag{2}$$

- Combining the two budget constraints into a single lifetime budget constraint yields

$$c_t^h(t) + \frac{c_t^h(t+1)}{r(t)} \leq \omega_t^h(t) + \frac{\omega_t^h(t+1)}{r(t)} \tag{3}$$

i.e. that discounted lifetime consumption must be less than or equal to the discounted lifetime endowments. The constraint is expressed in present value terms (=time good t).

- Need to show that these two sets of budget constraints are equivalent. It is trivial to show that (1) and (2) implies (3), given the way (3) was obtained. In order to show that (3) implies (1) and (2), we need to show that there exists some $l^h(t)$ that achieves this. Let $\left\{\tilde{c}_t^h(t), \tilde{c}_t^h(t+1)\right\}$ be such that equation (3) holds. Define $l^h(t)$ as

$$l^h(t) = \omega_t^h(t) - \tilde{c}_t^h(t)$$

i.e. such that (1) holds. Substitute this equation into eq. (3) and we have

$$\frac{\tilde{c}_t^h(t+1)}{r(t)} \leq l^h(t) + \frac{\omega_t^h(t+1)}{r(t)}$$

Rearrange and we get eq. (2).

2.4 Consumption decisions

- The competitive choice problem is to choose an affordable consumption basket maximizing utility, i.e. to solve

$$\max_{\left\{c_t^h(t), c_t^h(t+1)\right\}} u_t^h\left(c_t^h(t), c_t^h(t+1)\right)$$

subject to

$$c_t^h(t) + \frac{c_t^h(t+1)}{r(t)} \leq \omega_t^h(t) + \frac{\omega_t^h(t+1)}{r(t)}.$$

Alternatively, substitute $c_t^h(t+1)$ for $c_t^h(t)$ (using eq. (3)) in u_t^h and solve

$$\max_{\left\{c_t^h(t)\right\}} u_t^h\left(c_t^h(t), r(t)\left[\omega_t^h(t) - c_t^h(t)\right] + \omega_t^h(t+1)\right)$$

- The "first order condition" (FOC, and also called the "Intertemporal Euler equation") is

$$\frac{du_t^h}{dc_t^h(t)} = \frac{\partial u_t^h}{\partial c_t^h(t)} - r(t)\frac{\partial u_t^h}{\partial c_t^h(t+1)} = 0$$

- The utility maximizing choice for $c_t^h(t)$ is then given by the solution to the equation

$$r(t) = \frac{\frac{\partial u_t^h}{\partial c_t^h(t)}}{\frac{\partial u_t^h}{\partial c_t^h(t+1)}} = MRS \tag{4}$$

- The FOC (eq. (4)) implicitly defines a demand function for consumption of the young,

$$c_t^h(t) = \chi_t^h\left(r(t), \omega_t^h(t), \omega_t^h(t+1)\right). \tag{5}$$

Savings and assets

- Define savings of individual h of generation t as

$$s = \omega_t^h(t) - c_t^h(t)$$

- The savings function is then defined as

$$s_t^h\left(r(t), \omega_t^h(t), \omega_t^h(t+1)\right) = \omega_t^h(t) - \chi_t^h\left(r(t), \omega_t^h(t), \omega_t^h(t+1)\right) \tag{6}$$

2.5 Competitive equilibrium

Definition 1 *A competitive equilibrium is a set of prices and quantities that satisfies the following two conditions:*

(i) The quantities relevant for each individual maximizes that individual's utility in the set of all affordable quantities, given the prices and the individual's endowment, and

(ii) The quantities clear all markets at all dates.

- Steps in finding an equilibrium:

  1. Impose conditions (i) and (ii) (that is, solve the optimization problem for each agent and impose market clearing) and observe what conditions these imply for the sequence of prices and interest rates.
  2. Search for a sequence of prices and interest rates that fulfill these conditions.
  3. Check if the resulting quantities generated by the sequence of prices and interest rates indeed fulfill conditions (i) and (ii).

- Step (1a): Condition (ii) in Definition 1implies

  (a) a goods market clearing condition for each t:

  $$\sum_{h=1}^{N(t)} c_t^h(t) + \sum_{h=1}^{N(t-1)} c_{t-1}^h(t) = Y(t) = \sum_{h=1}^{N(t)} \omega_t^h(t) + \sum_{h=1}^{N(t-1)} \omega_{t-1}^h(t)$$

  (b) a loan market clearing condition for each t. Since no intergenerational trade occurs $\Rightarrow$ aggregate borrowing and lending within a generation must sum to zero in equilibrium:

  $$\sum_{h=1}^{N(t)} l^h(t) = 0$$

  Hence, aggregate consumption of the young must, in equilibrium, equal aggregate endowments of the young,

  $$\sum_{h=1}^{N(t)} c_t^h(t) = \sum_{h=1}^{N(t)} \omega_t^h(t) - \sum_{h=1}^{N(t)} l^h(t) = \sum_{h=1}^{N(t)} \omega_t^h(t) \tag{7}$$

  (and the same for the old, of course).

- Step (1b): Condition (i) in Definition 1 implies that individual consumption is given by χ_t^h in eq. (5)

  – Combined with (7), we obtain the following equilibrium condition:

  $$\sum_{h=1}^{N(t)} \chi_t^h\left(r(t), \omega_t^h(t), \omega_t^h(t+1)\right) = \sum_{h=1}^{N(t)} \omega_t^h(t)$$

  for all periods $t \geq 1$.

  – $\Rightarrow$ one equation with one endogenous variable for each period, $r(t)$, which can be solved for.

  – Alternatively, the equilibrium condition can, using the savings function in eq. (6), be defined in terms of aggregate savings:

  $$S_t(r(t)) = \sum_{h=1}^{N(t)} \omega_t^h(t) - \sum_{h=1}^{N(t)} \chi_t^h\left(r(t), \omega_t^h(t), \omega_t^h(t+1)\right)$$

  so market clearing implies for all $t \geq 1$,

  $$S_t(r(t)) = 0. \tag{8}$$

- Let us be formal about it: in an economy with the only assets being private borrowing and lending, the following proposition must hold

Proposition 1 *If the quantities and the sequence $r(t)$ is a competitive equilibrium, then $\{r(t)\}$ satisfies*

$$S_t(r(t)) = 0$$

for every t.

- Note that proposition 1 states a condition that holds for quantities and prices that are a competitive equilibrium. However, it says nothing about the existence of a competitive equilibrium: Steps 2 and 3. This is the purpose of the next proposition:

Proposition 2 *If $\{r(t)\}$ satisfies eq. (8) for every t, then there exist quantities such that they and the $r(t)$ sequence are a competitive equilibrium.*

Proof: Follow the logic suggested above: impose the conditions in Proposition 1 and see what conditions must hold on the sequence of gross real interest rates $\Rightarrow$ We need to show that any sequence of $r(t)$ fulfilling the condition $S_t(r(t)) = 0$, would generate the consumption allocations that both clear markets and are utility maximizing subject to the budget constraints. Consider some sequence $r(t)$ that fulfills (8) at all periods $t \geq 1$. The utility maximizing choice for individual h at time t given $r(t)$ is then given by the demand function of the young,

$$c_t^h(t) = \chi_t^h\left(r(t), \omega_t^h(t), \omega_t^h(t+1)\right).$$

The condition $S_t(r(t)) = 0$ and the definitions of aggregate and individual savings then imply that

$$S_t(r(t)) = \sum_{h=1}^{N(t)} s_t^h(t) = \sum_{h=1}^{N(t)} \omega_t^h(t) - \sum_{h=1}^{N(t)} c_t^h(t) = 0 \tag{9}$$

for the optimal choice of $c_t^h(t)$ (which is $c_t^h(t) = \chi_t^h\left(r(t), \omega_t^h(t), \omega_t^h(t+1)\right)$). From the budget constraint of the young we have

$$c_t^h(t) = \omega_t^h(t) - l^h(t),$$

which, together with equation (9) implies

$$\sum_{h=1}^{N(t)} l^h(t) = 0.$$

Thus, the market for borrowing and lending clears. This condition holds in every period and thus also for the old in period t, who were saving when young in period $t-1$. Summing the budget constraints of the old in period t, we have

$$\sum_{h=1}^{N(t-1)} c_{t-1}^h(t) = \sum_{h=1}^{N(t-1)} \omega_{t-1}^h(t) + r(t) \sum_{h=1}^{N(t-1)} l^h(t-1) = \sum_{h=1}^{N(t-1)} \omega_{t-1}^h(t). \tag{10}$$

Combining (9) and (10) we have

$$\sum_{h=1}^{N(t)} c_t^h(t) + \sum_{h=1}^{N(t-1)} c_{t-1}^h(t) = \sum_{h=1}^{N(t)} \omega_t^h(t) + \sum_{h=1}^{N(t-1)} \omega_{t-1}^h(t),$$

i.e. goods market clearing. We have thus shown that given a sequence of gross interest rates fulfilling the zero aggregate savings condition at every date, there exists quantities derived from utility maximization clearing the markets for goods and private borrowing and lending at every date.

QED

- NOTE: in general, a competitive equilibrium does not result in Pareto optimal allocations.