ECON 4310
MCandless and Wallace, Chapter 3[1]

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3 Introducing a government

- Purpose of lecture: introduce a government that can levy taxes, redistribute transfers, and issue government bonds.
- A government is viewed as an infinitely lived institution.
- Assume policies are time consistent:
  - Definition: a policy at time $t+k$ that seemed optimal at time t may must be optimal to carry out when period $t+k$ appears.
  - Examples where time consistency is violated: repeated elections (and possibly new government each period), crime and punishment.
  - Time consistency puts strong restrictions on future plans of the government.

3.1 Taxes

- A simple tax structure: a lump-sum tax on endowments ("transfer" = negative tax).
- Tax structure is commonly known at all dates.
- Individual h of generation t faces the following taxes over his/her lifetime:

$$t_t^h = \left\{ t_t^h(t), t_t^h(t+1) \right\}.$$

- With no government consumption or borrowing, the budget constraint of the government equals

$$\sum_{h=1}^{N(t)} t_t^h(t) + \sum_{h=1}^{N(t-1)} t_{t-1}^h(t) = 0.$$

[1]The lecture notes of the first part of the class (first 7-8 lectures) are largely based on McCandless and Wallace. Correspondance to kjetil.storesletten@econ.uio.no

3.1.1 Budget constraint and consumption decisions

If pre-tax endowments for individual h of generation t are

$$\omega_t^h = \left\{\omega_t^h(t), \omega_t^h(t+1)\right\},$$

then post-tax endowments are

$$\omega_t^h - t_t^h = \left\{\omega_t^h(t) - t_t^h(t), \omega_t^h(t+1) - t_t^h(t+1)\right\}.$$

The budget constraints are thus when young

$$c_t^h(t) \leq \omega_t^h(t) - t_t^h(t) - l^h(t)$$

and when old,

$$c_t^h(t+1) \leq \omega_t^h(t+1) - t_t^h(t+1) + r(t)l^h(t),$$

which implies a lifetime budget constraint

$$c_t^h(t) + \frac{c_t^h(t+1)}{r(t)} \leq \omega_t^h(t) - t_t^h(t) + \frac{\omega_t^h(t+1) - t_t^h(t+1)}{r(t)}.$$

The competitive choice problem is to choose an affordable utility-maximizing consumption basket, i.e. to solve

$$\max_{\left\{c_t^h(t)\right\}} u_t^h\left(c_t^h(t), r(t)\left(\omega_t^h(t) - t_t^h(t) - c_t^h(t)\right) + \omega_t^h(t+1) - t_t^h(t+1)\right)$$

The first order condition is still

$$\frac{du_t^h}{dc_t^h(t)} = \frac{\partial u_t^h}{\partial c_t^h(t)} - r(t)\frac{\partial u_t^h}{\partial c_t^h(t+1)} = 0$$

so the utility maximizing choice for $c_t^h(t)$ is still given by the solution to the equation

$$r(t) = \frac{\frac{\partial u_t^h}{\partial c_t^h(t)}}{\frac{\partial u_t^h}{\partial c_t^h(t+1)}} = MRS.$$

Note, however, that the optimization problem now depends on post-tax endowments, so the demand function the young can be written as

$$c_t^h(t) = \chi_t^h\left(r(t), \omega_t^h(t) - t_t^h(t), \omega_t^h(t+1) - t_t^h(t+1)\right),$$

and the savings function can be written as

$$s_t^h(r(t)) = \omega_t^h(t) - t_t^h(t) - \chi_t^h\left(r(t), \omega_t^h(t) - t_t^h(t), \omega_t^h(t+1) - t_t^h(t+1)\right).$$

Aggregate savings will thus equal

$$S(r(t)) = \sum_{h=1}^{N(t)} s_t^h\left(r(t), \omega_t^h(t) - t_t^h(t), \omega_t^h(t+1) - t_t^h(t+1)\right).$$

- Note that the general definition of a competitive equilibrium holds for an economy with transfers and taxes.

- Since there is no intergenerational borrowing and lending, aggregate savings of the young must still equal zero in equilibrium. This can be seen by summing up the budget constraints of the young

$$\sum_{h=1}^{N(t)} c_t^h(t) = \sum_{h=1}^{N(t)} \omega_t^h(t) - \sum_{h=1}^{N(t)} l^h(t) - \sum_{h=1}^{N(t)} t_t^h(t)$$

  Given $\sum_{h=1}^{N(t)} l^h(t) = 0$, we have

$$S(r(t)) = \sum_{h=1}^{N(t)} s_t^h(r(t)) = \sum_{h=1}^{N(t)} \left(\omega_t^h(t) - t_t^h(t) - c_t^h(t)\right) = 0.$$

  The equilibrium condition $S(r(t)) = 0$ is as before both necessary and sufficient.

- Pareto optimality: In general, one can show that if a competitive equilibrium is not Pareto optimal, then there exist a government tax-transfer policy that produces a Pareto superior allocation.

3.2 Government borrowing

- Assume the government issues one-period bonds; claims to one unit of the consumption good next period. Moreover, the government always honors its debt (as before, only the young are interested in purchasing bonds).

- Suppose the government issues $B(t)$ units of bonds in period t. There are four ways the government can finance repayment of the debt in period $t+1$:

  1. tax the young of generation $t+1$ a total of $B(t)$ units
  2. tax the old of generation t a total of $B(t)$ units
  3. issue $B(t+1)$ units of bonds that raise a total of $B(t)$ units
  4. some mix of 1-3.

- The time t budget constraint of the government is now

$$\sum_{h=1}^{N(t)} t_t^h(t) + \sum_{h=1}^{N(t-1)} t_{t-1}^h(t) + p(t)B(t) - B(t-1) = 0$$

  where $p(t)$ is the price of one government bond at time t.

- The government budget is said to be balanced when

$$\sum_{h=1}^{N(t)} t_t^h(t) + \sum_{h=1}^{N(t-1)} t_{t-1}^h(t) = 0.$$

  The government runs a deficit (surplus) when the right-hand side is negative (positive).

- One-period bonds have the same risk-characteristics as private lending within generations, and they must therefore yield the same return $r(t)$ as private lending (no-arbitrage equilibrium condition). Proof: With bonds, the budget constraints of the individuals are

$$\begin{aligned} c_t^h(t) &\leq \omega_t^h(t) - t_t^h(t) - l^h(t) - p(t)b^h(t) \\ c_t^h(t+1) &\leq \omega_t^h(t+1) - t_t^h(t+1) + r(t)l^h(t) + b^h(t). \end{aligned}$$

Consequently, the lifetime budget constraint is

$$\begin{aligned} c_t^h(t) + \frac{c_t^h(t+1)}{r(t)} &\leq \omega_t^h(t) - t_t^h(t) + \frac{\omega_t^h(t+1) - t_t^h(t+1)}{r(t)} \\ &\quad - b^h(t)\left[p(t) - \frac{1}{r(t)}\right]. \end{aligned}$$

This implies that the individual's demand for bonds equals

$$b^h(t) = \begin{cases} 0 & \text{if } 1/r(t) < p(t) \\ \infty & \text{if } 1/r(t) > p(t) \\ ? & \text{if } 1/r(t) = p(t) \end{cases}$$

Since neither $b^h(t) = 0$ nor $b^h(t) = \infty$ can be an equilibrium with positive bond holdings,

$$r(t) = 1/p(t)$$

must be an equilibrium condition.

  - The same return $\Rightarrow$ the individual is indifferent between the two types of savings, only the net position matters (indeterminacy).

3.2.1 The competitive equilibrium with a government

Sum the budget constraints of the young:

$$\sum_{h=1}^{N(t)} c_t^h(t) = \sum_{h=1}^{N(t)} \omega_t^h(t) - \sum_{h=1}^{N(t)} l^h(t) - \sum_{h=1}^{N(t)} t_t^h(t) - p(t)\sum_{h=1}^{N(t)} b^h(t)$$

Given $\sum_{h=1}^{N(t)} l^h(t) = 0$ and $B(t) = \sum_{h=1}^{N(t)} b^h(t)$, we have that aggregate savings equal

$$S(r(t)) = \sum_{h=1}^{N(t)} s_t^h(r(t)) = \sum_{h=1}^{N(t)} \left(\omega_t^h(t) - t_t^h(t) - c_t^h(t)\right) = p(t)B(t).$$

Summing up the constraints of the old (keeping in mind that all the debt in period $t-1$ was bought by generation $t-1$)

$$\begin{aligned} \sum_{h=1}^{N(t-1)} c_{t-1}^h(t) &= \sum_{h=1}^{N(t-1)} \left[\omega_{t-1}^h(t) - t_{t-1}^h(t) + r(t-1)l^h(t-1) + b^h(t-1)\right] \\ &= \sum_{h=1}^{N(t-1)} \left[\omega_{t-1}^h(t) - t_{t-1}^h(t)\right] + B(t-1) \end{aligned}$$

where $\sum_{h=1}^{N(t-1)} l^h(t-1) = 0$ and $\sum_{h=1}^{N(t-1)} b^h(t-1) = B(t-1)$. Summing the budget constraints of the young and the old we get

$$\begin{aligned}\sum_{h=1}^{N(t)} c_t^h(t) + \sum_{h=1}^{N(t-1)} c_{t-1}^h(t) &= \sum_{h=1}^{N(t)} \left[\omega_t^h(t) - t_t^h(t)\right] + \sum_{h=1}^{N(t-1)} \left[\omega_{t-1}^h(t) - t_{t-1}^h(t)\right] \\ &\quad -p(t)B(t) + B(t-1).\end{aligned}$$

Moreover, imposing the government budget constraint gives

$$\sum_{h=1}^{N(t)} t_t^h(t) + \sum_{h=1}^{N(t-1)} t_{t-1}^h(t) + p(t)B(t) - B(t-1) = 0,$$

which guarantee market clearing in the goods market, i.e.,

$$\sum_{h=1}^{N(t)} c_t^h(t) + \sum_{h=1}^{N(t-1)} c_{t-1}^h(t) = \sum_{h=1}^{N(t)} \omega_t^h(t) + \sum_{h=1}^{N(t-1)} \omega_{t-1}^h(t) = Y(t)$$

- Bottom line, the equilibrium condition for savings is simply

$$S_t(r(t)) = \frac{B(t)}{r(t)},$$

  where we have exploited the no-arbitrage condition on the price of bonds, i.e. $p(t) = 1/r(t)$.

3.3 Ricardian equivalence

Proposition 1 *Given an initial equilibrium under some pattern of lump-sum taxation and government borrowing, alternative (intertemporal) patterns of lump-sum taxation that keeps the present value (at the interest rate of the initial equilibrium) of each individual's tax liability equal to that of the initial equilibrium, are equivalent in the following sense: Corresponding to each alternative taxation pattern is a pattern of government borrowing such that the consumption allocation of the initial equilibrium, including consumption of the government, and the interest rates of the initial equilibrium, are an equilibrium under the alternative taxation pattern.*

3.4 Rolling over government debt

- Rolling over debt = financing payments on outstanding debt by issuing new bonds. At time t the equilibrium condition is

$$S_t(r(t)) = \frac{B(t)}{r(t)},$$

  At time $t+1$ the equilibrium condition is, if debt is rolled over,

$$S_{t+1}(r(t+1)) = \frac{B(t+1)}{r(t+1)} = B(t).$$

At time $t+2$ the equilibrium condition is, if debt is rolled over,

$$S_{t+2}(r(t+2)) = \frac{B(t+2)}{r(t+2)} = B(t+1) = r(t+1)B(t).$$

Hence, if debt is rolled over forever, the amount of bonds issued in period $t+j$ equals

$$\begin{aligned} B(t+j) &= r(t+j)B(t+j-1) \\ &= r(t+j)r(t+j-1)B(t+j-2) \\ &\quad ... \\ &= r(t+j)r(t+j-1)...r(t+1)B(t) \end{aligned}$$

or

$$B(t+j) = B(t)\prod_{k=1}^{j} r(t+k)$$

- Consider three different cases:

  1. Case 1: $r(t+k) = 1$ for all k

     $$B(t+j) = B(t).$$

     The amount of debt issued is constant over time (called a stationary equilibrium).
  2. Case 2: $r(t+k) \leq \underline{r} < 1$ for all k

     $$B(t+j) = B(t)\prod_{k=1}^{j} r(t+k) \leq B(t)\,\underline{r}^j$$

     As time moves on we have

     $$0 \leq \lim_{j\to\infty} B(t+j) \leq \lim_{j\to\infty} B(t)\,\underline{r}^j = 0,$$

     so government debt goes to zero in the long run (another stationary equilibrium, different from the one in Case 1).
  3. Case 3: $r(t+k) \geq \bar{r} > 1$ for all k

     $$B(t+j) = B(t)\prod_{k=1}^{j} r(t+k) \geq B(t)\,\bar{r}^j$$

     As time moves on we have

     $$\lim_{j\to\infty} B(t+j) \geq \lim_{j\to\infty} B(t)\,\bar{r}^j = \infty,$$

     so government debt goes to infinity (a bubble), which cannot be an equilibrium since, eventually, the required refinancing will exceed the aggregate endowment of the young.

- A bubble is an unsustainable price path for an asset. In our economy bubbles can never occur since they cannot be equilibrium paths. Do real-world examples exist? Pyramid-games, rare stamps, IT stocks?

- Knife-edge equilibria

3.5 Bonds vs tax-transfer schemes

Proposition 2 *An equilibrium with bonds can be duplicated (in terms of consumption allocations and prices) with a tax-transfer scheme balancing the budget of the government at all dates and having no government borrowing at any date.*

Proof: Consider the case with bonds only (no taxes or transfers). The budget constraints for the individual are then

$$\begin{aligned} c_t^h(t) &\leq \omega_t^h(t) - l^h(t) - p(t)b^h(t) && (1) \\ c_t^h(t+1) &\leq \omega_t^h(t+1) + r(t)l^h(t) + b^h(t). && (2) \end{aligned}$$

Alternatively, if only taxes and transfer exist (i.e. the government does not borrow or lend), the constraints are

$$\begin{aligned} c_t^h(t) &\leq \omega_t^h(t) - t_t^h(t) - l^h(t) && (3) \\ c_t^h(t+1) &\leq \omega_t^h(t+1) - t_t^h(t+1) + r(t)l^h(t). && (4) \end{aligned}$$

Note that if we set

$$\begin{aligned} t_t^h(t) &= \frac{b^h(t)}{r(t)} \\ t_t^h(t+1) &= -b^h(t) \end{aligned}$$

then equations (1)-(2) and (3)-(4) are equivalent, and the same consumption allocations will be achieved in equilibrium.

3.6 An application to pension systems

- All industrialized countries have mandatory pension schemes. Across countries, these systems have several features in common:
  - were put in place between 1930-1960 and expanded during 1960-1980.
  - pension contributions are, legally, a loan to the government from the worker, paying a particular return $h(t)$.
  - pension contributions are subtracted from earnings before the employer gets to pay the worker (a payroll tax).
  - pension systems contain an old-age component and a spouse component. In some countries the pension system also provide medical insurance and finance early retirement.
  - initially, the systems were all pay-as-you-go, or balanced within each period, i.e.,

$$\sum_{h=1}^{N(t)} t_t^h(t) + \sum_{h=1}^{N(t-1)} t_{t-1}^h(t) = 0.$$

- Due to the population transition (lower fertility after 1960 and longer longevity), most countries now promise a return and accumulate a pension fund to finance future pension liabilities for the "baby-boomers".
- The introduction of pension systems worked as a great transfer of wealth to the initial old.

- The gross return, $h(t)$, on pay-as-you-go pension contributions is, on average, the aggregate growth rate of labor earnings. In our simple economies, this return is simply

$$h(t) = \frac{N(t)}{N(t-1)} = n.$$

Thus, if the pension contributions for the young are taxed $t_t^h(t)$, the net taxes (or pension benefits) for individual h of generation t in period $t+1$ will be

$$t_t^h(t+1) = -h(t) t_t^h(t).$$

Consequently, if pension contributions are a fraction η of the endowment (when young), the consumption allocations will be

$$\begin{aligned} c_t^h(t) &\leq \omega_t^h(t) - \eta\omega_t^h(t) - l^h(t) \\ c_t^h(t+1) &\leq \omega_t^h(t+1) - h(t)\eta\omega_t^h(t) + r(t)l^h(t). \end{aligned}$$

Note that pension benefits are not tradable (and has no price), so the lifetime budget constraint becomes

$$\begin{aligned} c_t^h(t) + \frac{c_t^h(t+1)}{r(t)} &\leq (1-\eta)\,\omega_t^h(t) + \frac{\omega_t^h(t+1) + h(t)\eta\omega_t^h(t)}{r(t)} \\ &= \left(1 + \eta\left(\frac{h(t)}{r(t)} - 1\right)\right)\omega_t^h(t) + \frac{\omega_t^h(t+1)}{r(t)} \end{aligned}$$

- A pay-as-you-go pension system is, on the margin, a gain, in terms of the present value of consumption, if

$$r(t) < h(t)$$

for all $t \geq 1$. Conversely, it is a loss if

$$r(t) > h(t).$$

In this case, the pension system works as a tax (i.e. mandatory savings at a below-market rate of return). Finally, the budget constraint is not affected if $r(t) = h(t)$.

  - The aggregate annual growth rate of wages has been 3–4% in most OECD countries during the last 50 years (≈1–2% population growth rate and ≈2% growth rate in wages per worker).
  - The average "riskfree" rate of return has been, on average, 1–2% during the 20th century (compared to 5–9% average stock market return).

– $\Rightarrow$ this "free lunch" may have been a major motivation for the introduction of the pension systems. The leading alternative motivation for the introduction of the pension systems is paternalism, the belief that policy makers know better how mush individuals should save than do the individuals themselves.

– Now, however, the expected population growth is negative and the expected productivity growth is $\approx$ 1.5% In comparison, riskfree rate of return is $\approx$6% (on 10-year bonds).