ECON 4310
Pensions – continued[1]

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14 General equilibrium and the neoclassical growth model

- Message of previous lecture: Given the ageing of the baby-boom generation, the pension system must accumulate wealth or plan on a massive default on pensions of the baby-boomers (i.e. much lower pension benefits after 2020), combined with a surge in taxes on the young.
- So far, we have simplified our analysis by either
  (a) abstract from price changes and focus only on capital flows: the "storage model" with constant rate of return on savings. Abstraction: small open economy.
  (b) abstract from capital flows and focus only on the price changes: the "land model" of chapters 1-6, where physical storage is ruled out and intertemporal effects of e.g. pension transition is captured through changes in taxes and prices.

  A more realistic model is somewhere in between these two extremes.
- Suppose all agents have a *time-endowment* of 1 when young and zero when old, which they supply inelastically. Thus, total labor supply in period t is $N(t)$.
- Output is a function of both labor and capital;

$$Y(t) = F\left(K(t), N(t)\right) = K(t)^{\theta} \cdot N(t)^{1-\theta},$$

  where $\theta \in (0,1)$. Note that this production function implies an increasing product but falling *marginal* product in each input:

$$\frac{\partial}{\partial K}F > 0, \qquad \frac{\partial^2}{\partial K^2}F < 0$$
$$\frac{\partial}{\partial N}F > 0, \qquad \frac{\partial^2}{\partial N^2}F < 0$$

  Moreover, F exhibits constant return to scale, and the aggregate production function is interpreted as the production function of small competitive firms (who face free entry).

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- Competitive equilibrium: market for labor and capital must clear. Firms rent capital and labor on competitive markets at rental prices r and w. Optimality condition: rent so much that the marginal profit is zero:

$$\begin{aligned} \frac{\partial}{\partial K}\{F(K,N) - rK - wN\} &= 0 \\ \frac{\partial}{\partial N}\{F(K,N) - rK - wN\} &= 0 \end{aligned}$$

Optimality condition:

$$\begin{aligned} r = F_1(K,N) = \theta(K/N)^{\theta-1} \\ w = F_2(K,N) = (1-\theta)(K/N)^{\theta} \end{aligned}$$

Note: since $\theta Y = rK$, one can interpret θ as the share of output that is paid to capital owners.

14.1 Equilibrium in the production economy

- Consider an economy with constant population growth, $N(t) = (1+n)N(t-1)$ and preferences given by $u_t^h = c_t^h(t)\left[c_t^h(t+1)\right]^{\beta}$.
- An individual's savings are given by

$$s\left(r(t)\right) = \frac{w(t)}{1+\beta} - \frac{0}{(1+\beta)r(t)}$$

Thus, $K(t+1) = \frac{1}{1+\beta}w(t)N(t)$.

- We can then figure out the dynamics of the capital stock per worker k:

$$\begin{aligned} k_t &= \frac{K(t+1)}{N(t+1)} \\ &= \frac{1}{1+\beta}\frac{N(t)}{N(t+1)}w(t) \\ &= \frac{1-\theta}{(1+\beta)(1+n)}\left(\frac{K(t)}{N(t)}\right)^{\theta} = \frac{(1-\theta)}{(1+\beta)(1+n)}k_t^{\theta} \end{aligned} \tag{1}$$

- Dynamics: convergence to a steady-state. Set $k_{t+1} = k_t = k^*$ in equation (**??**):

$$\begin{aligned} k^* &= \frac{1-\theta}{(1+\beta)(1+n)}\left(k^*\right)^{\theta} \\ &\Rightarrow \\ k^* &= \left(\frac{1-\theta}{(1+\beta)(1+n)}\right)^{1/(1-\theta)} \end{aligned}$$

- Note: $n \uparrow$ implies $k \downarrow$ (since $1/(1-\theta) > 0$), and therefore $w \downarrow$ and $r \uparrow$.
- Consider now a situation with a fall in the population growth rate (baby-bust), i.e. $n \downarrow$. This should then induce the wage to increase several periods until it converges at the new high k.

15 Conclusion

Outline of course:

1. Define an equilibrium. Understand the equilibrium conditions. Note also that the competitive equilibrium can be Pareto-inferior (so the First welfare theorem can break down in an overlapping generations economy).

2. Introduce a government. Note that government debt is a transfer from future generations to the currently living. equivalence between an economy with debt and a tax-transfer economy without debt.

3. Introduce altruistic concerns for descendants $\Rightarrow$ bequests. Note that Ricardian equivalence holds for all dynasties who give bequests.

4. Long-term debt and economies with land: Understand the concept of *perfect foresight*. Also: understand the expectation hypothesis for interest rates, as well as exploring equilibria with (and their equivalence to economies with pay-as-you-go pensions)

5. Asset pricing with risk (CCAPM) - understanding pricing of risk and insurance.

6. Economies with storage: allow capital to be stored between periods. Application: pensions (including a neoclassical example)

7. Labor supply (important distinction: different response to short-term and permanent tax changes).