ECON 4310
MCandless and Wallace, Chapter 6[1]

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5 Infinitely-lived assets

- Purpose of lecture:
    1. Theory: introduce infinitely-lived assets in order to complete the equilibrium notion.
    2. Applications: (i) international macro/finance, and (ii) consumption-based asset pricing model (in next lecture).
- Some assets do not have a finite maturity date, for example land.
- Introduce "land" in the model: let A units of land give a total crop of $D(t)$ units of time t good in each period t.
- Let $d(t)$ denote the amount of crop per unit of land:

$$d(t) = \frac{D(t)}{A}$$

  and assume that the sequence of future crops, $\{d(t)\}_{t=1}^{\infty}$, is known in advance.
- Assume that the initial old own all the land, collect the crop at no cost, and then sell the land to the young at price $p(t)$ per unit of land, where $p(t)$ is the ex-dividend price.

5.1 Temporary equilibrium with land

- As in lecture 4, we will find the competitive equilibrium by first solving for a temporary equilibrium. For simplicity, assume that the only assets are land and private borrowing and lending. The budget constraints are then

$$\begin{aligned} c_t^h(t) &\leq \omega_t^h(t) - l^h(t) - p(t)a^h(t) \\ c_t^h(t+1) &\leq \omega_t^h(t+1) + r(t)l^h(t) + \left(p^{h,e}(t+1) + d(t+1)\right) a^h(t). \end{aligned}$$

[1] The lecture notes of the first part of the class (first 7-8 lectures) are largely based on McCandless and Wallace. Correspondance to kjetil.storesletten@econ.uio.no

- Expected wealth: the lifetime budget constraint is

$$c_t^h(t) + \frac{c_t^h(t+1)}{r(t)} \leq \omega_t^h(t) + \frac{\omega_t^h(t+1)}{r(t)} - a^h(t)\left[p(t) - \frac{d(t+1) + p^{h,e}(t+1)}{r(t)}\right].$$

Thus, the demand of individual h for k-period bonds equals

$$a^h(t) = \begin{cases} 0 & \text{if } p(t) > \frac{d(t+1)+p^{h,e}(t+1)}{r(t)} \\ \infty & \text{if } p(t) < \frac{d(t+1)+p^{h,e}(t+1)}{r(t)} \\ ? & \text{if } p(t) = \frac{d(t+1)+p^{h,e}(t+1)}{r(t)} \end{cases}$$

Proposition 1 *If there exists unanimity of expectations such that*

$$p^{h,e}(t+1) = p^e(t+1),$$

for all h of period t, and if some land exists, then in any equilibrium,

$$p(t) = \frac{d(t+1) + p^e(t+1)}{r(t)}.$$

- The proposition states that the present value of next period's crop plus the value of the land next period must equal today's price.

Definition 1 *Given $\left\{u_t^h(.,.), \omega_t^h, d(t+1), p^e(t+1)\right\}$, a time t temporary equilibrium with land is a pair of prices $[r(t), p(t)]$ such that the following equilibrium conditions are fulfilled*

(i) $p(t) = \frac{d(t+1)+p^e(t+1)}{r(t)}$

(ii) $S_t(r(t)) = p(t)A$

- To show that the second condition is an equilibrium condition: Market clearing on the goods market implies

$$\sum_{t=1}^{N(t)} c_t^h(t) + \sum_{h=1}^{N(t-1)} c_{t-1}^h(t) = \sum_{h=1}^{N(t)} \omega_t^h(t) + \sum_{h=1}^{N(t-1)} \omega_{t-1}^h(t) + D(t)$$

Total consumption of the old equals

$$\sum_{h=1}^{N(t-1)} c_{t-1}^h(t) = \sum_{h=1}^{N(t-1)} \omega_{t-1}^h(t) + D(t) + p(t)A.$$

Total consumption of the young then equals

$$\sum_{h=1}^{N(t)} c_t^h(t) = \sum_{h=1}^{N(t)} \omega_t^h(t) - p(t)A.$$

Rearranging the above equation, aggregate savings can be expressed as

$$S_t(r(t)) = \sum_{h=1}^{N(t)} s_t^h(r(t)) = \sum_{h=1}^{N(t)} \left(\omega_t^h(t) - c_t^h(t)\right) = p(t)A.$$

- The prices defining a temporary equilibrium can be found, as before, by solving for $r(t)$:

$$S_t(r(t)) = \frac{d(t+1) + p^e(t+1)}{r(t)} A.$$

If $S_t(.)$ is increasing in $r(t)$ there is at least one $r(t)$ that solves this equation. Alternatively, solving for $p(t)$,

$$S_t\left(\frac{d(t+1) + p^e(t+1)}{p(t)}\right) = p(t)A,$$

and a price function can be defined as

$$p(t) = f_t\left(p^e(t+1), d(t+1), A\right). \tag{1}$$

5.2 Perfect foresight

Definition 2 *A perfect foresight competitive equilibrium with land is an infinite sequence of prices $p(t)$ and $r(t)$ and endogenous variables such that the time t values are a temporary equilibrium satisfying*

$$p^e(t+1) = p(t+1).$$

- From now on, a "perfect foresight competitive equilibrium" is simply referred to as an equilibrium.

- Finding an equilibrium:

  1. Guess and verify
     (a) Guess a price $p(t)$ and check if the equilibrium conditions are satisfied for the $p(t+1)$, $p(t+2), \ldots$ implied by the equilibrium condition, expressed as equation (1).
     (b) Restrict attention to stationary equilibria where $p(t) = p(t+1)$ is constant over time and thus might be easily guessed at.
  2. Perform a graphical analysis using the pricing function (1).

- The economy impose some natural restrictions on the price sequence, such as ruling out negative prices or price sequences that are explosive – i.e. there typically exists some upper bound on how large prices can be.

5.3 International capital movements

- Does it matter who owns the assets in a country?

- Consider a world with I different countries, each possessing a certain amount of land, i.e.

$$A_1 < A_2 < \ldots < A_I$$

where each unit of land yields $d(t)$ units of crop, independently of country. Assume that the crop can be transported at no cost between countries.

Three regimes:

1. In a portfolio autarchy regime, individuals are restricted to buy domestic land only and to act in their local private borrowing market.
2. In a laissez-faire regime, individuals can buy land anywhere and act on the private borrowing market in any country.
3. In an international borrowing regime, individuals are restricted to buy domestic land only, but can act on the private borrowing market in any country.

The laissez-faire regime

- Global (unregulated) markets for loans and land.
- A world market for land:
$$p^i(t) = p(t).$$
- A world market for private loans:
$$r^i(t) = r(t).$$
- Equilibrium condition (i): is, as before, $p(t) = \frac{d(t+1)+p^e(t+1)}{r(t)}$, while equilibrium condition (ii) amounts to
$$\sum_{i=1}^{I} S_t^i(r(t)) = p(t) \sum_{i=1}^{I} A_i$$

The international borrowing regime

- Global (unregulated) markets for loans, but no trade in ownership of land.
- A world market for private loans:
$$r^i(t) = r(t).$$
- In a stationary equilibrium, $p^i(t) = p^i(t+1)$ in any country i. Thus, from equilibrium condition (i) it must be that
$$\begin{aligned} r(t) &= \frac{d(t+1) + p^i(t)}{p^i(t)} \\ &= \frac{d(t+1)}{p^i(t)} + 1 = \frac{d(t+1)}{p^j(t)} + 1 \end{aligned}$$
for any countries $i, j \in \{1, 2, ..., I\}$. Thus, the prices of land must be equalized across countries (despite no trade).

- The budget constraints for individual h in country i are (assuming perfect foresight and $p^i(t) = p(t)$):

$$\begin{aligned} c_t^{ih}(t) &\leq \omega_t^{ih}(t) - l^{ih}(t) - p(t)a^{ih}(t) \\ c_t^{ih}(t+1) &\leq \omega_t^{ih}(t+1) + r(t)l^{ih}(t) + (p(t+1) + d(t+1))\, a^{ih}(t) \end{aligned}$$

  or expressed in terms of savings:

$$s_t^{ih}\left(r(t)\right) = l^{ih}(t) + p(t)a^{ih}(t).$$

  Aggregating over individuals in country i, we express aggregate national savings as

$$S_t^i\left(r(t)\right) = \sum_{h=1}^{N_i(t)} \left[l^{ih}(t) + p(t)a^{ih}(t)\right] = L^i(t) + p(t)A_i$$

  where $L^i(t)$ is net international lending.

- Market clearing on the international loan market requires

$$\sum_{i=1}^{I} L^i(t) = 0$$

- Aggregate savings in the world must then satisfy

$$\sum_{i=1}^{I} S_t^i\left(r(t)\right) = \sum_{i=1}^{I} \left[L^i(t) + p(t)A_i\right] = p(t)\sum_{i=1}^{I} A_i$$

  which is the same equilibrium condition as under the laissez-faire regime. Hence, international borrowing and lending overcomes the constraints on land ownership. This is a version of factor price equalization.

- The equality between the laissez-faire regime and the international borrowing regime breaks down if the country-specific crops are stochastic.