# ECON 4310
# MCandless and Wallace, Chapter 8[1]

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## 7 A storage technology

- Purpose of lecture: introduce storage into the model to allow the society to transfer resources over time.
- Application: investigate the effects on savings of capital taxation and a pension system.
- The storage technology is a linear, constant returns to scale, intertemporal technology that transforms time $t$ goods into time $t+1$ goods. For every unit of goods stored, $\lambda$ units are returned in the next period, where $\lambda \geq 0$. Let $K(t+1)$, called capital, be the amount of time $t$ goods put into storage.

**Definition 1** *A path of total consumption, $\{C(t)\}_{t=1}^{\infty}$ is feasible for an economy with storage if, given $K(1) \geq 0$, there exist a non-negative sequence $\{K(t)\}_{t=2}^{\infty}$ that satisfies, for all $t \geq 1$,*

$$C(t) + K(t+1) \leq Y(t) + \lambda K(t)$$

- Let $k^h(t+1)$ denote the amount of time $t$ goods put into storage by individual $h$. Budget constraints (assuming unanimity of expectations) are given by:

$$\begin{aligned} c_t^h(t) &\leq \omega_t^h(t) - l^h(t) - p(t)a^h(t) - k^h(t+1) \\ c_t^h(t+1) &\leq \omega_t^h(t+1) + r(t)l^h(t) + \\ &\quad + [d(t+1) + p(t+1)]\, a^h(t) + \lambda k^h(t+1). \end{aligned}$$

[1] The lecture notes of the first part of the class (first 7-8 lectures) are largely based on McCandless and Wallace. Correspondance to kjetil.storesletten@econ.uio.no

- The lifetime budget constraint is

$$
\begin{aligned}
c_t^h(t) + \frac{c_t^h(t+1)}{r(t)} \leq\; & \omega_t^h(t) + \frac{\omega_t^h(t+1)}{r(t)} \\
& - a^h(t)\left[p(t) - \frac{d(t+1) + p^e(t+1)}{r(t)}\right] \\
& - k^h(t+1)\left[1 - \frac{\lambda}{r(t)}\right]
\end{aligned}
$$

Thus, in equilibrium we must have

$$
\begin{aligned}
p(t) &= \frac{d(t+1) + p^e(t+1)}{r(t)} \\
r(t) &\geq \lambda \\
0 &= k^h(t+1)\left[r(t) - \lambda\right].
\end{aligned}
$$

- Market clearing requires

$$
\begin{aligned}
\sum_{t=1}^{N(t)} l^h(t) &= 0 \\
\sum_{t=1}^{N(t)} a^h(t) &= A \\
\sum_{t=1}^{N(t)} c_t^h(t) + \sum_{h=1}^{N(t-1)} c_{t-1}^h(t) + \sum_{h=1}^{N(t)} k^h(t+1) &= \\
= \sum_{h=1}^{N(t)} \omega_t^h(t) + \sum_{h=1}^{N(t-1)} \omega_{t-1}^h(t) + d(t)A + \lambda \sum_{h=1}^{N(t-1)} k^h(t) \\
C(t) + K(t+1) &= Y(t) + \lambda K(t)
\end{aligned}
$$

- As before, market clearing is ensured by clearing in the private lending market and a constraint on aggregate savings:

$$
\begin{aligned}
S_t(r(t)) &= \sum_{h=1}^{N(t)} \left(\omega_t^h(t) - c_t^h(t)\right) = \sum_{h=1}^{N(t)} \left(l^h(t) + p(t)a^h(t) + k^h(t+1)\right) \\
&= \sum_{h=1}^{N(t)} \left(p(t)a^h(t) + k^h(t+1)\right) = p(t)A + K(t+1).
\end{aligned}
$$

- As before, perfect foresight implies

$$p^e(t+1) = p(t+1)$$

**Definition 2** *A perfect foresight competitive equilibrium for any economy with storage and land is a non-negative sequence* $\{p(t), r(t), K(t+1)\}_{t=1}^{\infty}$*, that for all* $t \geq 1$ *satisfies*

- $p(t) = \frac{d(t+1)+p^e(t+1)}{r(t)}$
- $S_t(r(t)) = p(t)A + K(t+1)$
- $r(t) \geq \lambda$
- $K(t+1)[r(t) - \lambda] = 0,$

*given sequences* $\left\{D(t), \left\{\omega_t^h\right\}_{h=1}^{N(t)}\right\}_{t=1}^{\infty}$ *and initial conditions for the old* $\left\{a^h(1), k^h(1), \omega\right.$

## 7.1 Finding a competitive equilibrium

- 1. Guess and verify
    - (a) Guess a price $p(t)$ and check if the implied prices at $t+1$ obey the equilibrium conditions.
    - (b) Restrict attention to stationary equilibria where $p(t) = p(t+1)$ is constant over time and guess at this equilibrium.
  2. Perform a graphical analysis using some pricing function and the storage technology.

### 7.1.1 Solving for a stationary equilibrium

- A stationary equilibrium satisfies

$$\begin{aligned} p &= \frac{d+p}{r} && (1) \\ S(r) &= pA + K && (2) \\ r &\geq \lambda \\ K\left[1 - \frac{\lambda}{r}\right] &= 0 \end{aligned}$$

- Three possibilities:

  1. Assume that $K = 0$. Solve for $r$ and $p$ using equations (1) and (2) (as in Lecture 5). Check that $r \geq \lambda$.
  2. Assume that $r = \lambda$: Solve for $p$ using equation (1) and check if (2). implies $K \geq 0$, i.e. that $S(r) - pA \geq 0$.
  3. If neither 1 nor 2, then there does not exist a stationary competitive equilibrium.

#### 7.1.2 A graphical analysis

- With $K(t+1) \geq 0$ we have

$$S_t\left(\frac{d(t+1)+p(t+1)}{p(t)}\right) \geq p(t)A,$$

or a price function that satisfies

$$p(t) \leq f_t\left(p^e(t+1), d(t+1), A\right). \tag{3}$$

Equilibrium conditions (i) and (iii) implies

$$\lambda \leq r(t) = \frac{d(t+1)+p^e(t+1)}{p(t)}$$

or, equivalently,

$$p(t) \leq \frac{d(t+1)+p(t+1)}{\lambda} \tag{4}$$

Combined, equations (3) and (4) implies that the price of land satisfies

$$p(t) = \min\left\{\frac{d(t+1)+p(t+1)}{\lambda},\ f_t\left(p^e(t+1), d(t+1), A\right)\right\}$$

- Assume stationary environment (but not necessarily stationary equilibrium). Analyze two cases:

**Case 1** $D(t) = D > 0$:

(a) $\lambda$ sufficiently low $\Rightarrow$ a unique stationary equilibrium with $r \geq \lambda$, $K = 0$ and $p = f(p, d, A)$.

(b) $\lambda$ sufficiently high $\Rightarrow$ a unique stationary equilibrium with $r = \lambda$, $K \geq 0$ and $p = \frac{d+p}{\lambda}$.

**Case 2** $D(t) = D = 0$:

(a) $\lambda > 1 \;\Rightarrow$ a unique stationary equilibrium with $r = \lambda$, $K \geq 0$ and $p = \frac{d+p}{\lambda}$.

(b) $\lambda < 1 \;\Rightarrow$

(i) A stationary equilibrium with $p = 0$ and either $r = \lambda$ and $K \geq 0$, or $r \geq \lambda$ and $K = 0$.

(ii) A stationary equilibrium with $r > \lambda$, $K = 0$ and $p = f\left(p, 0, A\right) \equiv \bar{p}$.

(iii) A continuum of equilibria with $p(1) \in (0, \bar{p})$ and $\lim_{t\to\infty} p(t) = 0$. These equilibria may start without storage ($K(2) = 0$) but start using storage later, when the return on land is sufficiently low.

1. (a) i. Storage takes place when $p(t) \in \left(0, \underline{p}\right)$, where $\underline{p} < \bar{p}$ and $\underline{p}$ satisfies

$$\underline{p} = \lambda f\left(\underline{p}, 0, A\right).$$

## 7.2 Application 1: understanding saving

- Motives for saving:

  1. Consumption-smoothing . One example of this is life-cycle saving.
  2. Intertemporal substitution (if an agent's subjective discount rate differs from the rate of return on saving)
  3. Precautionary saving (accumulate buffer-stock as a precaution for bad future events)
  4. Bequest motive.
  5. Saving for downpayment onlumpy investment (e.g. minimal downpayment for buying a house).
  6. Love of wealth.

- Consider a version of our standard stationary economy: Endowments are $\omega = [\omega_1, \omega_2]$, storage gives a return $\lambda$, there are no government taxes, and preferences are given by

$$u_t^h = \log c_t^h(t) + \beta \log c_t^h(t+1)$$

  Budget constraints:

$$\begin{aligned} c_t(t) &= \omega_1 - k(t) - l(t) \\ c_t(t+1) &= \omega_2 + \lambda k(t) + r(t) l(t) \end{aligned}$$

  Suppose that $r(t) = \lambda$ (must verify afterwards that $k(t) > 0$). Then, individual savings are given by

$$s_t^h = \frac{\beta \omega_1}{1+\beta} - \frac{\omega_2}{(1+\beta)\lambda}$$

- Concave and time-additive utility $\Rightarrow$ agents like to smooth consumption. Example of consumption-smoothing: Suppose $\omega = [\omega + \varepsilon, \omega - \varepsilon]$ and $r = \lambda = 1/\beta$. Then

$$\begin{aligned} s_t^h &= \frac{\beta(\omega+\varepsilon)}{1+\beta} - \frac{\omega - \varepsilon}{(1+\beta)\frac{1}{\beta}} \\ &= \frac{2\beta}{1+\beta}\varepsilon. \end{aligned}$$

So savings increases as $\varepsilon$ increases (and the need for consumption-smoothing increases).

- Example of intertemporal substitution: Suppose $\omega_1 = \omega_2 = \omega$. The, savings are given by

$$\begin{aligned} s_t^h &= \frac{\beta\omega}{1+\beta} - \frac{\omega}{(1+\beta)\,r} \\ &= \frac{\omega}{(1+\beta)\,r}\left(\beta - \frac{1}{r}\right) \end{aligned}$$

So larger $r$ yields larger saving.

- Precautionary savings. Setting:
  - Suppose endowments when old are subject to an *individual-specific* shock (different from the *aggregate* shock to land crop that we looked at before). For example, suppose $\omega_1 = \omega_2 = \omega$ and

$$\omega_2^h = \omega + \tilde{\varepsilon}$$

    where $\tilde{\varepsilon} \in [-\varepsilon, \varepsilon]$ and half the population gets the low outcome and half the population gets the high outcome. However, agents do not know, when young, who will get the good shock.
  - If there were complete markets, agents would insure each other and all would consume $\omega$ when old.
  - Assume that $\tilde{\varepsilon}$ is not verifyable, so that insurance markets do not exist (markets are "incomplete"). Can the agent "self-insure"?
  - Solving the problem under incomplete markets:

$$\begin{aligned} & \max_k \left\{\log c_t^h\,(t) + \beta E \log c_t^h\,(t+1)\right\} \\ = & \max_k \left\{\log(\omega - k) + \beta E \log(\omega + \tilde{\varepsilon} + \lambda k)\right\} \end{aligned}$$

    The FOC yields

$$\begin{aligned} 0 &= \frac{\partial}{\partial k}\left\{\log(\omega_1 - k) + \beta E \log(\omega_2 + \tilde{\varepsilon} + \lambda k)\right\} \\ &= -\frac{1}{\omega - k} + \beta\lambda E \frac{1}{\omega + \tilde{\varepsilon} + \lambda k} \\ &= -\frac{1}{\omega - k} + \frac{\beta\lambda/2}{\omega + \varepsilon + \lambda k} + \frac{\beta\lambda/2}{\omega - \varepsilon + \lambda k} \end{aligned}$$

Assume $\beta = 1/\lambda$. Then the optimal solution is

$$0 = -\frac{2}{\omega - k} + \frac{1}{\omega + \varepsilon + \lambda k} + \frac{1}{\omega - \varepsilon + \lambda k}$$

$$k = \frac{1}{2\lambda(1+\lambda)}\left(\sqrt{\omega^2(\lambda+1)^2 + (\lambda+1)4\lambda\varepsilon^2} - \omega(1+\lambda)\right) \geq 0$$

### 7.3 Application 2: optimal taxation

- Purpose of application: explore which composition of taxes will minimize the tax distortions of a society.
- Environment: economy with private lending and a storage technology (for simplicity there is no land). A government needs to finance government consumption $G(t)$ each period through taxation of government borrowing.
- Consider effects on savings, labor input and welfare of using each of three tax instruments: $\tau_k$, $\tau_L$, and $\tau_c$ (for simplicity, we assume the tax rates to be constant over time).
- Tax instruments:
  1. Capital taxation applies to lending and storage, and must therefore be collected from the old generation only. Total capital tax revenues in period $t$ are given by

     $$\tau_k \sum_{h=1}^{N(t-1)} \left[r(t)l^h(t-1) + \lambda k^h(t)\right] = \tau_k \lambda K(t),$$

     provided the private lending market clears each period ($\sum_{h=1}^{N(t-1)} l^h(t-1) = 0$).
  2. Labor income taxation $\tau_L$. We will consider two types of economies here:
     (a) Economies with labor income as exogenous endowments (which we have considered so far). In this case, total labor tax revenues are given by

     $$\tau_L \left(\sum_{h=1}^{N(t)} \omega_t^h(t) + \sum_{h=1}^{N(t-1)} \omega_{t-1}^h(t)\right).$$

(b) Economies with labor supply (i.e. "hours worked") as an endogenous choice.

3. Consumption taxation $\tau_c$, i.e. a "value added" tax on consumption. Total consumption tax revenues are given by

$$\tau_c \left( \sum_{h=1}^{N(t)} c_t^h(t) + \sum_{h=1}^{N(t-1)} c_{t-1}^h(t) \right) = \tau_c \left( C_t(t) + C_{t-1}(t) \right).$$

#### 7.3.1 Economies with exogenous labor supply

Consumption tax versus labor income tax

- Consider first an economy with taxes on labor income and consumption, but with $\tau_k = 0$. The following proposition summarizes the results:

**Proposition 1** *In an economy with storage and exogenous labor supply, taxes on labor income and consumption are equivalent, in the sense that they give rise to the same competitive equilibrium prices and allocations, provided the present value of taxes is held constant.*

Proof:

- Budget constraints of an individual $h$ is given by

$$\begin{aligned} (1+\tau_c)\, c_t^h(t) &\leq (1-\tau_L)\, \omega_t^h(t) - l^h(t) - k^h(t+1) \\ (1+\tau_c)\, c_t^h(t+1) &\leq (1-\tau_L)\, \omega_t^h(t+1) + r(t) l^h(t) + \lambda k^h(t+1). \end{aligned}$$

Combine these two constraints into one:

$$\begin{aligned} (1+\tau_c)\left[c_t^h(t) + \frac{c_t^h(t+1)}{r(t)}\right] &\leq (1-\tau_L)\left[\omega_t^h(t) + \frac{\omega_t^h(t+1)}{r(t)}\right] \\ &\quad -k^h(t+1)\left[1 - \frac{\lambda}{r(t)}\right] \\ &\Leftrightarrow \\ c_t^h(t) + \frac{c_t^h(t+1)}{r(t)} &\leq \frac{1-\tau_L}{1+\tau_c}\left[\omega_t^h(t) + \frac{\omega_t^h(t+1)}{r(t)}\right] \\ &\quad -\frac{k^h(t+1)}{1+\tau_c}\left[1 - \frac{\lambda}{r(t)}\right] \end{aligned}$$

Recall equilibrium condition (iv) for a competitive equilibrium with storage is

$$k^h(t+1)\left[1-\frac{\lambda}{r(t)}\right]=0.$$

Thus, in equilibrium, the budget constraint is

$$c_t^h(t)+\frac{c_t^h(t+1)}{r(t)}\leq\frac{1-\tau_L}{1+\tau_c}\left[\omega_t^h(t)+\frac{\omega_t^h(t+1)}{r(t)}\right].$$

Consequently, as long as $r(t)$ and the fraction

$$\psi=\frac{1-\tau_L}{1+\tau_c}$$

remain constant, the agents face the same constraint and must, consequently, make the same consumption choices.

- To complete the proof of Proposition 1, we need to prove this fact, which is the purpose of the following lemma:

**Lemma 1** *For different combinations of taxes on consumption and labor, the fraction* $\psi=\frac{1-\tau_L}{1+\tau_c}$ *is constant, provided each tax-combination raise the same present value of taxes from each generation.*

**Proof**: Suppose the government wants to extract tax revenues from generation $t$, amounting to $G_t$ in present value Consider now the following two financing alternatives:

1. Labor income taxes only: $\tau_c=0$ and $\tau_L$ has to satisfy the following equation:

$$\begin{aligned}G_t &= \tau_L^*\left[\omega_t^h(t)+\frac{\omega_t^h(t+1)}{r(t)}\right]\\ &\Leftrightarrow \\ \tau_L^* &= \frac{G_t}{\omega_t^h(t)+\frac{\omega_t^h(t+1)}{r(t)}}\end{aligned}$$

   This implies

$$\psi=\frac{1-\tau_L}{1+\tau_c}=1-\tau_L^*=1-\frac{G_t}{\omega_t^h(t)+\frac{\omega_t^h(t+1)}{r(t)}}.$$

2. Consumption taxes only: $\tau_L = 0$ and $\tau_c$ has to satisfy the following equation:

$$\begin{aligned}
G_t &= \tau_c^* \left[ c_t^h(t) + \frac{c_t^h(t+1)}{r(t)} \right] \\
&= \frac{\tau_c^*}{1+\tau_c^*} \left[ \omega_t^h(t) + \frac{\omega_t^h(t+1)}{r(t)} \right] \\
&\Leftrightarrow \\
\tau_c^* &= \frac{G_t \left( \omega_t^h(t) + \frac{\omega_t^h(t+1)}{r(t)} \right)}{\omega_t^h(t) + \frac{\omega_t^h(t+1)}{r(t)} - G_t}
\end{aligned}$$

This implies

$$\psi = \frac{1-\tau_L}{1+\tau_c} = \frac{1}{1+\tau_c^*} = 1 - \frac{G_t}{\omega_t^h(t) + \frac{\omega_t^h(t+1)}{r(t)}}.$$

QED

- Digression: Pension system. In this economy, a pay-as-you-go pension system crowds out storage (as long as storage occurs): one unit more for financing pension benefits means one unit less in savings.

Capital income tax versus labor income tax

**Definition 3** *A perfect foresight competitive equilibrium for an economy with taxes, storage and exogenous labor supply is a non-negative sequence* $\{r(t), K(t+1)\}_{t=1}^{\infty}$*, that for all* $t \geq 1$ *satisfies*

- $S_t\left((1-\tau_k)\, r(t)\right) = K(t+1)$
- $r(t) \geq \lambda$
- $K(t+1)\left[1 - \frac{\lambda}{r(t)}\right] = 0,$

*given sequences* $\left\{\left\{\omega_t^h\right\}_{h=1}^{N(t)}\right\}_{t=1}^{\infty}$ *and initial conditions for the old* $\left\{k^h(1), \omega_0^h(1)\right\}_{h=1}^{N(0)}$*, and where the savings function,* $S_t(.)$*, incorporates the taxes on consumption and labor income.*

- Consider now an economy with taxes on labor income and capital income (since taxes on consumption and endowments are equivalent – see Proposition 1 – we set the consumption tax to zero, $\tau_c = 0$).

- Budget constraints of an individual $h$ is given by

$$\begin{aligned} c_t^h(t) &\leq (1-\tau_L)\,\omega_t^h(t) - l^h(t) - k^h(t+1) \\ c_t^h(t+1) &\leq (1-\tau_L)\,\omega_t^h(t+1) + (1-\tau_k)\,r(t)l^h(t) + (1-\tau_k)\,\lambda k^h(t+1). \end{aligned}$$

  Combine these two constraints into one:

$$\begin{aligned} c_t^h(t) + \frac{c_t^h(t+1)}{(1-\tau_k)\,r(t)} &\leq (1-\tau_L)\left[\omega_t^h(t) + \frac{\omega_t^h(t+1)}{(1-\tau_k)\,r(t)}\right] \\ &\quad -k^h(t+1)\left[1 - \frac{\lambda}{r(t)}\right] \end{aligned}$$

- Consider only the case when positive storage occurs even in the case when all tax revenues come from capital income (i.e. $\tau_k > 0 \Rightarrow K(t) > 0$). Motivation: if zero storage occurs in equilibrium, no tax revenues on capital income can be collected, so $\tau_k$ is irrelevant. Hence,

$$r(t) = \lambda \text{ for all } t \geq 1.$$

- Assume the government needs to raise taxes on each generation equal to $G$ in present value (to use for government consumption, say). Note that with present value we mean tax revenues discounted with the rate of return before tax, i.e. $\lambda$. Assume that $N(t) = N$ for all $t$ and that endowments for all individuals $h$ of all generations $t$ are given by

$$\omega_t^h = [\omega_1, \omega_2].$$

  Restrict attention to stationary equilibria. Compute now the tax rates required to finance this consumption

  1. Suppose all tax revenues are collected on labor income, i.e. $\tau_k = 0$. The tax rate on labor income is then given by the solution to

$$\begin{aligned} G &= \tau_L\left[\omega_1 + \frac{\omega_2}{\lambda}\right]N \\ &\Leftrightarrow \\ \tau_L &= \frac{G}{N\left[\omega_1 + \frac{\omega_2}{\lambda}\right]}. \end{aligned}$$

2. Suppose all tax revenues are collected on capital income, i.e. $\tau_L = 0$. The tax rate on capital income is then given by the solution to

$$G = \tau_k S\left(\left(1-\tau_k\right)\lambda\right).$$

- Example:

$$u_t^h\left(c_t^h\left(t\right), c_t^h\left(t+1\right)\right) = c_t^h\left(t\right)c_t^h\left(t+1\right)$$

In this case, the savings function becomes

$$s_t^h(t) = \frac{\left(1-\tau_L\right)\omega_1}{2} - \frac{\left(1-\tau_L\right)\omega_2}{2\left(1-\tau_k\right)\lambda}$$

and aggregate savings is then given by

$$S\left(\left(1-\tau_k\right)\lambda\right) = \frac{\left(1-\tau_L\right)}{2}\omega_1 N - \frac{\left(1-\tau_L\right)}{2\left(1-\tau_k\right)\lambda}\omega_2 N.$$

A necessary condition for this to be an equilibrium is that positive storage occurs, i.e. that

$$\begin{aligned} 0 &\leq S\left(\left(1-\tau_k\right)\lambda\right) \\ &\Leftrightarrow \\ \omega_1 &\geq \frac{\omega_2}{\left(1-\tau_k\right)\lambda}. \end{aligned}$$

Let us assume that this condition is satisfied.

- Welfare analysis. Assume $[\omega_1, \omega_2] = [2, 1]$. Consider the following numerical results:

| | Savings | $u$ | $\tau_L$ | $\tau_k$ | $\lambda$ | $G$ |
|---|---|---|---|---|---|---|
| $\tau_L$ only | 0.654 | 2.568 | 0.0188 | 0 | 1.5 | .05 |
| $\tau_k$ only | 0.638 | 2.563 | 0 | 0.078 | 1.5 | .05 |
| $\tau_L$ only | 0.492 | 2.176 | 0.0167 | 0 | 1 | .05 |
| $\tau_k$ only | 0.435 | 2.168 | 0 | 0.115 | 1 | .05 |
| $\tau_L$ only | 0.405 | 2.077 | 0.0157 | 0 | 0.85 | .05 |
| $\tau_k$ only | 0.288 | 2.058 | 0 | 0.173 | 0.85 | .05 |

Results might not be surprising, given that a tax on capital income crowds out savings, while a tax on labor income is, essentially, a lump sum tax.

### 7.3.2 Economies with endogenous labor supply

- Assume that individuals obtain utility from leisure and offer labor services endogenously at a constant exogenous wage rate $W$. Let labor supply of an individual $h$ of generations $t$ be given by

$$L_t^h = \left[L_t^h(t), L_t^h(t+1)\right],$$

where we assume that $0 \leq L_t^h(s) \leq 1$. The total labor tax revenues in period $t$ are given by

$$\tau_L \, W \left( \sum_{h=1}^{N(t)} L_t^h\,(t) + \sum_{h=1}^{N(t-1)} L_{t-1}^h\,(t) \right) = \tau_L \, W L(t).$$

The utility of an agent $h$ of generation $t$ is now

$$u_t^h \left( c_t^h(t), c_t^h(t+1), L_t^h(t), L_t^h(t+1) \right),$$

where the same assumptions on $u_t^h$ are maintained: differentiability, monotonicity, and convexity.

- Budget constraints of an individual $h$ is given by

$$\begin{aligned} c_t^h(t) &\leq W\,(1-\tau_L)\, L_t^h(t) - l^h(t) - k^h(t+1) \\ c_t^h(t+1) &\leq W\,(1-\tau_L)\, L_t^h(t+1) + (1-\tau_k)\, r(t) l^h(t) \\ &\quad + (1-\tau_k)\, \lambda k^h(t+1). \end{aligned}$$

Combine these two constraints into one:

$$\begin{aligned} c_t^h\,(t) + \frac{c_t^h\,(t+1)}{(1-\tau_k)\, r(t)} &\leq W\,(1-\tau_L) \left[ L_t^h\,(t) + \frac{L_t^h\,(t+1)}{(1-\tau_k)\, r(t)} \right] \\ &\quad - k^h(t+1) \left[ 1 - \frac{\lambda}{r(t)} \right] \end{aligned}$$

- Definition of competitive equilibrium does not need any alterations, except that feasibility now requires that

$$\begin{aligned} \sum_{t=1}^{N(t)} c_t^h\,(t) + \sum_{h=1}^{N(t-1)} c_{t-1}^h\,(t) + K(t+1) &= \\ &= W \left( \sum_{h=1}^{N(t)} L_t^h\,(t) + \sum_{h=1}^{N(t-1)} L_{t-1}^h\,(t) \right) + \lambda K(t) \end{aligned}$$

- Again, consider only the case when positive storage occurs even in the case when all tax revenues come from capital income, so

$$r(t) = \lambda \text{ for all } t \geq 1.$$

- Again, restrict attention to stationary equilibria. Assume the government needs to raise taxes on each generation equal to $G$ in present value, that $N(t) = N$ for all $t$ and that endowments for all individuals $h$ of all generations $t$ are given by

$$\omega_t^h = [\omega_1, \omega_2].$$

Compute now the tax rates required to finance this consumption

1. Suppose all tax revenues are collected on labor income, i.e. $\tau_k = 0$. The tax rate on labor income is then given by the solution to

$$\begin{aligned} G &= w\tau_L \left[ L_t^h(t) + \frac{L_t^h(t+1)}{\lambda} \right] N \\ &\Leftrightarrow \\ \tau_L &= \frac{G}{wN \left[ L_t^h(t) + \frac{L_t^h(t+1)}{\lambda} \right]}. \end{aligned}$$

2. Suppose all tax revenues are collected on capital income, i.e. $\tau_L = 0$. The tax rate on capital income is then given by the solution to

$$G = \tau_k S\left((1 - \tau_k)\lambda\right).$$

- Welfare analysis:

1. Optimal composition of taxes depend on the sensitivity of savings and labor supply to $\tau_k$ and $\tau_L$.
2. Economists often argue that
    (a) Labor supply is not very sensitive to taxation.
    (b) Savings is quite sensitive to taxation.
        – portfolio composition is especially sensitive (since several assets are tax exempt, international "capital flight" is feasible, and only realized capital gains are subject to taxation).

  - capital taxation in connection with inflation can mean extremely high tax on real capital income.

(c) In a horse-race between capital taxes and labor income taxes, labor income taxes usually comes out as less distortionary, but less redistributive.

3. If agents live many periods, an attractive option emerges for the government: tax capital heavily in the first period and create a fund from which future government expenditures can be financed and future taxes can remain low. Bottom line: tax initial capital (which cannot be distorted) and keep $\tau_k = \tau_L = 0$ in the future.