ECON4310 Fall 2010 Seminar 1

Week 41

1 Ramsey's growth model

Extracted and adapted from the 2009 exam.

In this question we shall be looking at a discrete-time version of Ramsey's growth model. The social planner maximizes:

$$U = \sum_{t=0}^{\infty} \beta^t u(\bar{c}_t) \tag{1}$$

where $\bar{c}_t$ is consumption per capita and $0 < \beta < 1$ is a subjective discount factor. The period utility function is

$$u(c) = c^{1-\theta}/(1-\theta) \tag{2}$$

where $\theta > 1$.

The aggregate production function is

$$Y_t = F(K_t, A_t L_t) \tag{3}$$

where Y_t is output, K_t is the capital stock, L_t the labor force and A_t a labor-augmenting productivity factor. F is assumed to be homogeneous of degree one. (If you need additional assumptions about F to answer the questions below, you should state them as part of your answer). L_t is exogenous and grows with a rate n per period ($L_{t+1} = (1+n)L_t$). Similarly, A_t is exogenous and grows with rate g. The capital stock grows according to

$$K_{t+1} = K_t - \delta K_t + Y_t - C_t \tag{4}$$

where δ is the depreciation rate, and $C_t = L_t \bar{c}_t$ is aggregate consumption. For every t it is required that $K_t \geq 0$ and $C_t \geq 0$.

1. Define $k_t = K_t/A_t L_T$ and $c_t = \bar{c}_t/A_t$. Explain in words what these variables measure. Show that the social planner's problem can be expressed as

$$\max U = \sum_{t=0}^{\infty} \beta^t \frac{(A_t c_t)^{1-\theta}}{1-\theta} \tag{5}$$

given that

$$(1+n)(1+g)k_{t+1} = (1-\delta)k_t + f(k_t) - c_t \qquad t = 0, 1, 2, \ldots \tag{6}$$

and given k_0.

2. Use the Bellman equation to show that the first-order condition for an internal optimum can be written

$$c_t^{-\theta} = \beta c_{t+1}^{-\theta} \frac{1 + f'(k_{t+1}) - \delta}{(1+g)^{\theta}(1+n)} \tag{7}$$

   Interpret this condition. What does it say about the growth rate of consumption?

3. Explain what is meant by a balanced growth path (a steady state). Does a balanced growth path exist in the present model? What determines the values of k and c along a balanced growth path? What role does intertemporal substitution play for these steady-state values?

4. Compare the values of k and c along the balanced growth paths for two economies that have different population growth n. What does reduced population growth mean for the level of the real wage?

5. What is meant by stability of the steady state? Is the steady state in the present model stable? How are the initial levels of k and c determined? Illustrate with a graph.

6. Suppose the economy is initially on a balanced growth path. Discuss with the help of a graph the effect of a decrease in n on the whole time path for k_t.

2 The Golden Rule of Accumulation and Dynamic Inefficiency

1. Explain what is meant by the*golden rule of accumulation.* Why does the steady state in the Ramsey model above deviate from the golden rule?

2. What is meant by *dynamic inefficiency* and how can it arise? Can it occur in the Ramsey-model discussed above?